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Weekly price prediction of garlic and ginger using complex exponential smoothing

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Abstract

Garlic and ginger crops are commercially significant spices with high export value. Over the last few years, there are more fluctuation in price, which has put growers at more risk and indirectly reduced production. Furthermore, the advanced price forecasts for both crops became a valuable input for policymakers in order to stabilize prices and a key factor of effective market intelligence service. Therefore, the study aims to identify the appropriate forecasting model for the weekly prices of garlic and ginger. For this purpose, the study used the 331 weekly observations of wholesale prices for garlic and ginger crops which were collected from the 22nd week of 2016 to the 35th week of 2022 for the Varanasi market. The statistical tests identified the presence of non-normal, nonstationary and nonlinearity features in both price series. The perusal of the literature suggested that the ARIMA and Exponential smoothing models were implied as benchmark models. Based on the presence of characteristics, the complex exponential smoothing model and Time delay neural network were selected as candidate models. From this comparative analysis, the results of accuracy measures suggested that complex exponential smoothing showed better performance compared to benchmark models and Time delay neural networks. This study recommended that complex exponential smoothing is a suitable model for forecasting the future values of garlic and ginger prices.

Keywords: Spices, price forecasting, nonlinearity, high frequency, complex exponential smoothing

1. Introduction

India is the largest producer, consumer, and export of spices at the global level. As a result, India is known as the "Spice Bowl of the World". This is not just because of their production, but also due to their excellent flavour, rich aroma, and rich texture. Among key spices, Ginger and garlic had the greater share of 30% and 19% of India's total spices production respectively. Furthermore, India is the second largest exporter of ginger after China. Both crops are well-known spices and are being cultivated for fresh vegetables as well as dried spices (Mathew *et al.*, 2018) [8]. Garlic and ginger prices showed higher price fluctuation in the past few years, exposing the growers to more risk. Moreover, these fluctuations are subject to climate risks impacting production and market arrivals (Jhade & Singh, 2021) [4]. The awareness of various market channels, crop weather advisory and market intelligence services will help farmers to reduce risk. Among these, market intelligence services become efficient if the price information is accurate and timely available. Additionally, it helps policymakers to make policies regarding stabilization prices.

The price of agricultural commodities exhibits the characteristics such as high volatility, non-stationarity, nonlinearity, seasonality, and calendar effects (Wang *et al.*, 2022) [13]. It poses a great challenge in the price forecasting task of both crops. Moreover, the prices depend on many factors which could not be quantified. Therefore, historical data are used for modeling the characteristics and forecasting future value. The most popular univariate method is ARIMA which is utilized to predict the future values of garlic in previous studies (Wang, 2018) [12]. To handle the nonlinear characteristics of prices, the machine learning method especially ANN got great attention in the price forecasting domain (Jha & Sinha, 2014; Lama *et al.*, 2016; Singh, 2021; Rajpoot *et al.*, 2022) [3, 6, 10, 9]. Li *et al.*, (2015) [7] suggested the prediction of weekly data is important compared to monthly because the price fluctuation occurs over a week.

Svetunkov *et al.*, (2022) ^[11] mentioned that Complex exponential smoothing performed better in high-frequency data such as weekly, daily and hourly data. The involvement of complex numbers in this model captures the complex mixture of nonlinearity and seasonality. Therefore, in this study, ARIMA and Exponential Smoothing were used as benchmark models. Complex exponential smoothing and Time delay neural network were selected as the candidate models. The study aims to identify the appropriate model to forecast the future prices of garlic and ginger.

2. Materials and Methods

Weekly price data of ginger and garlic crops from the 22nd week of 2016 to the 35th week of 2022 were collected for the Varanasi market from the website <http://www.upkrishivipran.in/>. Each series consist of 313 observations. The data was split into a training set and a test set. The data from the 22nd week of 2016 to the 9th week of 2022 were used as a training set to build the model and the 10th week of 2022 to the 35th week of 2022 was used as a test set to check the performance of the models.

2.1 ARIMA

ARIMA, which was introduced by Box Jenkins, is the most well-known method of analyzing univariate time series. In the time series process $\{y_t\}$, present values are related to its past values. Generally, Time series is generated by a linear aggregation of random shock. In practice, the successful model should have parameters parsimoniously. It is possible to accomplish this with a small number of autoregressive and moving average terms. ARIMA can be expressed as

$$(B)(1 - B)^d y_t = \theta(B)\varepsilon_t \tag{1}$$

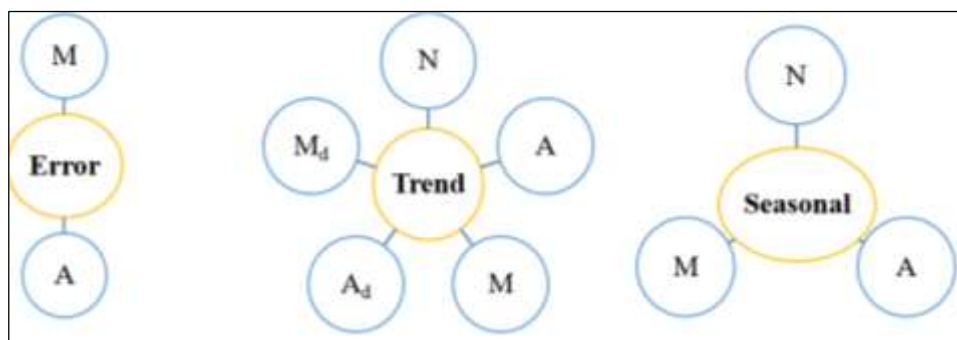
Where, $(1-B)^d$ is a number of differences, $\varepsilon_t = N(0, \sigma^2)$ or white noise. In the consideration of seasonality, Seasonal Autoregressive integrated moving average can be implied. It can be written as

$$\phi(B)\Phi(B^s)(1 - B^s)^D(1 - B)^d y_t = \theta(B)\Theta(B^s)\varepsilon_t \tag{2}$$

Where, s is seasonal frequency, $(1 - B^s)^D$ is number of seasonal differences, $(1 - B)^d$ is number of differences, $\varepsilon_t = WN(0, \sigma^2)$ indicating white noise. The box Jenkins methodology was used to identify the best ARIMA model for future forecasts. The automatic algorithm for identifying the appropriate ARIMA model was achieved with the help of the forecast package in R.

2.2 Exponential Smoothing

Brown (1959) ^[14], Holt (1957) ^[15] and Winters (1960) ^[16] are the pioneers to develop the exponential smoothing model. Forecast values are determined by weighted averages of prior data, with the weights declining exponentially from present to past observations in the exponential smoothing method. The time series data are split into trend, seasonal, and error components by exponential smoothing. There are five types of Trends, three types of seasonality, and two types of error which are presented in Figure 1.



N-None, A-Additive, M-Multiplicative, A_d-Additive damped, M_d-Multiplicative damped

Fig 1: Types of Error, Trend, and Seasonality involved in the Exponential smoothing model

The exponential model is represented by the three-letter abbreviation ETS (Error, Trend, Seasonal). Thirty exponential smoothing models are produced by various combinations of the three components (Hyndman *et al.*, 2002) ^[2]. A recursive form is provided as the general form for 30 exponential smoothing models.

$$y_t = w(x_{t-1}) + r(x_{t-1})\varepsilon_t \tag{3}$$

$$x_t = f(x_{t-1}) + g(x_{t-1})\varepsilon_t \tag{4}$$

Where $x_t = (l_t, b_t, s_t, s_{t-1}, \dots, s_{t-m+1})$ is the state vector, $\{\varepsilon_t\}$ is a Gaussian white noise process with variance σ^2 and $\mu_t = w(x_{t-1})$. The model with additive errors has $r(x_{t-1}) = 1$, therefore $y_t = \mu_t + \varepsilon_t$. The model with multiplicative errors has $r(x_{t-1}) = \mu_t$, so $y_t = \mu_t(1 + \varepsilon_t)$. Hyndman & Khandakar, (2008) ^[1] have created the smooth package in R for the automatic selection of the exponential smoothing model.

2.3 Complex exponential smoothing

Svetunkov *et al.*, (2016) ^[17] proposed a new approach to exponential smoothing which is based on the new term “information potential”, an unobserved time series component. In this method, the observed values of a time series are converted to complex numbers by combining the information potential. Instead of real variables in the exponential model, the complex variable was applied in the following equation by using information potential.

$$\hat{y}_{t+1} + i\hat{p}_{t+1} = (\alpha_0 + i\alpha_1)(y_t + ip_t) + (1 - \alpha_0 + i - i\alpha_1)(\hat{y}_t + i\hat{p}_t) \quad (5)$$

Where \hat{y}_t is the estimated value of the time series, \hat{p}_t is the estimated value of information potential and $\alpha_0 + i\alpha_1$ is the complex smoothing parameter. The complex-valued function can be represented by a system of two real-valued functions as following equations.

$$\hat{y}_{t+1} = (\alpha_0 y_t + (1 - \alpha_0)\hat{y}_t) - (\alpha_1 p_t + (1 - \alpha_1)\hat{p}_t) \quad (6)$$

$$\hat{p}_{t+1} = (\alpha_1 y_t + (1 - \alpha_1)\hat{y}_t) + (\alpha_0 p_t + (1 - \alpha_0)\hat{p}_t) \quad (7)$$

The final forecast of CES is the combination of the above two parts. It revealed that CES is the nonlinear method that is connected with first and second equations and changes simultaneously based on the complex smoothing parameter value. The recursive form of complex exponential smoothing was given below

$$y_t = w'x_{t-1} + \varepsilon_t \quad (8)$$

$$x_t = Fx_{t-1} + qp_t + g\varepsilon_t' \quad (9)$$

Where $x_t = \begin{pmatrix} l_t \\ c_t \end{pmatrix}$ is the state vector, $F = \begin{pmatrix} 1 & -(1 - \alpha_1) \\ 1 & 1 - \alpha_0 \end{pmatrix}$ is the transition matrix, $g = \begin{pmatrix} \alpha_0 \\ \alpha_1 \end{pmatrix}$ is the persistence vector, $q = \begin{pmatrix} -\alpha_1 \\ \alpha_0 \end{pmatrix}$ is the information potential persistence vector and $w = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ is the measurement vector. In this model, model selection is done among non-seasonal CES and Seasonal CES with the help of minimum AIC and BIC values. The complex exponential smoothing model of given data is performed using the smooth package of R.

2.4 Time delay Neural Network (TDNN)

ANNs are a type of model that is replicated of the structure of neurons in the brain. Conventional models rely on assumptions like linearity, normality, and stationarity. In the case of ANN, it does not rely on a prior assumption of a data-generating process and is capable of understanding numerous nonlinear structures and complicated relationships contained in the data. In this architecture, the input layer, hidden layer, and output layer were the most essential terms. In univariate analysis, the number of input layers was determined using the lagged values of the same variable. This crucial number could be determined by examining the structure of autocorrelation. A single hidden layer has been frequently used in forecasting time series. Choosing the output layer is a simple task because we often just require one output. The time delay neural network's general statement is stated as

$$y_t = \alpha_0 + \sum_{j=1}^q \alpha_j g(\beta_{0j} + \sum_{i=1}^p \beta_{ij} y_{t-p}) + \varepsilon_t \quad (4)$$

Where, α_j ($j = 0, 1, 2, \dots, q$) and β_{ij} ($i = 0, 1, 2, \dots, p, j = 0, 1, 2, \dots, q$) are the model parameters or connection weights, p is the number of input nodes and q is the number of hidden nodes. Each node in the hidden layer receives the weighted sum of all inputs and bias terms, the value of which is always one. Each hidden node transforms this weighted sum of input variables using the activation function, which is defined as the nonlinear relationship between a network's inputs and outputs. Sigmoid functions are normally employed as activation functions for hidden layer transfer functions.

$$g(x) = \frac{1}{1 + \exp(-y)} \quad (5)$$

Similar to the input node, the output node also receives the weighted sum of the output from every hidden node and converts the weighted total using its activation function to produce an output. It can be described as

$$y_t = f(y_{t-1}, y_{t-2}, \dots, y_{t-p}, w) + \varepsilon_t \quad (6)$$

Where w is a vector of network parameters and f is a function of the network structure. This structure behaves similarly to a nonlinear autoregressive model. Neural networks have adaptive nature like learning by doing. For this purpose, the data in this study was divided into two sets: a training set and a test set. The training set is used to estimate parameters and build the network, whereas the test set is used to determine sample performance. The TDNN was executed by forecast package in R studio.

2.5 Accuracy measures

To measure the effectiveness of models for vegetable price forecasting, widely applied accuracy measures (Koutsandreas *et al.*, 2022) ^[5], including Root Mean Square Error (RMSE), Mean Absolute Error (MAE) and Mean Absolute Percentage Error (MAPE), are calculated by the following equations.

$$RMSE = \sqrt{\frac{1}{N} \sum_{t=1}^N (y_t - \hat{y}_t)^2} \quad (7)$$

$$MAE = \frac{1}{N} \sum_{t=1}^N |y_t - \hat{y}_t| \quad (8)$$

$$MAPE = \frac{1}{N} \sum_{t=1}^N \left| \frac{y_t - \hat{y}_t}{y_t} \right| \quad (9)$$

3. Results and Discussion

The descriptive statistics of both series utilized in the study are provided in Table 1 which indicates that the garlic price varies from Rs. 2683 to Rs. 11953 per quintal and the ginger price ranges from Rs.2275 and Rs. 8644 per quintal of Varanasi market. The coefficient of variation is 43.65% and 36.87% for the wholesale price of garlic and ginger, respectively which showed that the data are heterogeneous in nature. The skewness and kurtosis showed that these series are positively skewed and platykurtic. The time plot and boxplot of both series were presented in Figure 2. The pattern of the time plots reveals that both series exhibited nonstationary and nonlinearity characteristics and the boxplot confirms that these series had the non-normality without outliers.

The data were tested by statistical tests to confirm graphical identified characteristics, and the results were presented in Table 2. According to the p-value, the Shapiro-Wilk test and Anderson-Darling test reject the null hypothesis which confirms that the data do not follow the normal distribution. The stationarity was tested by the ADF test and KPSS test. The null hypothesis of the ADF test is about the presence of a unit root, while the null hypothesis of the KPSS test is about the absence of a unit root. The result of both tests ensures that both series are nonstationary. The BDS test was used to confirm the presence of a nonlinearity pattern in the selected series and the results are given in table 3. The p-value for embedding dimensions 2 and 3 are lesser than 0.01, which proved that the selected series had nonlinear patterns.

In this experiment, the models ARIMA, exponential Smoothing, complex exponential smoothing, and time delay neural network are selected as candidate models for comparison purposes. The model-building process of these models was attained through the training set and validation was done by the test sample. The training sample consisted of 305 weekly observations and 26 observations were kept as the test set.

Table 1: Summary statistics of selected price series in the study

Crops	Garlic	Ginger
Count	331	331
Minimum	1683	2275
Mean	5637.14	4292.28
Median	5660.83	3560
Maximum	11953	8644
Standard Deviation	2460.61	1582.69
CV (%)	43.65	36.87
Skewness	0.34	0.82
Kurtosis	-0.75	-0.33

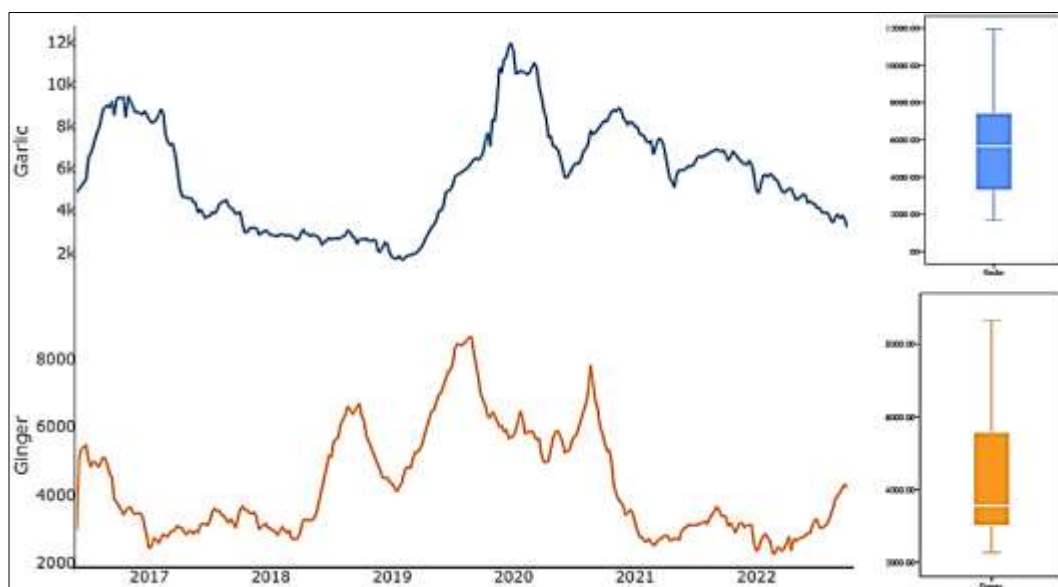


Fig 2: Time plot and Box plot for garlic and ginger series of Varanasi market

Table 2: The results of normality and stationarity tests for the Garlic and Ginger price series

Characteristics	Name of the test	Garlic		Ginger	
		Statistic	p-value	Statistic	p-value
Normality	Shapiro-Wilk	0.96	<0.001	0.89	<0.001
	Anderson-Darling	3.35	<0.001	13.38	<0.001
Stationarity	ADF	-2.28	0.46	-2.33	0.44
	KPSS	0.65	0.02	0.86	<0.001

Table 3: The result of the nonlinearity test for the Garlic and Ginger price series

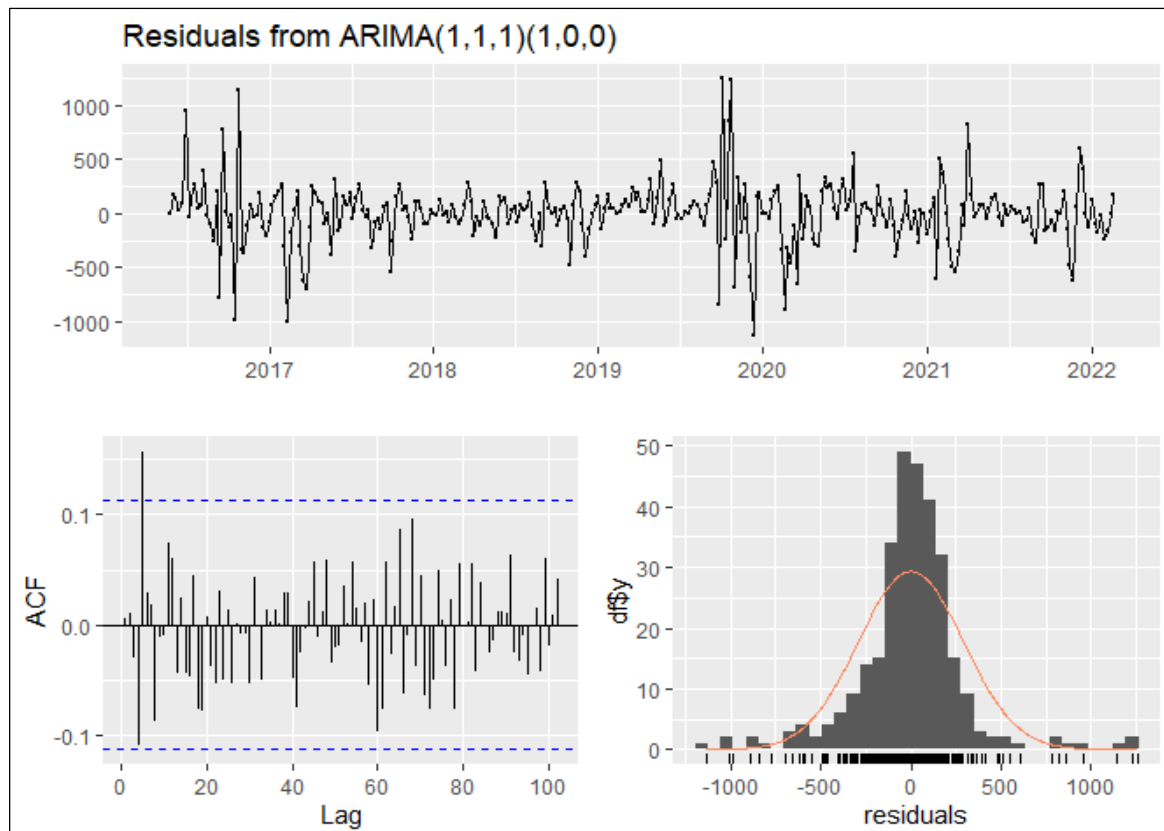
Crops	M	Eps[1]		Eps[2]		Eps[3]		Eps[4]	
		Statistic	p-value	Statistic	p-value	Statistic	p-value	Statistic	p-value
Garlic	2	351.36	<0.001	135.17	<0.001	72.60	<0.001	56.84	<0.001
	3	673.44	<0.001	175.59	<0.001	78.52	<0.001	55.79	<0.001
Ginger	2	68.62	<0.001	85.17	<0.001	58.02	<0.001	42.64	<0.001
	3	107.51	<0.001	108.50	<0.001	63.05	<0.001	41.64	<0.001

As mentioned in methodology, based on the lowest AIC and BIC values, seasonal ARIMA (1,1,1)(1,0,0)₅₂ and (2,1,1)(0,1,1)₅₂ were found to be an appropriate model for the price series of garlic and ginger. The parameter of the selected model was estimated by the maximum likelihood method which was given in Table 4 along with residual analysis. The residuals of the fitted model were tested by the Ljung box test and graphical representation which were provided in Table 4 and Figure 3. The p values of the test were 0.87 and 0.74, which confirmed that the residuals were not auto-correlated and the residual plots with ACF and histogram were presented in Figure 3 which revealed that the residuals were white noise. The selected models were adequate for both series which were used for validation purposes.

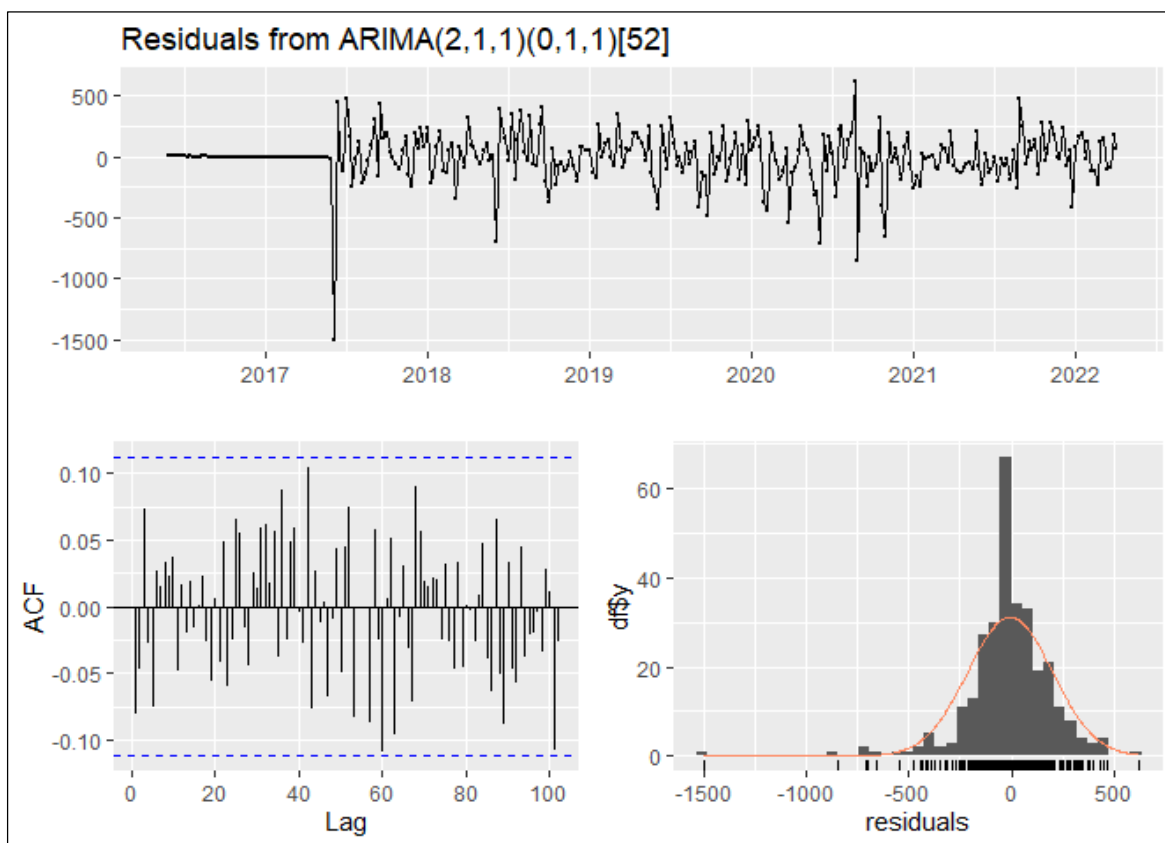
Table 4: The parameters of the selected ARIMA model

Crop	Model	AIC	BIC	Box test
Garlic				
ARIMA (1,1,1) (1,0,0) ₅₂	AR(1)	0.87**	4320.17	4335.03
	MA(1)	0.69**		
	SAR(1)	0.04**		
Ginger				
ARIMA (2,1,1) (0,1,1) ₅₂	AR(1)	1.27**	3489.49	3507.13
	AR(2)	-0.37**		
	MA(1)	0.87**		
	SMA(1)	-0.65**		

** Significant at 1% level of significance



(a) Garlic price



(b) Ginger price

Fig 3: Correlogram and Histogram of residuals from the selected models

Similar way, the best exponential smoothing model was selected based on the minimum AIC and BIC. Among thirty models, ETS (M, M_d, N) and ETS (A, A_d, N) were found as the best fit for the garlic and ginger series respectively. The identified model failed to capture the seasonality in weekly data. The estimated parameters of models were depicted in Table 5 along with residual analysis. According to the p-value of the Ljung Box test, the residuals were independent.

Table 5: The specifications of the selected exponential smoothing model

Crop	Model	α	β	ϕ	AIC	BIC	Box test
Garlic	ETS (MM _d N)	1.00	0.14	0.8968	4255.79	4274.40	0.053
Ginger	ETS (AA _d N)	1.00	1.00	0.3532	4027.49	4042.37	0.107

The seasonal complex smoothing model was chosen from the model-building step, based on the lowest value of AIC and BIC. The estimated parameters of the seasonal complex exponential model were shown in Table 6. In both price series, the parameter α_1 was greater than one. Therefore, the model is expected to produce an exponential trajectory.

Table 6: The specifications of selected complex exponential smoothing

Crop	$a_0 + ia_1$	$b_0 + ib_1$	AIC	BIC
Garlic	2.0002 + 1.0001i	1.0198 + 0.991i	4324.42	4343.02
Ginger	2.0005 + 1.0003i	1.0369 + 0.9908i	4131.415	4132.59

In the machine learning model, the best fit is determined by the lowest RMSE and MAPE. Based on this, the appropriate model for the garlic price series was chosen with six lagged, one seasonal lagged, and four hidden nodes (7:4s:11). Likewise, the model with one lagged, one seasonal lagged, and two hidden nodes was found to be the best fit for the price series of ginger. With this model, the sigmoid function and linear function are added as activation functions of the hidden layer and output layer.

Table 7: The description of the TDNN model

Crop	Model	Input		Parameters	Training RMSE	Training MAPE
		lag	Seasonal lag			
Garlic	7:4s:11	6	1	37	212.21	3.10
Ginger	2:2s:11	1	1	9	189.60	3.23

Table 8: Forecasting performance of different models using accuracy measures

	RMSE	MAE	MAPE
Garlic			
ARIMA	758.02	622.87	16.47
ES	716.17	610.99	15.67
CES	713.85	593.83	16.00
TDNN	1495.52	1338.15	34.60
Ginger			
ARIMA	609.28	535.76	16.75
ES	809.26	614.76	17.00
CES	457.42	353.48	10.86
TDNN	596.83	409.06	11.03

The forecasts of twenty-six weeks are obtained using one step ahead forecast of the selected model. The difference between test values and forecast values is measured by Root Mean Square Error, Mean Absolute Error and Mean Absolute Percentage Error, which helps to compare the forecasting performance of different models. The forecasting performance of ARIMA, Exponential Smoothing, CES and TDNN is given in Table 8. For the price series of Garlic, Complex exponential smoothing showed the smallest value of RMSE and MAE but not in MAPE. In the case of the Ginger series, the smallest value of three accuracy measures confirmed that the Complex exponential smoothing showed superiority among these four models. Generally, the seasonality in weekly data is less consistent than the seasonality in monthly data. Even though, the ARIMA model, CES and TDNN capture the seasonality except exponential smoothing. The results recommended that the complex exponential smoothing model performed well in the high-frequency data with nonstationary and nonlinear characteristics.

4. Conclusion

In this study, a different forecasting model has been implied to predict the weekly wholesale price of garlic and ginger for the Varanasi market. The study aims to identify a stable approach for weekly data with nonstationary and nonlinear characteristics. The challenges faced in weekly data of garlic and ginger are high frequency, less consistency in seasonality, and high fluctuations. The Classical model ARIMA and the Exponential Smoothing model were used as benchmark models. The previous literature suggested that the Complex Exponential Smoothing and Time Delay Neural Networks were able to handle nonlinearity and high-frequency data. From comparative analysis, Complex exponential smoothing was captured and modelled the price series of garlic in a good manner. In the case of the ginger price series, Complex exponential smoothing showed dominant performance compared to benchmark models followed by a Time delay neural network. Based on the results obtained in this study, the statistical model namely the complex exponential smoothing model showed significant performance compared to the machine learning method. This study also helps policymakers to make decisions about the relative price of garlic and ginger.

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