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Expansion of the polynomial set $S_n(x_1, x_2, x_3)$ in terms of Jacobi polynomials

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Abstract
In the present paper, an effort has been made to give Expansion of The Polynomial Set $S_n(x_1, x_2, x_3)$ in Terms of Jacobi Polynomials. Many interesting new results may be obtained on specializing the respective parameters in which some of them are believed to be new.
AMS Subject Classification: 33c.

Keywords: Jacobi polynomials, generalized, hypergeometric polynomial, generating relation

1. Introduction

We defined the classical generalized hypergeometric polynomial set $Sn(x1, x2, x3)$ by means of the generating relation.

$$(1-\mu t)^{-\nu} F\left[\begin{matrix}\mu_1;(a_g) \\ \lambda_1 x_2^{e_1} t^{e_1} \\ (b_h)\end{matrix}\right] F\left[\begin{matrix}(G_r);(A_p);(C_u)(\alpha_k) \\ \lambda x_1^{e_4} t, \lambda_2 x_2^{e_2} t^{e_2}, \lambda_3 x_3^{e_3} t^{e_3} \\ (H_s);(B_q);(D_\nu);(\beta_w)\end{matrix}\right]$$

$$= \sum_{n=0}^{\infty} S^{\nu;\mu;\mu_1;\lambda;\lambda_1;\lambda_2;\lambda_3;(a_g);(G_r);(A_p);(C_u);(\alpha_k);}_{n,e;e_1;e_2;e_3;e_4;(b_h);(H_s)(B_q);(D_\nu);(\beta_w)} \left(x_1, x_2, x_3\right) t^n \tag{1}$$

Where ν, μ, μ1, λ, λ1, λ2, λ3 are real and e, $e1$ and $e4$ are non-negative integer and $e2$, $e3$ are natural numbers.
The left-hand side of (1.1) contains the product of generalized hypergeometric function and Lauricella function in the notation of Burchnall and Chaundy [2]. The polynomial set contains number of parameters, for simplicity we shall denote

$$S^{\nu;\mu;\mu_1;\lambda;\lambda_1;\lambda_2;\lambda_3;(a_g);(G_r);(A_p);(C_u);(\alpha_k);}_{n,e;e_1;e_2;e_3;e_4;(b_h);(H_s)(B_q);(D_\nu);(\beta_w)}(x_1, x_2, x_3) \text{ by } S_n(x_1, x_2, x_3)$$

Where n is the order of the polynomial set.
After little simplification (1.1) gives,

$$S_n\left(x_1, x_2, x_3\right) = \sum_{m=0}^{\left[\frac{n}{e}\right]} \sum_{m_1=0}^{\left[\frac{n-em}{e_1}\right]} \sum_{m_2=0}^{\left[\frac{n-em-e_1m_1}{e_2}\right]} \sum_{m_3=0}^{\left[\frac{n-em-e_1m_1-e_2m_2-e_3m_3}{e_3}\right]}$$

$$\times\frac{\left[(G_r)\right]_{n-em-e_1m_1-(e_2-1)}}{\left[(H_s)\right]_{n-em-e_1m_1-(e_2-1)}}\times\frac{\left[(a_g)\right]_{m_1}\left[(C_u)\right]_{m_2}\left[(\alpha_k)\right](\nu)_m\,\mu^m(\mu_1)_{m_1}\lambda_1^{m_1}\lambda_2^{m_2}}{\left[(b_h)\right]_{m_1}\left[(D_\nu)\right]_{m_2}\left[(\beta_w)\right]_{m_3}\,m!\,m_1!\,x^{e_1m+e_2m_2}\,m_2!}$$

$$\times\frac{\left(\lambda_3x_3^{e_3}\right)^{m_3}\left(\lambda x_1^{e_4}\right)^{n-em-e_1m_1-e_2m_2-e_3m_3}}{m_3!\left(n-em-e_1m_1-e_2m_2-e_3m_3\right)!} \tag{2}$$

2. Notations

A. i) $n = 1, 2, 3, \ldots\ldots n-1, n$.

ii) $(ap) = a1, a2, a3, \ldots\ldots ap$.

iii) $[(ap)] = a1, a2, \ldots\ldots ap$.

iv) $\left[(a_p)\right]_n=\prod_{i=1}^{p}(a_i)_n=(a_1)_n(a_2)_n(a_3)_n\ldots\ldots(a_p)_n.$

B. i) $\Delta(a;b)=\frac{b}{a},\frac{b+1}{a},\frac{b+2}{a},\ldots\ldots\frac{b+a-1}{a}.$

ii) $\Delta_k\left[a(i);b\right]=\frac{b}{a},\frac{b+1}{a},\ldots\ldots\frac{b+a-2}{a}$

iii) $\Delta_k\left[m;(a_p)\right]=\left(\frac{a_i+r-1}{m}\right)_k \quad \begin{matrix} r=1,2\ldots\ldots m\\ i=1,2,\ldots\ldots p\end{matrix}$

iv) $\Delta(a;\, b\pm c\pm d)=\Delta(a;\, b+c+d)\,\Delta(a;\, b+c-d)$, $\Delta(a;\, b-c+d)$, $\Delta(a;\, b-c-d)$.

C. i) $\Delta_k\left[a,b\right]=\prod_{r=1}^{a}\left(\frac{b+r-1}{a}\right)_k=\left(\frac{b}{a}\right)_k,\left(\frac{b+1}{a}\right)_k\ldots\ldots\left(\frac{b+a-2}{a}\right)_k$

ii) $\Delta_k\left[m;(a_p)\right]=\prod_{i=1}^{p}\prod_{r=1}^{m}\left(\frac{a;+r-1}{m}\right)_k.$

D. i) $\Gamma\left[(a_p)\right]=\prod_{i=1}^{p}\Gamma(a_i).$

ii) $\Gamma\left[a+\frac{(m)}{m}\right]=\prod_{r=1}^{m}\Gamma\left(a+\frac{r}{m}\right).$

iii) $\Gamma\left[(a;b)\right]=\prod_{r=1}^{a}\Gamma\left(\frac{b+r-1}{a}\right).$

vi) $\Gamma\left[\Delta\left(m;(a_p)\right)\right]=\prod_{i=1}^{p}\prod_{r=1}^{m}\left(\frac{a_i+r-1}{m}\right).$

E. i) $\Gamma_*(a\pm b)=\Gamma(a+b)\Gamma(a-b).$

ii) $\Gamma_{**}(a+1)=\Gamma(a+b)\Gamma(a-b).$

$$M_1=\frac{\left[(G_r)\right]_n\left[(A_p)\right]_n\lambda^n}{\left[(H_s)\right]_n\left[(B_q)\right]_n n!}$$

3. *Sn*(*x*1, *x*2, *x*3) in Terms of Jacobi Polynomials

We have from (1.2)

$$\sum_{n=0}^{\infty}S_n(x_1,x_2,x_3)=\sum_{n=0}^{\infty}\sum_{m,m_1,m_2,m_3=0}^{\infty}\frac{\left[(G_r)\right]_{n+m_2+m_3}\left[(A_p)\right]_n\left[(a_g)\right]_{m_1}}{\left[(H_s)\right]_{n+m_2+m_3}\left[(B_q)\right]_n\left[(b_h)\right]_{m_1}}$$

$$\times \frac{\left[(C_u)\right]_{m_2} \left[(\alpha_k)\right]_{m_3} (\nu)_m \, \mu^m \, (\mu_1)_m \, \lambda_1^{m_1} \, \lambda^n \left(x_1^{e_4}\right)^n \lambda_2^{m_2} \, \left(\lambda_3 x_2^{e_3}\right)^{m_3} t^{n+m+m_1+m_2+m_3}}{\left[(D_v)\right]_{m_2} \left[(\beta_w)\right]_{m_3} \, m! \, \, m_1! \, x_2^{e_1 m_1 + e_2 m_2} m_2! \, \, m_3!} \tag{3}$$

We have from [1]

$$\left(x_1^{e_4}\right)^n = n!(n+c)! \sum_{i=0}^{n} \frac{(-1)^i \, (2i+c+d+1) P_i^{(c,\alpha)} \left(1+2x_1^{e_4}\right)}{(n-i)! \qquad (c+d+i+1)_{n+1}}$$

Hence, (3.1) can be thrown in to the form

$$\sum_{n=0}^{\infty} S_n \left(x_1, x_2, x_3\right) = \sum_{n=0}^{\infty} \sum_{m,m_1,m_2,m_3=0}^{\infty} \sum_{i=0}^{n} \frac{\left[(G_r)\right]_{n+m_2+m_3}}{\left[(H_s)\right]_{n+m_2+m_3}}$$

$$\times \frac{\left[(A_p)\right]_n \left[(a_g)\right]_{m_1} \left[(C_u)\right]_{m_2} \left[(\alpha_k)\right]_{m_3} (\nu)_m \, \mu^m \, (\mu_1)_m \, \lambda_1^{m_1}}{\left[(B_q)\right]_n \left[(b_h)\right]_{m_1} \left[(B_v)\right]_{m_2} \left[(\beta_w)\right]_{m_3} \, m! \, \, m_1! \, x_2^{e_1 m_1 + e_2 m_2}}$$

$$\times \frac{\lambda_2^{m_2} \left(\lambda_3 x_3^{e_3}\right)^{m_3} (-1)^i \, (2i+c+d+1) P_i^{(c,\alpha)} \left(1+2x_1^{e_4}\right) t^{n+m+m_1+m_2+m_3}}{m_2! \qquad m_3! \qquad (n-i)! \qquad (c+d+i+1)_{n+1}}$$

$$= \sum_{n=0}^{\infty} \sum_{i=0}^{n} \sum_{m=0}^{\left[\frac{n}{e}\right]} \sum_{m_1=0}^{\left[\frac{n-em}{e_1}\right]} \sum_{m_2=0}^{\left[\frac{n-em-e_1 m_1}{e_2}\right]} \sum_{m_3=0}^{\left[\frac{n-em-e_1 m_1 - e_2 m_2}{e_3}\right]}$$

$$\times \frac{\left[(G_r)\right]_{n-em-e_1 m_1 -(e_2-1)m_2-(e_3-1)m_3} \left[(A_p)\right]_{n-em-e_1 m_1 - e_2 m_2 - e_3 m_3}}{\left[(H_s)\right]_{n-em-e_1 m_1 -(e_2-1)m_2-(e_3-1)m_3} \left[(B_q)\right]_{n-em-e_1 m_1 - e_2 m_2 - e_3 m_3}}$$

$$\times \frac{\left[(a_g)\right]_{m_1} \left[(C_u)\right]_{m_2} \left[(\alpha_k)\right]_{m_3} (\nu)_m \, (\mu_1)_{m_1} \, \mu^m \, \lambda_1^{m_1}}{\left[(b_h)\right]_{m_1} \left[(D_v)\right]_{m_2} \left[(\beta_w)\right]_{m_3} \, m! \, m_1! \, x^{e_1 m + e_2 m_2}}$$

$$\times \frac{\lambda_2^{m_2} \quad \left(\lambda_3 x_3^{e_3}\right)^{m_3} \, \lambda^{n-em-e_1 m_1 - e_2 m_2 - e_3 m_3} \, (-1)^i}{m_2! \quad m_3! \, \left(n-i-em-e_1 m_1 - e_2 m_2 - e_3 m_3\right)!}$$

$$\times \frac{(2i+c+d+1)\left(n+c-em-e_1 m_1 - e_2 m_2 - e_3 m_3\right)! \, P_i^{(c,\alpha)} \left(1+2x_1^{e_4}\right)}{(c+d+i+1)_{n+1-em-e_1 m_1 - e_2 m_2 - e_3 m_3}} t^n \tag{4}$$

Equating the co-efficient of *tn* from both sides in (3.2), and after little simplification, we finally achieve (For *e*2 > 1, *e*3 > 1)

$$S_n(x_1,x_2,x_3)=M_1\sum_{i=0}^{n}\sum_{m,m_1,m_2,m_3=0}^{\infty}\frac{(n+c)!(-1)^i(2i+c+d+1)}{(n-i)!(c+d++i+1)_{n+1}}$$

$$\times P_i^{(c,\alpha)}\left(1+2x_1^{e_4}\right)\frac{\left[1-(H_s)-n\right]_{em+e_1m_1+(e_2-1)m_2+(e_3-1)m_3}}{\left[1-(G_r)-n\right]_{em+e_1m_1+(e_2-1)m_2+(e_3-1)m_3}}$$

$$\times\frac{\left[1-(B_q)-n\right]_{em+e_1m_1+e_2m_2+e_3m_3}\left[(a_g)\right]_{m_1}\left[(C_u)\right]_{m_2}\left[(\alpha_k)\right]_{m_3}}{\left[1-(A_p)-n\right]_{em+e_1m_1+e_2m_2+e_3m_3}\left[(b_h)\right]_{m_1}\left[(D_v)\right]_{m_2}\left[(\beta_w)\right]_{m_3}}$$

$$\times\frac{(\nu)_m(\mu_1)_m\,\mu^m\,\lambda_1^{m_1}\,\lambda_2^{m_2}\left(\lambda_3x_3^{e_3}\right)^{m_3}}{m!\,m_1!\,m_2!\,x_2^{e_1m_1+e_2m_2}\,m_3!\,\lambda^{em+e_1m_1+e_2m_2+e_3m_3}}$$

$$\times\frac{(-n+i)_{em+e_1m_1+e_2m_2+e_3m_3}(-c-d-i-n+1)_{em+e_1m_1+e_2m_2+e_3m_3}}{(-n-c)_{em+e_1m_1+e_2m_2+e_3m_3}}$$

$$\times(-1)^{e(r+s+p+q+1)^m}(-1)^{e_1(r+s+p+q+1)^{m_1}}(-1)^{\{e_2(r+s+p+q+1)+r+s\}^{m_2}}(-1)^{\{e_3(r+s+p+q+1)+r+s\}^{m_3}} \quad (5)$$

Where *c* is non-integer.a

The single terminating facter $(-n+i)_{em+e_1m_1+e_2m_2+e_3m_3}$ makes all summation in (5) runs up to ∞.

Hence

Corollary I: For *e*2 > 1 and *e*3 > 1, we have

$$S_n(x_1,x_2,x_3)=M_1\sum_{i=0}^{n}\frac{(n+c)!(-1)^n(2i+c+d+1)P_i^{(c,d)}\left(1+2x_1^{e_4}\right)}{(n-i)!\;(c+d+i+1)_{n+1}}$$

$$\times F^{1+q+s:g:u:k}_{p+r:h:v:w}\left[\begin{matrix}\left[(-n+i):e,e_1,e_2,e_3\right]\left[(-c-d-i-n-1):e:e_1,e_2,e_3\right]\\ \left[(-n-c):e,e_1,e_2,e_3\right]\qquad\rule{4em}{0.4pt}\end{matrix}\right.$$

$$\begin{matrix}\left[(1-(H_s)-n);e,e_1,e_2-1,e_3-1\right],\left[(1-(B_q)-n);e,e_1,e_2,e_3)\right]\left[(a_g):1\right]\\ \left[(1-(G_r)-n);e,e_1,e_2-1,e_3-1\right],\left[1-(A_p)-n);e,e_1,e_2,e_3\right]\left[(b_q):1\right]\end{matrix}$$

$$\begin{matrix}\left[(C_u):1\right],\left[(\alpha_k):1\right],\left[\nu:1\right];\left[\mu_1:1\right]\\ \left[(D_v):1\right],\left[(\beta_w):1\right];\end{matrix};\frac{\mu(-1)^{e(r+s+p+q+1)}}{\lambda^e},\frac{\lambda_1(-1)^{e_1(r+s+p+q+1)}}{(\lambda x_2)^{e_1}},$$

$$\left.\frac{\lambda_2(-1)^{e_2(r+s+p+q+1)+r+s}}{(\lambda x_2)^{e_2}},\frac{\lambda_3(-1)^{e_3(r+s+p+q+1)+r+s}\,x^{e_3}}{\lambda^{e_3}}\right] \quad \ldots (3.4)$$

Corollary II: For *e*2 = 1 and *e*3 > 1, we arrive at

$$S_n\left(x_1,x_2,x_3\right)=M_1\sum_{i=0}^{n}\frac{(n+c)!(-1)^n\,(2i+c+d+1)P_i^{(c,d)}\left(1+2x_1^{e_4}\right)}{(n-i)!\;(c+d+i+1)_{n+1}}$$

$$\times F_{p+r:h:\text{v}:w}^{1+q+s:g:u:k}\left[\begin{array}{l}\left[(-n+i):e,e_1,1,e_3\right]\left[(-c-d-i-n-1):e:e_1,1,e_3\right]\\\left[(-n-c):e,e_1,1,e_3\right]\end{array}\right.$$

$$\left[(1-(H_s)-n);e,e_1,0,e_3-1\right],\left[\left(1-\left(B_q\right)-n);e,e_1,1,e_3\right)\right]\left[\left(a_g\right):1\right]$$
$$\left[(1-(G_r)-n);e,e_1,0,e_3-1\right],\left[1-\left(A_p\right)-n);e,e_1,1,e_3\right]\left[\left(b_q\right):1\right]$$

$$\begin{array}{l}\left[(C_u):1\right],\left[(\alpha_k):1\right],[\text{v}:1];[\lambda_1:1]\\\left[(D_\text{v}):1\right],\left[(\beta_w):1\right];\end{array};\frac{\mu(-1)^{e(r+s+p+q+1)}}{\lambda^e},\frac{\lambda_1(-1)^{e_1(r+s+p+q+1)}}{(\lambda x_2)^{e_1}},$$

$$\left.\frac{\lambda_2(-1)^{2(r+s)+p+q+1}}{(\lambda x_2)^{e_2}},\frac{\lambda_3x_3^{e_3}(-1)^{e_3(r+s+p+q+1)+r+s}}{\lambda^{e_3}}\right]\tag{6}$$

Corollary III: For $e2 > 1$ and $e3 = 1$, we achieve

$$S_n\left(x_1,x_2,x_3\right)=M_1\sum_{i=0}^{n}\frac{(n+c)!(-1)^n\,(2i+c+d+1)P_i^{(c,d)}\left(1+2x_1^{e_4}\right)}{(n-i)!\;(c+d+i+1)_{n+1}}$$

$$\times F_{p+r:h:\text{v}:w}^{1+q+s:g:u:k}\left[\begin{array}{l}\left[(-n+i):e,e_1,e_2,1\right]\left[(-c-d-i-n-1):e:e_1,e_2,1\right]\\\left[(-n-c):e,e_1,e_2,1\right]\end{array}\right.$$

$$\left[(1-(H_s)-n);e,e_1,e_2-1,0\right],\left[\left(1-\left(B_q\right)-n);e,e_1,e_2,1\right)\right]\left[\left(a_g\right):1\right]$$
$$\left[(1-(G_r)-n);e,e_1,e_2-1,0\right],\left[1-\left(A_p\right)-n);e,e_1,e_2,1\right]\left[\left(b_q\right):1\right]$$

$$\begin{array}{l}\left[(C_u):1\right],\left[(\alpha_k):1\right],[\text{v}:1];[\lambda_1:1]\\\left[(D_\text{v}):1\right],\left[(\beta_w):1\right];\end{array};\frac{\mu(-1)^{e(r+s+p+q+1)}}{\lambda^e},\frac{\lambda_1(-1)^{e_1(r+s+p+q+1)}}{(\lambda x_2)^{e_1}},$$

$$\left.\frac{\lambda_2(-1)^{e_2(r+s+p+q+1)+r+s}}{(\lambda x_2)^{e_2}},\frac{\lambda_3x_3^{e_3}(-1)^{2(r+s)+p+q+1}}{\lambda}\right]$$

$$S_n\left(x_1,x_2,x_3\right)=M_1\sum_{i=0}^{n}\frac{(n+c)!(-1)^n\,(2i+c+d+1)P_i^{(c,d)}\left(1+2x_1^{e_4}\right)}{(n-i)!\;(c+d+i+1)_{n+1}}$$

$$\times F_{p+r:h:\text{v}:w}^{1+q+s:g:u:k}\left[\begin{array}{l}\left[(-n+i):e,e_1,1,1\right]\left[(-c-d-i-n-1):e:e_1,1,1\right]\\\left[(-n-c):e,e_1,1,1\right]\end{array}\right.$$

$$\left[(1-(H_s)-n);e,e_1,0,0\right],\left[\left(1-\left(B_q\right)-n);e,e_1,1,1\right)\right]\left[\left(a_g\right):1\right]$$
$$\left[(1-(G_r)-n);e,e_1,0,0\right],\left[1-\left(A_p\right)-n);e,e_1,1,1\right]\left[\left(b_q\right):1\right]$$

$$\left[(C_u):1\right],\left[(\alpha_k):1\right],[\nu:1];[\lambda_1:1] ; \frac{\mu(-1)^{e(r+s+p+q+1)}}{\lambda^e}, \frac{\lambda_1(-1)^{e_1(r+s+p+q+1)}}{(\lambda x_2)^{e_1}},$$
$$\left[(D_\nu):1\right],\left[(\beta_w):1\right];$$

$$\left.\frac{\lambda_2(-1)^{2(r+s)+p+q+1}}{\lambda x_2}, \frac{\lambda_3 x_3(-1)^{2(r+s+p+q+1)}}{\lambda}\right] \tag{7}$$

Corollary IV: For $e2 = 1$ and $e3 = 1$, we obtain

$$\times F^{1+q+s:g:u:k}_{p+r:h:\nu:w}\left[\begin{array}{l}\left[(-n+i):e,e_1,1,1\right]\left[(-c-d-i-n-1):e:e_1,1,1\right] \\ \left[(-n-c):e,e_1,1,1\right] \end{array}\right.$$

$$\left[(1-(H_s)-n);e,e_1,0,0\right],\left[(1-(B_q)-n);e,e_1,1,1)\right]\left[(a_g):1\right]$$
$$\left[(1-(G_r)-n);e,e_1,0,0\right],\left[1-(A_p)-n);e,e_1,1,1\right]\left[(b_q):1\right]$$

$$\left[(C_u):1\right],\left[(\alpha_k):1\right],[\nu:1];[\lambda_1:1] ; \frac{\mu(-1)^{e(r+s+p+q+1)}}{\lambda^e}, \frac{\lambda_1(-1)^{e_1(r+s+p+q+1)}}{(\lambda x_2)^{e_1}},$$
$$\left[(D_\nu):1\right],\left[(\beta_w):1\right];$$

$$\left.\frac{\lambda_2(-1)^{2(r+s)+p+q+1}}{\lambda x_2}, \frac{\lambda_3 x_3(-1)^{2(r+s+p+q+1)}}{\lambda}\right] \tag{8}$$

Particular Cases of (3.4)

i) On taking $r = 0 = s = p = q = g = h$; $\mu 1 = 1 + \lambda 2$; $\lambda = 1 = e = e4 = x2$; $\lambda 1 = -1$ and writing $x1 = y1$, in (3.4), we arrive at

$$A_n^{\lambda_2}(y) = \frac{y^n}{n!} M_1 \sum_{i=0}^{n} \frac{(n+c)!(-1)^n (2i+c+d+1) P_i^{(c,d)}\left(1+2x_1^{e_4}\right)}{(n-i)!\ (c+d+i+1)_{n+1}}$$

$$\times F\left[\begin{array}{ll} -n+i,-c-d-i-n-1,-n,\lambda_2+1; & \\ & \frac{1}{y} \\ -n-c; & \end{array}\right]$$

Where $A_n^{\lambda_2}(y)$ are the Srivastava Polynomials [4]

ii) On putting $r = 0 = s = p = g$; $q = 1 = h = e = e4 = \nu = \mu 1$; $\lambda = \frac{1}{2} = \lambda_1$; $\beta 1 = 1 + \beta 1$; $b1 = 1 + \alpha$; and $x_1 = \frac{x+1}{x-1}$ in (3.4), we get

$$P_n^{(\alpha,\beta)}(x) = \frac{(1+\alpha)_n \left(\frac{x+1}{2}\right)^n}{n!}$$

$$\times M_1 \sum_{i=0}^{n} \frac{(n+c)!(-1)^i (2i+c+d+1) P_i^{(c,d)}\left(1+2x_1^{e_4}\right)}{(n-i)! \qquad (c+d+i+1)_{n+1}}$$

$$\times F\left[\begin{matrix}-n,-\beta-n,-n+i-c-d-i-n-1; & \\ & \frac{x-1}{x+1} \\ -n-c,1+\alpha; & \end{matrix}\right]$$

Where $P_n^{(\alpha,\beta)}(x)$ are the Jacobi Polynomials.

iii) On taking $r = 0 = s = p = g$; $q = 1 = h = e = e4 = \nu = \mu1$; $x2 = 1$; $\lambda=\frac{1}{2}=\lambda_1$; $\beta1 = 1 + \alpha$; $b1 = 1 + \beta$ and $x_1=\frac{x-1}{x+1}$ in (3.4), we achieve

$$P_n^{(\alpha,\beta)}(x)=\frac{(1+\alpha)_n\left(\frac{x+1}{2}\right)^n}{n!}$$

$$\times M_1\sum_{i=0}^{n}\frac{(n+c)!(-1)^i(2i+c+d+1)P_i^{(c,d)}\left(1+2x_1^{e_4}\right)}{(n-i)!\qquad(c+d+i+1)_{n+1}}$$

$$\times F\left[\begin{matrix}-n,-\alpha-n,-n+i-c-d-i-n-1; & \\ & \frac{x+1}{x-1} \\ 1+\beta,-n-c; & \end{matrix}\right]$$

Where $P_n^{(\alpha,\beta)}(x)$ are Jacobi Polynomials.

iv) For $r = 0 = s = p = q$; $h = 1$, or 2, $b1 = 1$, $b2 = 1 + \alpha$; $x2 = g = 1 = e = e4 = \nu = \mu1$; $a1 = 1 + \beta$, and writing $\frac{1}{x}$ for $x1$, we get

$$\phi_n(x)=\frac{\alpha^{-n}}{n!}\sum_{i=0}^{n}\frac{(n+c)!(-1)^i(2i+c+d+1)P_i^{(c,d)}\left(1+2x_1^{e_4}\right)}{(n-i)!\qquad(c+d+i+1)_{n+1}}$$

$$\times F\left[\begin{matrix}-n,1+\lambda,-n+i-c-d-i-n-1; & \\ & x \\ -n-c;1,1+\alpha; & \end{matrix}\right]$$

Where $\phi_n(x)$ [3] reduces to Simple Laguerre Polynomials for $\alpha = \beta$.

v) On making the substitution $p = 0 = q = r = s = g$; $h = \{1, 2\} = b1 = 1 + \alpha$, $b2 = 1 + \beta$; $e4 = 1 = e = n = \mu1 = 2F = e1 = \lambda = \lambda1$; and writing $\frac{1}{x}$ for $x1$, we achieve

$$f_n(x)=\frac{x^{-n}}{n!}\sum_{i=0}^{n}\frac{(n+c)!(-1)^i(2i+c+d+1)P_i^{(c,d)}\left(1+2x_1^{e_4}\right)}{(n-i)!\qquad(c+d+i+1)_{n+1}}$$

$$\times F\left[\begin{matrix}-n+i,-c-d-i-n-1; & \\ & x \\ -n-c;1,1+\alpha,1+\beta; & \end{matrix}\right]$$

Where $fn(x)$ are [3]

vi) On taking $r = 0 = s = p = g$; $q = 1 = h = \nu = \mu1 = x2 = e = e2 = e4$; $\lambda=\frac{1}{2}=\lambda_1$, $\beta_1=\lambda+\frac{1}{2}=b_1$; and $\frac{x+1}{x-1}$ for $x1$ in (5), we have

$$C_n^{\lambda}(x)=\frac{(2\lambda)_n\left(\frac{x+1}{2}\right)}{n!}\sum_{i=0}^{n}\frac{(n+c)!(-1)^i\,(2i+c+d+1)\,P_i^{(c,d)}\left(1+2x_1^{e_4}\right)}{(n-i)!\qquad(c+d+i+1)_{n+1}}$$

$$\times F\left[\begin{matrix}-n,-n+i,\frac{1}{2}-\lambda-n-c-d-i-n-1; & \\ & \frac{x-1}{x+1}\\ -n-c;\lambda+\frac{1}{2}; & \end{matrix}\right]$$

Where $C_n^{\lambda}(x)$ are the Gegenbauer Polynomials.

vii) For taking $r = 0 = s = p = q = g$; h = {1, 2} $b1 = u + 1$, $b_2 = \text{v}+1+\frac{u}{2}$, $e1 = e = \nu = \mu 1 = \lambda = x2$; $e4 = 2$; $\lambda 1 = -1$ and $x_1 = \frac{1}{z}$, we get

$$J_n^{(u,\text{v})}(z)=\frac{\Gamma\left(\text{v}+n+1+\frac{\mu}{2}\right)z^u}{\Gamma(u+1)\Gamma\left(v+1+\frac{u}{2}\right)}$$

$$\times\sum_{i=0}^{n}\frac{(n+c)!(-1)^i\,(2i+c+d+1)\,P_i^{(c,d)}\left(1+2x_1^{e_4}\right)}{(n-i)!\qquad(c+d+i+1)_{n+1}}$$

$$\times F\left[\begin{matrix}-n,-n+i,-n-c-d-i-n-1; & \\ & z^2\\ -n-c;u+1,\text{v}+1+\frac{u}{2}; & \end{matrix}\right]$$

Where $J_n^{(u,\text{v})}(z)$ are the Bateman Polynomials.

viii) If we set $r = 0 = s = g$; $p = 1 = g = h = \nu = \mu 1 = e4 = e = e1 = \lambda 1 = x2$; $\lambda = -1$ and $\beta 1 = c = b1$; $A1 = 1$ and writing $x_1 = \frac{1}{y}$, we arrive at

$$\Psi_n(c,-1,y)=\frac{(-1)^n\left(\frac{x+1}{2}\right)^n}{n!\qquad(c)_n}$$

$$\times\sum_{i=0}^{n}\frac{(n+c)!(-1)^i\,(2i+c+d+1)\,P_i^{(c,d)}\left(1+2x_1^{e_4}\right)}{(n-i)!\qquad(c+d+i+1)_{n+1}}$$

$$\times F\left[\begin{matrix}-n,-n+i,-c-d-i-n-1,1-c-n; & \\ & y\\ -n-c,-n,c; & \end{matrix}\right]$$

Where $\Psi n(c, -1, y)$ are the Bateman's Polynomials.

3. Reference

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