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Some basic concepts of algebraic calculus in tensors

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Abstract
In this paper, we discuss algebraic calculus from the tensors. Some of the basic problems are lying in the basic nature of that underlying the space with geometry. We made to survey certain instances of finite dimensional spaces and also cover instances of infinite dimensional vector spaces.

Keywords: Vector spaces, n-dimensional space, autonomous system, manifolds

1. Introduction
In this paper, we discuss how problems are related to the vector space of manifolds in tensor from the algebraic. We know that vector space is a geometric attribute if the underlying space is Euclidean then we know that the vector of the space is zero. In other words, all Euclidean spaces are flat. Otherwise, the vector space comes to fore geometry is all about vector. If the entity is an algebraic calculus then we would like to know how this algebra in space. Algebraic calculus more over time has delved upon the issue of tensor while studying the problems arising from analysis and topology. The theory of relativity by regarding space time as a continuum. Thus, vector space dominated investigating the problems associated with the space and methods employed to understand its nature came from the methods of calculus. The study in this regard also emphasized the importance of topology and manifolds. Halmos [2] around 1953's initiated tensor and their algebraic calculus studies for the problems. Some of the findings are the most noteworthy developments that have taken place in the past few decades. Guiding many researcher to continue their study on problems arising from these backgrounds. Here, we combine the algebraic calculus in tensor of the underlying space and study evolution of the space itself. We are drawn to relate geometric such evolutionary processes. In the next two sections we give some basic terminologies associated with the algebraic structures along with vector spaces of the underlying manifolds (or space). Subsequent sections deal with our work.

2. Preliminary
In this section, various conventions concerning notation and terminology which are used in the sequel are epitomized. Except where the contrary is stated explicitly, it is understood that these conventions are used throughout the paper. Precise definitions of the various terms used in this section appear in later portions of the paper.
Terminology, (i) In this paper, two different metrics occur: the usual Riemann metric of Riemannian geometry and the new metric of conformal geometry. The Riemannian metric of a space is frequently referred to simply as the "metric" of the space and geometric objects which are constructed relative to the Riemannian metric are referred to as "metric geometric objects." (ii) However, when no ambiguity is possible, the adjective "conformal" used in referring to various geometric objects is omitted. For example, the "conformal measure tensors" are usually called the "measure tensors." (iii) The words "space" and "surface" are used to distinguish geometric objects defined relative to an enveloping Riemann space or to a subspace embedded in it. When there is no danger of ambiguity, these words are omitted. For example, the "surface conformal Riemann tensor" may be referred to as the "conformal Riemann tensor." General conventions, (i) The usual conventions of tensor notation are assumed. For example, a tensor equation in which an index is not summed is valid for each value of the index within its range.

When the same letter appears in any term as a subscript and superscript, it is understood that this letter is summed for all values within its range. Sometimes summation is also denoted by Σ. (ii) The same kernel letter is used to indicate the contravariant and covariant components of the same vector. For example, π^{α} and π_{α} refer to the same vector. A similar assumption is made for any tensor. (iii) Space and surface components of the same tensor are also indicated by the same kernel letter. For example, the space vector λ^{α} and the space tensor $\pi^{\alpha k}_{\beta l}$ have the surface components λ^{i} and π^{ik}_{jl} respectively. (iv) The properties of a general tensor are usually illustrated by means of the tensor π^{α}_{β} or π^{i}_{j} or $\pi^{\alpha i}_{\beta j}$. Here α and i represent any contravariant (covariant) space and surface indices respectively. (v) As a general rule, lower case Roman and Greek kernel letters represent Riemannian metric geometric objects, capital Greek kernel letters represent relative conformal geometric objects and capital Roman kernel letters represent conformal geometric objects. (vi) References, indicated by brackets, refer to the bibliography at the end of the paper.

Indices: The set of indices $\alpha,\beta,\gamma,\delta,\varepsilon,\zeta; g,h,i,j,k,l;\tau,\omega$ have the ranges $1,2,\cdots,m; 1,2,\cdots,n; n+1,n+2,\cdots,m$ respectively. The powers and subscripts u,v,w,a,b have arbitrary ranges. Indices such as α_{w} and i_{v} have the same range as α and i respectively. The symbol (i_{w}) means $i_1,i_2,\ldots,i_w$ The letters m,n,p and q,r,s represent dimension numbers and class numbers respectively. They are not indices.

Spaces: The symbols V_m, R_m, S_m are used to represent an m-dimensional Riemann space, Euclidean space, and space of constant curvature respectively. The symbol $V^{(p)}$ refers to a p-dimensional vector space. However $I_1, I_{12},\cdots,I_{12\cdots M}$ and $I_2, I_3,\cdots,I_M$ refer to the conformal osculating vector spaces and conformal normal vector spaces of V_n.

Conformal correspondence. A Riemann space conformal to V_m is denoted by $\overline{V_m}$. Thus $\overline{R_m}$ signifies a conformally Euclidean space. A geometric object in $\overline{V_m}$ corresponding to the geometric object $\Im$ in V_m is denoted by $\overline{\Im}$.

Operations. (ï) The projection of a vector π^{α} in a vector space $V^{(p)}$ is denoted by $*\pi^{\alpha}$. The precise space $V^{(p)}$ into which π^{α} is projected is indicated in the context.

(ii) Covariant differentiation with respect to $a_{\alpha\beta}$, g_{ij} and $\overline{a_{\alpha\beta}}$, $\overline{g_{ij}}$ are denoted by a comma (,) and semicolon (;) respectively. The symbol for conformal covariant differentiation is the colon (:). The conformal derivative (along a curve) with respect to 5 is denoted by $\upsilon / \upsilon S$.

(iii) A cyclic sum of tensors is indicated by a plus sign (+) and an alternating sum by a minus sign (—).

2.1 Definitions: A vector space of V is a subset of V which is itself a vector space under the induced laws in real numbers of $\mathbb{R}^n$. The vectors $\lambda_1,\lambda_2,\ldots\lambda_p$ from a autonomous system of order p. when the relation

$$x^1\lambda_1 + x^2\lambda_2 + \ldots + x^n\lambda_p = 0$$

Implies that all the co-efficient x^i are zero. In the case the vectors are said to be linearly independent vectors which we conclude that vectors of notion is standard from which can't accept in autonomous system. In this system notion are involve it as linearly dependent.

In this vector are admits autonomous systems of arbitrarily are emphasizes in the bounded. So the vector V is a finite dimensional. In some case there a positive integer 'n' but the autonomous system of order is 'n'. When vector spaces are in finite dimensional so$dim V = n$ is a basis.

2.2 Theorem: A set of vectors is a basis of V if and only if
1. The vectors are linearly independent.
2. Every vector of v is expressible as a linear combination of these vectors.

2.3 Proposition: The number of elements in one basis of a finite dimensional vector space is the same as that in any basis.

Proof: Let $u, v \in V$ is a vector space of dimension over the field $\mathbb{R}^n$ and let their sum $u + v$ is defined by $u = (u_1,u_2,\ldots,u_n)$ and $v = (v_1,v_2,\ldots,v_n)$ then

$$u + v = (u_1 + v_1, u_2 + v_2, \ldots, u_n + v_n)$$

For $\mathbb{R}^n$. Let α times V satisfy.
It is obvious that $u + v$ and αv both belong to V hence

$\alpha V\ (u_1, u_2, \ldots, u_n) = \alpha(v(u_1), v(u_2), \ldots, v(u_n))$ To examine the dimensionally we suppose that V has a basis $(e_1, e_2, \ldots, e_n)$.

$$u = \sum_{j=1}^{n} \alpha_i^j e_j \tag{1}$$

and

$$v = \sum_{h=1}^{n} \beta_h^k e^h \tag{2}$$

if (1) and (2) is solved uniquely for a given initial condition

$$V(u_{i_1}^*, \ldots, u_{i_r}^*, v_{k_1}^*, \ldots, v_{k_s}^*) = \sum_{j_1,\ldots,j_r} \sum_{h_1,\ldots,h_s} \beta_h^k \alpha_i^j V(e_{j_1}, \ldots, e_{j_r}, e^{h_1}, \ldots, e^{h_s})$$

Where $V(e_{j_1}, \ldots, e_{j_r}, e^{h_1}, \ldots, e^{h_s})$ are the components of V relative to the basis $\{e_i\}$ of V.

Proposition1 4: Let a autonomous system of order r is given $(r < n)$ the system can be completed by means of $(n - r)$ other vectors to from s basis for V.

Proof: Let 'n' vectors forms a basis for V will be written as $u_1, u_2, \ldots, u_n$ and the set that is the basis u_i and any vector 'λ' is uniquely expressible in the from

$$\lambda = \sum_{i=1}^{n} \lambda^i u_i \tag{3}$$

The numbers λ^i where $i = 1, 2, \ldots, n$ are called components of λ. Now we specifying its components relative to any one basis. If μ^i are the another components of vector μ relative to the same basis the components of the vector.

$$\mu = \sum_{j=1}^{n} \mu^j u_i' \tag{4}$$

adding $\lambda + \mu$ in the one basis of 'i' components

$$\lambda + \mu = \sum_{i,j=1}^{n} (\lambda^i + \mu^j) u_i u_i'$$

It means that any arbitrary number of 'a' the components of the vector $a\lambda$ are $a\lambda^i$.

3. The dual space

In n-dimensional vector space V is a contravariant vectors is associated with Vis another vector space are V* are covariant vectors. But the element of tensors will appears in between of contravariant and covariant vectors as far as possible it will be used in terms of systemically, the process of vector space of tensor are using the algebraic terms of symbolically in tensors yields.

3.1 Theorem: Suppose V is a vector space and W is a subset of V. Then, W is a subspace V.

3.2 Definition: Suppose V is a vector space. A non-empty subset V* of V is called a subspace of V, if V* is a vector space under the addition and scalar multiplication in V.

3.3 Proposition: The subspace contravariant of covariant is a vector space.

Proof: If α, β are any two covariant vectors define their sum $\alpha + \beta$ to be the covariant vector 'γ' such that

$$\gamma(\lambda)=\alpha(\lambda)+\beta(\lambda) \tag{5}$$

Here $\gamma(\lambda)$ are real numbers of sum and 'λ' are all of contravariant. The product $a\alpha$ of a covariant vector 'α' by the real number 'a' is defined to be the covariant vector δ such that

$$\delta(\lambda)=a\alpha(\lambda) \tag{6}$$

If (1)and (2) are implies the dual space of V and is denoted by V*. Let μ be any arbitrary of a real valued function of over the space V and the basis of the component is e_i in (2) to shown that

$$\mu(\lambda)=\mu\left(\sum_{i=1}^{n}\lambda^i e_i\right)=\sum_{j=1}^{n}\left(\sum_{i=1}^{n}\lambda_i e_i\right)\mu_j e_i^{'}$$

$$=\sum_{i,j=1}^{n}\lambda_i\mu_j(e_i e_i^{'})=\sum_{i,j=1}^{n}\lambda_i\mu_j=\lambda_i\mu_j$$

4. Covariant and Contravariant vectors in tensor

Let $(x^1,x^2,\ldots,x^n)$ be co-ordinates of a point in X-coordinate system and $\left(\bar{x}^1,\bar{x}^2,\ldots,\bar{x}^n\right)$ be co-ordinates of the same point in Y-coordinate system. Let λ^i, $(\lambda^1,\lambda^2,\ldots,\lambda^n)$ be nth functions of coordinates then X-coordinate system is x^i and Y-coordinate system $\bar{x}^i$. According to some basic laws of tensor.

$$\bar{\lambda}^i=\frac{\partial\bar{x}^i}{\partial x^j}\lambda^j \text{ or } \lambda^j=\frac{\partial x^j}{\partial\bar{x}^i}\bar{\lambda}^i$$

Then λ^i are called components of contravariant vector.

Let λ_i, $(\lambda_1,\lambda_2,\ldots,\lambda_n)$ be nth functions of coordinates theX-coordinate system is x_i and Y-coordinate system $\bar{x}_i$. According to some basic laws of tensor.

$$\bar{\lambda}_i=\frac{\partial x^j}{\partial\bar{x}^i}\lambda_j \text{ or } \lambda_j=\frac{\partial\bar{x}^i}{\partial x^j}\bar{\lambda}_i$$

Then λ_i are called components of covariant vector.

4.1 Proposition: Thecomponent of tangent vector on the curve in n-dimensional space are component of contravarient vector.

Proof: Let $\frac{dx^1}{dt},\frac{dx^2}{dt},\ldots,\frac{dx^n}{dt}$ be the component of the tangent vector of the point $(x^1,x^2,\ldots,x^n)$ that is $\frac{dx^i}{dt}$ be the component of the tangent vector in X-coordinate system.

Let the component of tangent vector of the point $\left(\bar{x}^1,\bar{x}^2,\ldots,\bar{x}^n\right)$ in Y-coordinate system are $\frac{d\bar{x}^i}{dt}$. Then $\bar{x}^1,\bar{x}^2,\ldots,\bar{x}^n$ being a function of $x^1,x^2,\ldots,x^n$ which is a function of t. So,

$$\frac{d\bar{x}^i}{dt}=\frac{\partial\bar{x}^i}{\partial x^1}\frac{dx^1}{dt}+\frac{\partial\bar{x}^i}{\partial x^2}\frac{dx^2}{dt}+\cdots+\frac{\partial\bar{x}^i}{\partial x^n}\frac{dx^n}{dt}$$

$$\frac{d\bar{x}^i}{dt}=\frac{\partial\bar{x}^i}{\partial x^j}\frac{dx^j}{dt}$$

So, $\frac{dx^i}{dt}$ is component of contravarient vector that is the component of the tangent vector on the curve in n-dimensional space are component of contravarient vector.

4.2 Proposition: The component of tangent vector on the curve in n-dimensional space are component of covariant vector.

5. Proof: Let $\frac{dx^1}{dt}, \frac{dx^2}{dt}, \ldots, \frac{dx^n}{dt}$ be the component of the tangent vector of the point $(x^1, x^2, \ldots, x^n)$ that is $\frac{dx^i}{dt}$ be the component of the tangent vector in X-coordinate system. Let the component of tangent vector of the point $\left(\bar{x}^1, \bar{x}^2, \ldots, \bar{x}^n\right)$ in Y-coordinate system are $\frac{d\bar{x}^i}{dt}$. Then $\bar{x}^1, \bar{x}^2, \ldots, \bar{x}^n$ being a function of $x^1, x^2, \ldots, x^n$ which is a function of t. So,

$$\frac{d\bar{x}^i}{dt} = \frac{\partial x^j}{\partial \bar{x}^1}\frac{d\bar{x}^1}{dt} + \frac{\partial x^j}{\partial \bar{x}^2}\frac{d\bar{x}^2}{dt} + \cdots + \frac{\partial x^j}{\partial \bar{x}^n}\frac{d\bar{x}^n}{dt}$$

$$\frac{d\bar{x}^i}{dt} = \frac{\partial x^j}{\partial \bar{x}^i}\frac{d\bar{x}^i}{dt}$$

So, $\frac{dx^i}{dt}$ is component of covariant vector that is the component of tangent vector on the curve in n-dimensional space are component of covariant vector.

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