International Journal of Statistics and Applied Mathematics 2023; 8(3): 191-198

International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452
Maths 2023; 8(3): 191-198

https://www.mathsjournal.com
Received: 03-02-2023
Accepted: 05-03-2023

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An exact formula for ruin probability in generalized risk model with homogeneous Markov chains

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DOI: https://doi.org/10.22271/maths.2023.v8.i3c.1016

Abstract
This paper considers a general risk model with the impact of interest rates. The purpose is to build a formula to calculate the ruin probability of that model. The assumption we expand in this paper is that the series of insurance premiums, the series of insurance claims and the series of interest rates are homogeneous and independent Markov chains. The sequences of random variables studied in this paper all have positive integer values. Using the properties of probability theory, we can build a formula for calculating ruin probability of the research model.
2000 MSC: primary62P05, 62E10, secondary 60F05.

Keywords: Ruin probability, generalized risk model, Non –ruin probability, homogeneous Markov chain

1. Introduction

Insurance models have been studied by many authors in terms of formulating the ruin probability estimation formula for those models. The second approach is to build a formula for calculating the ruin probability, in Claude Lefèvre and Stéphane Loisel [1], the authors have built a formula of ruin probability for compound biomial and compound Poisson risk models. The result is extended. the formula for calculating the ruin probability of Pircard and Lefèvre [9]. For the general risk model with the impact of interest rates determined by (1.1) below.

$$Z_t = Z_{t-1}(1+I_t) + X_t - Y_t; t = 1, 2, ... \quad (1)$$

With

Z_t is the capital at period t, Z_o= z is the capital of the first period of the insurance company; u; t takes the value in the set $N^* = \{1; 2; 3; .....\}$; $X = \{X_i\}_{i\geq 1}$ are premiums; $Y = \{Y_j\}_{j\geq 1}$ are claims and $I = \{I_k\}_{k\geq 1}$ are interests.

The approaches of Claude Lefèvre và Stéphane Loisel do not build a formula for calculating the ruin probability. To build the formula for calculating the ruin probability for model (1.1), we assume that the random variable sequences are Markov chains with positive integer values, which is consistent with practice.

The article is structured as follows: in Section 2 we introduce model, asumptions; in Section 3, we study the model (1.1) to establish the formula for calculating the ruin probability and the non ruin probability; Finally, in section 4, we introduce the conclusion of this paper.

2. Model and Assumptions

To study the model posed for the problem of this paper, we make the following assumptions:

Assumption 1. $Z_o = z$, t takes the value in the set $N^* = \{1; 2; 3; .....\}$.

Assumption 2. X_n is premium amounts from the n[th] premium;
$X = \{X_n\}_{n\geq 1}$ is a Markov chain and is homogeneous,
X_n take values in $T_X = \{x_1, x_2, ..., x_K\} (0 < x_1 < x_2 < ... < x_K)$.

Let $q_{ij} = P\left[X_{n+1} = x_j \middle| X_n = x_i\right], (n \in N); x_i, x_j \in T_X \left(0 \leq q_{ij} \leq 1, \sum_{j=1}^{K} q_{ij} = 1\right)$ and $q_i = P(X_1 = x_i)(x_i \in T_X)$.

We make the following convention

$$P(0 < X_n \leq x_K < +\infty) = 1,$$

$$P(X_n = 0) = 0 \text{ if } and \text{ if } P(X_n > 0) = 1.$$

Assumption 3. Y_n is claim amounts from the n[th] claim,
$Y = \{Y_n\}_{n\geq 1}$ is a Markov chain and is homogeneous,

Y_n take values in $T_Y = \{y_1, y_2, ..., y_P\} (0 < y_1 < y_2 < ... < y_P)$.

Let $p_{ij} = P\left[Y_{m+1} = y_j \middle| Y_m = y_i\right], (m \in N); y_i, y_j \in T_Y \left(0 \leq q_{ij} \leq 1, \sum_{j=1}^{P} q_{ij} = 1\right)$ and $p_i = P(Y_1 = y_i)(y_i \in T_Y)$.

We make the following convention

$$P(1 \leq Y_m \leq y_P < +\infty) = 1,$$

$$P(Y_m = 0) = 0 \text{if } and \text{ if } P(Y_m > 0) = 1.$$

Assumption 4. I_n is interest amounts from the n[th] interest,
$I = \{I_n\}_{n\geq 1}$ is a Markov chain and is homogeneous,

I_n take values in $T_I = \{i_1, i_2, ..., i_Q\} (0 \leq i_1 < i_2 < ... < i_Q)$.

Let $r_{lh} = P\left[I_{n+1} = r_l \middle| I_n = r_h\right], (n \in N); r_l, r_h \in T_I \left(0 \leq r_{lh} \leq 1, \sum_{h=1}^{Q} r_{lh} = 1\right)$ and $r_k = P(I_1 = r_k)(r_k \in T_I)$.

We make the following convention

$$P(0 \leq I_n \leq i_Q < +\infty) = 1.$$

a. X, Y and I are sequences of independent random variables.
From (1.1), we have:

$$Z_t = z.\prod_{k=1}^{t}(1 + I_k) + \sum_{k=1}^{t-1}\left((X_k - Y_k)\prod_{j=k+1}^{t}(1 + I_j)\right) + X_t - Y_t. \quad (2)$$

The ruin time is defined by $T_z = \inf\left\{j : Z_j < 0\right\}$, where $\inf \phi = \infty$.
With assumption 2.1 to assumption 2.5:
The finite time ruin probabilities of model (1.1) is defined as:

$$\psi_t(z) = P(T_z \le t) = P\left(\bigcup_{j=1}^{t}(Z_j < 0)\right) \quad (3)$$

The finite time non ruin probabilities of model (1.1) is defined as:

$$\varphi_t(z) = 1 - \psi_t(z) = P(T_z \ge t+1) = P\left(\bigcap_{j=1}^{t}(Z_j \ge 0)\right). \quad (4)$$

The problem posed in this paper is to build formulas for $\psi_t(z)$ and $\varphi_t(z)$. We first prove the following lemma.

Lemma 2.1. Assuming z, $x_i\ (i=\overline{1,t}), y_i\ (i=\overline{1,t})$ are positive integers và $i_k\ (k=\overline{1,t})$ is non - negative number. If n is a possitive integer number that $1 \le n \le t-1$ satisfies:

$$y_n \le z\prod_{k=1}^{n}(1+i_k) + \sum_{k=1}^{n-1}(x_k - y_k)\prod_{j=k+1}^{n}(1+i_j) + x_n \quad (5)$$

Then

$$z\prod_{k=1}^{n+1}(1+i_k) + \sum_{k=1}^{n}(x_k - y_k)\prod_{j=k+1}^{n+1}(1+i_j) + x_{n+1} \ge 1 \quad (6)$$

Proof

Use (2.4), we infer

$$y_n \le z\prod_{k=1}^{n}(1+i_k) + \sum_{k=1}^{n-1}(x_k - y_k)\prod_{j=k+1}^{n}(1+i_j) + x_n$$

$$\Leftrightarrow x_n - y_n \ge -z\prod_{k=1}^{n}(1+i_k) - \sum_{k=1}^{n-1}(x_k - y_k)\prod_{j=k+1}^{n}(1+i_j)$$

We have

$$z\prod_{k=1}^{n+1}(1+i_k) + \sum_{k=1}^{n}(x_k - y_k)\prod_{j=k+1}^{n+1}(1+i_j) + x_{n+1}$$

$$= z\prod_{k=1}^{n+1}(1+i_k) + \sum_{k=1}^{n-1}(x_k - y_k)\prod_{j=k+1}^{n+1}(1+i_j) + (x_n - y_n)(1+i_{n+1}) + x_{n+1}$$

$$\ge z\prod_{k=1}^{n+1}(1+i_k) + \sum_{k=1}^{n-1}(x_k - y_k)\prod_{j=k+1}^{n+1}(1+i_j) + \left[-z\prod_{k=1}^{n}(1+i_k) - \sum_{k=1}^{n-1}(x_k - y_k)\prod_{j=k+1}^{n}(1+i_j)\right](1+i_{n+1}) + x_{n+1} = x_{n+1} \ge 1.$$

So (2.5) is correct.

Lemma 2.1 has been proved □

3. The main result

The main result of the paper is that we will build a formula for the ruin probability (2.2) and a formula for the non-ruin probability (2.3).

Theorem 3.1. Consider model (1.1) with Assumption 2.1 to Assumption 2.5, then finite time non ruin probability of model (1.1) is defined by

$$\varphi_t^{(1)}(z) = \sum_{c_1,c_2,...,c_t=1}^{Q} \sum_{m_1,m_2,...,m_t=1}^{K} r_{d_1} r_{d_1d_2} ... r_{d_{t-1}d_t} q_{m_1} q_{m_1m_2} ... q_{m_{t-1}m_t} \left(\sum_{1\le n_1 \le f_1} \sum_{1\le n_2 \le f_2} ... \sum_{1\le n_t \le f_t} p_{n_1} p_{n_1n_2} ... p_{n_{t-1}n_t} \right), \quad (7)$$

Where

$$f_1 = \max\left\{ n_1 : y_{n_1} \le \min\left\{ \left[u \prod_{k=1}^{1} (1+i_{d_k}) + x_{m_1} \right], y_P \right\} \right\},$$

$$f_2 = max\left\{ n_2 : y_{n_2} \le \min\left\{ \left[u \prod_{k=1}^{2} (1+i_{d_k}) + \sum_{k=1}^{1} (x_{m_k} - y_{n_k}) \prod_{j=k+1}^{2} (1+i_{d_j}) + x_{m_2} \right], y_P \right\} \right\},$$

$$f_t = max\left\{ n_t : y_{n_t} \le \min\left\{ \left[u \prod_{k=1}^{t} (1+i_{d_k}) + \sum_{k=1}^{t-1} (x_{m_k} - y_{m_k}) \prod_{j=k+1}^{t} (1+i_{d_j}) + x_{m_t} \right], y_P \right\} \right\},$$

and $[a]$ is integer part of a.

Proof.
Firtly, we let

$$A := \bigcap_{j=1}^{t} (Z_j \ge 0) = \left(Y_1 \le z \prod_{k=1}^{1} (1+I_k) + X_1 \right) \cap \left(Y_2 \le z \prod_{k=1}^{2} (1+I_k) + \sum_{k=1}^{1} (X_k - Y_k) \prod_{j=k+1}^{2} (1+I_j) + X_2 \right) \cap$$

$$\left(Y_3 \le z \prod_{k=1}^{3} (1+I_k) + \sum_{k=1}^{2} (X_k - Y_k) \prod_{j=k+1}^{3} (1+I_j) + X_3 \right) \cap ...... \cap \left(Y_t \le z \prod_{k=1}^{t} (1+I_k) + \sum_{k=1}^{t-1} (X_k - Y_k) \prod_{j=k+1}^{t} (1+I_j) + X_t \right) \quad (8)$$

By Asumption 2.4, we have $I_1 = i_{d_1}, I_2 = i_{d_2}, ..., I_t = i_{d_t}$ with $i_{d_1}, i_{d_2}, ..., i_{d_t}$ are non-negative numbers statisfy:

$0 \le i_{d_1}, i_{d_2}, ..., i_{d_t} \le i_Q$.

Let $B_{d_1d_2...d_t} = \left(I_1 = i_{d_1}\right) \cap \left(I_2 = i_{d_2}\right) \cap ... \cap \left(I_t = i_{d_t}\right)$.

Because $I = \{I_n\}_{n\ge1}$ is a Markov chain and it is homogeneous then

$$P(B_{d_1d_2...d_t}) = P\left[\left(I_1 = i_{d_1}\right) \cap \left(I_2 = i_{d_2}\right) \cap ... \cap \left(I_t = i_{d_t}\right)\right]$$

$$= P\left(I_1 = i_{d_1}\right).P\left(I_2 = i_{d_2} \middle| I_1 = i_{d_1}\right)...P\left(I_t = i_{d_t} \middle| I_{t-1} = i_{d_{t-1}}\right) = r_{d_1} r_{d_1d_2} ... r_{d_{t-1}d_t} \quad (9)$$

By Asumption 2.2, we have $X_1 = x_{m_1}, X_2 = x_{m_2}, ..., X_t = x_{m_t}$ with $x_{m_1}, x_{m_2}, ..., x_{m_t}$ are possitive numbers statisfy:

$0 < x_{m_1}, x_{m_2}, ..., x_{m_t} \le x_M$ Define $C_{m_1m_2...m_t} = \left(X_1 = x_{m_1}\right) \cap \left(X_2 = x_{m_2}\right) \cap ... \cap \left(X_t = x_{m_t}\right)$.

Because $X = \{X_n\}_{n\ge1}$ is a Markov chain and it is homogeneous then

$$P(C_{m_1m_2...m_t}) = P\left[\left(X_1 = x_{m_1}\right) \cap \left(X_2 = x_{m_2}\right) \cap ... \cap \left(X_t = x_{m_t}\right)\right]$$

$$= p_{m_1} p_{m_1m_2} ... p_{m_{t-1}m_t} \quad (10)$$

Firsly, we have $I_1 = i_{d_1} (d_1 = \overline{1,Q})$ then (3.2) is given

$$A \overset{as}{=} \bigcup_{d_1=1}^{Q} \left(I_1 = i_{d_1}\right) \cap \left(\left(Y_1 \le z \prod_{k=1}^{1} (1+i_{d_k}) + X_1 \right) \cap \left(Y_2 \le z(1+i_{d_1}) \prod_{k=2}^{2} (1+I_k) + \sum_{k=1}^{1} (X_k - Y_k) \prod_{j=k+1}^{2} (1+I_j) + X_2 \right) \cap \right.$$

$$\left. \left(Y_3 \le z(1+i_{d_1}) \prod_{k=2}^{3} (1+I_k) + \sum_{k=1}^{2} (X_k - Y_k) \prod_{j=k+1}^{3} (1+I_j) + X_3 \right) \cap \dots \dots \cap \left(Y_t \le z(1+i_{d_1}) \prod_{k=2}^{t} (1+I_k) + \sum_{k=1}^{t-1} (X_k - Y_k) \prod_{j=k+1}^{t} (1+I_j) + X_t \right) \right)$$

Where

$A \overset{as}{=} B$ if $P(A \Delta B) = 0$ and $A \Delta B = (A \setminus B) \cup (B \setminus A)$.

Similarly, we let $I_2 = i_{d_2}, \dots, I_t = i_{d_t} (d_2, \dots, d_t = \overline{1,Q})$, (3.2) is defined as

$$A \overset{as}{=} \bigcup_{c_1,c_2,\dots,c_t=1}^{Q} \left\{ \left(I_1 = i_{d_1}\right) \cap \left(I_2 = i_{d_2}\right) \cap \dots \cap \left(I_t = i_{d_t}\right) \right\} \cap \left(\left(Y_1 \le z \prod_{k=1}^{1} (1+i_{d_k}) + X_1 \right) \cap \right.$$

$$\left(Y_2 \le z \prod_{k=1}^{2} (1+i_{d_k}) + \sum_{k=1}^{1} (X_k - Y_k) \prod_{j=k+1}^{2} (1+i_{d_j}) + X_2 \right) \cap \left(Y_3 \le z \prod_{k=1}^{3} (1+i_{d_k}) + \sum_{k=1}^{2} (X_k - Y_k) \prod_{j=k+1}^{3} (1+i_{d_j}) + X_3 \right) \cap \dots$$

$$\left. \dots \cap \left(Y_t \le z \prod_{k=1}^{t} (1+i_{d_k}) + \sum_{k=1}^{t-1} (X_k - Y_k) \prod_{j=k+1}^{t} (1+i_{d_j}) + X_t \right) \right)$$

Next, we let $X_1 = x_{m_1} (m_1 = \overline{1,K})$, then

$$A \overset{as}{=} \bigcup_{d_1,d_2,\dots,d_t=1}^{Q} \left(\left\{ \left(I_1 = i_{d_1}\right) \cap \left(I_2 = i_{d_2}\right) \cap \dots \cap \left(I_t = i_{d_t}\right) \right\} \cap \left(\bigcup_{x_1=1}^{K} (X_1 = x_{m_1}) \cap \left(\left(Y_1 \le u \prod_{k=1}^{1} (1+i_{d_k}) + x_{m_1} \right) \cap \right. \right. \right.$$

$$\left(Y_2 \le z \prod_{k=1}^{2} (1+i_{d_k}) + \sum_{k=1}^{1} (x_{m_k} - Y_k) \prod_{j=k+1}^{2} (1+i_{d_j}) + X_2 \right) \cap$$

$$\left(Y_3 \le z \prod_{k=1}^{3} (1+i_{d_k}) + \left[(x_{m_1} - Y_1) + \sum_{k=2}^{2} (X_k - Y_k) \right] \prod_{j=k+1}^{3} (1+i_{d_j}) + X_3 \right) \cap \dots$$

$$\left. \left. \left. \dots \cap \left(Y_t \le z \prod_{k=1}^{t} (1+i_{d_k}) + \left[(x_{m_1} - Y_1) + \sum_{k=2}^{t-1} (X_k - Y_k) \right] \prod_{j=k+1}^{t} (1+i_{d_j}) + X_t \right) \right) \right) \right)$$

Similarly, we let $X_2 = x_{m_2}, \dots, X_t = x_{m_t} (m_2, \dots, m_t = \overline{1,K})$, (3.2) is defined as

$$A \overset{as}{=} \bigcup_{d_1,d_2,\dots,d_t=1}^{Q} \left(\left\{ \left(I_1 = i_{d_1}\right) \cap \left(I_2 = i_{d_2}\right) \cap \dots \cap \left(I_t = i_{d_t}\right) \right\} \cap \left(\bigcup_{m_1,m_2,\dots,m_t=1}^{K} \left\{ (X_1 = m_1) \cap (X_2 = m_2) \cap \dots \cap (X_t = m_t) \right\} \cap \right. \right.$$

$$\left(\left(Y_1 \le z \prod_{k=1}^{1} (1+i_{d_k}) + x_{m_1} \right) \cap \left(Y_2 \le z \prod_{k=1}^{2} (1+i_{d_k}) + \sum_{k=1}^{1} (x_{m_k} - Y_k) \prod_{j=k+1}^{2} (1+i_{d_j}) + x_{m_2} \right) \cap \right.$$

$$\left(Y_3 \le z\prod_{k=1}^{3}(1+i_{d_k}) + \sum_{k=1}^{2}(x_{m_k} - Y_k)\prod_{j=k+1}^{3}(1+i_{d_j}) + x_{m_3}\right)\cap\ldots\ldots\cap\left(Y_t \le z\prod_{k=1}^{t}(1+i_{d_k}) + \sum_{k=1}^{t-1}(x_{m_k} - Y_k)\prod_{j=k+1}^{t}(1+i_{d_j}) + x_{m_t}\right)$$

$$\stackrel{as}{=} \bigcup_{d_1,d_2,\ldots,d_t=1}^{Q}\left(\left\{(I_1 = i_{d_1})\cap(I_2 = i_{d_2})\cap\ldots\cap(I_t = i_{d_t})\right\}\cap\bigcup_{m_1,m_2,\ldots,m_t=1}^{K}\left(\left\{(X_1 = x_{m_1})\cap(X_2 = x_{m_2})\cap\ldots\cap(X_t = x_{m_t})\right\}\cap D_{m_1m_2\ldots m_t}^{d_1d_2\ldots d_t}\right)\right)$$

$$\stackrel{as}{=} \bigcup_{d_1,d_2,\ldots,d_t=1}^{Q}\left(\bigcup_{m_1,m_2,\ldots,m_t=1}^{K}\left\{A_{d_1d_2\ldots d_t}\cap B_{m_1m_2\ldots m_t}\cap D_{m_1m_2\ldots m_t}^{d_1d_2\ldots d_t}\right\}\right) \tag{11}$$

Where

$$D_{m_1m_2\ldots m_t}^{d_1d_2\ldots d_t} \stackrel{as}{=} \left(Y_1 \le z\prod_{k=1}^{1}(1+i_{d_k}) + x_{m_1}\right)\cap\left(Y_2 \le z\prod_{k=1}^{2}(1+i_{d_k}) + \sum_{k=1}^{1}(x_{m_k} - Y_k)\prod_{j=k+1}^{2}(1+i_{d_j}) + x_{m_2}\right)\cap$$

$$\left(Y_3 \le z\prod_{k=1}^{3}(1+i_{d_k}) + \sum_{k=1}^{2}(x_{m_k} - Y_k)\prod_{j=k+1}^{3}(1+i_{d_j}) + x_{m_3}\right)\cap\ldots\ldots\cap\left(Y_t \le z\prod_{k=1}^{t}(1+i_{d_k}) + \sum_{k=1}^{t-1}(x_{m_k} - Y_k)\prod_{j=k+1}^{t}(1+i_{d_j}) + x_{m_t}\right) \tag{12}$$

By Assumption 2.3, we have $Y_1 = y_{n_1}, Y_2 = y_{n_2}, \ldots, Y_{t-1} = y_{n_{t-1}}; Y_t = y_t$ with $y_1, y_2, \ldots, y_{t-1}; y_t$ are positive numbers satisfy: $0 < y_1, y_2, \ldots, y_t \le y_P$.

Thus, (3.6) is defined as

$$D_{m_1m_2\ldots m_t}^{d_1d_2\ldots d_t} \stackrel{as}{=} \bigcup_{y_1\le\left(z\prod_{k=1}^{1}(1+i_{d_k})+x_{m_1}\right)}\left(\bigcup_{y_2\le\left(z\prod_{k=1}^{2}(1+i_{d_k})+\sum_{k=1}^{1}(x_{m_k}-y_{n_k})\prod_{j=k+1}^{2}(1+i_{d_j})+x_{m_2}\right)}\left(\bigcup_{y_3\le\left(u\prod_{k=1}^{3}(1+i_{d_k})+\sum_{k=1}^{2}(x_{m_k}-y_{n_k})\prod_{j=k+1}^{3}(1+i_{d_j})+x_{m_3}\right)}\ldots\right.\right.$$

$$\left.\left.\ldots\left(\bigcup_{y_t\le\left(z\prod_{k=1}^{t}(1+i_{d_k})+\sum_{k=1}^{t-1}(x_{m_k}-y_{n_k})\prod_{j=k+1}^{t}(1+i_{d_j})+x_{m_t}\right)}\left\{(Y_1 = y_{n_1})\cap\ldots\cap(Y_t = y_{n_t})\right\}\right)\ldots\right)\right) \tag{13}$$

Where, by Lemma 2.1, $z\prod_{k=1}^{1}(1+i_{d_k}) + x_{m_1}$,

$z\prod_{k=1}^{2}(1+i_{d_k}) + \sum_{k=1}^{1}(x_{m_k} - y_{n_k})\prod_{j=k+1}^{2}(1+i_{d_j}) + x_{m_1}, \ldots, z\prod_{k=1}^{t}(1+i_{d_k}) + \sum_{k=1}^{t}(x_{m_k} - y_{n_k})\prod_{j=k+1}^{t}(1+i_{d_j}) + x_{m_2}$ are integer

numbers and $0 < y_{m_1}, y_{m_2}, \ldots, y_{n_t} \le y_P$ then, we let

$$f_1 = max\left\{n_1 : y_{n_1} \le \min\left\{\left[z\prod_{k=1}^{1}(1+i_{d_k}) + x_{m_1}\right], y_P\right\}\right\},$$

$$f_2 = max\left\{n_2 : y_{n_2} \le \min\left\{\left[z\prod_{k=1}^{2}(1+i_{d_k}) + \sum_{k=1}^{1}(x_{m_k} - y_{n_k})\prod_{j=k+1}^{2}(1+i_{d_j}) + x_{m_2}\right], y_P\right\}\right\}$$

$$f_t = max\left\{ n_t : y_t \le \min\left\{ \left[z\prod_{k=1}^{t}(1+i_{d_k}) + \sum_{k=1}^{t-1}(x_{m_k} - y_{n_k})\prod_{j=k+1}^{t}(1+i_{d_j}) + x_{m_t} \right], y_P \right\} \right\}$$

and $[a]$ is integer part of a, Thus, (3.7) is defined as

$$D_{m_1m_2...m_t}^{d_1d_2...d_t} \overset{as}{=} \bigcup_{1\le n_1\le f_1}\bigcup_{1\le n_2\le f_2}...\bigcup_{1\le n_t\le f_t}\left\{ \left(Y_1 = y_{n_1}\right)\cap\left(Y_2 = y_{n_2}\right)\cap...\cap\left(Y_t = y_{n_t}\right) \right\} \quad (14)$$

Because $Y = \{Y_n\}_{n\ge1}$ is a Markov chain and it is homogeneous then

$$P\left[\left(Y_1 = y_{n_1}\right)\cap\left(Y_2 = y_{n_2}\right)\cap...\cap\left(Y_t = y_{n_t}\right)\right] = q_{n_1}q_{n_1n_2}...q_{n_{t-1}n_t}$$

In the other hand, because system of events $\left\{\left(Y_1 = y_{n_1}\right)\cap\left(Y_2 = y_{n_2}\right)\cap...\cap\left(Y_t = y_{n_t}\right)\right\}$ in (3.8) be incompatible then

$$P(D_{m_1m_2...m_t}^{d_1d_2...d_t}) = \sum_{1\le n_1\le f_1}\sum_{1\le n_2\le f_2}...\sum_{1\le n_t\le f_t} q_{n_1}q_{n_1n_2}...q_{n_{t-1}n_t} \quad (15)$$

Under the assumption that X, Y and I are sequences of independent random variables then $B_{d_1d_2...d_t}, C_{m_1m_2...m_t}, D_{m_1m_2...m_t}^{d_1d_2...d_t}$ are independent events. Simultaneously, system of events $\left\{B_{d_1d_2...d_t} \cap C_{m_1m_2...m_t} \cap D_{m_1m_2...m_t}^{d_1d_2...d_t}\right\}$ in (2.10) be incompatible. Therefore, combining (3.3), (3.4) and (3.9), we have.

$$\varphi_t(z) = P(A) = \sum_{d_1,d_2,...,d_t=1}^{Q}\left(\sum_{m_1,m_2,...,m_t=1}^{K} P\left\{B_{d_1d_2...d_t} \cap C_{m_1m_2...m_t} \cap D_{m_1m_2...m_t}^{d_1d_2...d_t}\right\}\right)$$

$$= \sum_{d_1,d_2,...,d_t=1}^{Q}\left(\sum_{m_1,m_2,...,m_t=1}^{K} P\left(B_{d_1d_2...d_t}\right).P\left(C_{m_1m_2...m_t}\right).P\left(D_{m_1m_2...m_t}^{d_1d_2...d_t}\right)\right)$$

$$= \sum_{d_1,d_2,...,d_t=1}^{Q}\sum_{m_1,m_2,...,m_t=1}^{K} r_{d_1}r_{d_1d_2}...r_{d_{t-1}d_t}\,p_{m_1}p_{m_1m_2}...p_{m_{t-1}m_t}\left(\sum_{1\le n_1\le f_1}\sum_{1\le n_2\le f_2}...\sum_{1\le n_t\le f_t} q_{n_1}q_{n_1n_2}...q_{n_{t-1}n_t}\right) \quad (15)$$

Theorem 3.1. Has been proved.

Corollary 3.1. Consider model (1.1) with assumption 2.1 to assumption 2.5, then finite time ruin probability of model (1.1) is defined as

$$\psi_t(z) = 1 - \varphi_t(z)$$

$$= 1 - \sum_{d_1,d_2,...,d_t=1}^{Q}\sum_{m_1,m_2,...,m_t=1}^{K} r_{d_1}r_{d_1d_2}...r_{d_{t-1}d_t}\,p_{m_1}p_{m_1m_2}...p_{m_{t-1}m_t}\left(\sum_{1\le n_1\le f_1}\sum_{1\le n_2\le f_2}...\sum_{1\le n_t\le f_t} q_{n_1}q_{n_1n_2}...q_{n_{t-1}n_t}\right) \quad (16)$$

Remark 3.1. Formula (3.11) allows to calculate the ruin probability of the model (1.1) assuming that the series of insurance premium, the series of insurance claim, and the series of interest rates are sequences of random variables that depend on Markov and receive the value positive integers.

Formula (3.1) allows to calculate the non-ruin probability of the model (1.1) assuming that the series of insurance premium, the series of insurance claim, the series of interest rates are sequences of random variables that depend on Markov and receive the value positive integers.

4. Conclusion

Using the tools of probability theory, we have built a formula to calculate the ruin probability (The non-ruin probability) of a general insurance model with a series of Markov dependent random variables and receive the following values: positive integers. Our main result in this paper is Theorem 3.1.

5. References

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