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# An empirical assessment of asset value function for capital market price changes 

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#### Abstract

The stochastic analysis on the valuation of asset value function was examined in this paper; where multiplicative inverse effects, additive effects, additive inverse effects and multiplicative effects were used extensively in the model to obtain stock rate of returns. The problem was solved analytical by adopting Ito's theorem which the analytical closed form solution showed: (i) increase in stock market prices, increases the assets values. (ii) a multiplicative inverse stock return is the best in terms of precision and decision making. (iii) a little increase in stock volatility also increases the value of asset.(iv) additive inverse effects has the highest rate of returns throughout the trading days which informs an investors on proper way based on decision making. To this end, the validity of analytical solutions was clearly confirmed using empirical stock prices.


Keywords: Stock prices, stochastic analysis, asset value, return rates and SDE

## 1. Introduction

Funds are been invested in businesses so that the return rates will be suitably used to cover for the routine transacting cost or expenditure in capital assets. In order to improve its transacting events, an investor may need extra capital investments. Therefore, capital assets are set up on the basis of getting extra capital investments for growth which allows the business to increase in unit production, generate modern concepts on products or even introduce values to the business and to examine technological upgrades so as to enhance productiveness and lessen expenditures and to supplement the debilitated assets. Trading business, in the absence of capital assets, will certainly have challenging time getting off the ground. The outcome of stock market activities and performances has increased substantially in the growth of financial market as businessmen and businesswomen secure their usual incomes to secure a livelihood.
Nevertheless, asset transaction is usually noted for its considerable revenues. Returns with respect to investments for stock market are a means of appropriately assessing profit of an investment. It is the main parameter frequently utilized by investors to manage the usefulness of expenditure ${ }^{[1]}$. The dynamics of stock prices in the stock market are reproduced by unpredictable flow or movement of their value over time. For this reason, it is better to have an apt comprehension of the type of physical quantities to model so as to have pragmatic outcomes of the stock variables. For example ${ }^{[2]}$, studied on the stochastic model of the changes or variations of stock market prices. Conditions for finding out the market clarity price, adequate circumstances for robust steadiness and convergence to stability of the growth rate of the value function of equities. Furthermore ${ }^{[3]}$, worked on stochastic model of price changes on the floor of stock market. Hitherto, the equilibrium price and the market growth rates of shares were determined ${ }^{[4]}$. Considered the stochastic analysis of Markov Chain in Finite state. The transition Matrix simulated the application of 3-State transition probability matrix which allows them to present exact condition of getting predicted mean rate of returns of specific asset.
Similarly ${ }^{[5]}$, looked at the stochastic analysis of stock market expected returns and GrowthRates. The exact conditions for obtaining the drifts, volatilities, Growth-Rates of four different stocks were also considered ${ }^{[6]}$. Worked on the problem of share value changes applying stochastic differential equations, principal component analysis and KS goodness of fit test were studied.

Details of the analytical solutions were given; the computational and graphical results were presented and discussed respectively ${ }^{[7]}$. Looked at a combination of deterministic and stochastic systems with its random parameters in the model. A detailed presentation of the analytical solution to the proposed model was made, which determined the insurance quantities or variables.
Nonetheless ${ }^{[8]}$, studied the fluctuating characteristics of stock market effects applying suggested differential equation model. The research in ${ }^{[9]}$ considered stability analysis of stochastic model of price fluctuations on the stock exchange floor. Precise conditions in this analysis were derived, which determined the price stability and growth-rate of stock shares.
The research in ${ }^{[10]}$ looked at stochastic analysis of the behavior of asset's market values. Outcomes show an efficiency in predicting asset market values, using the proposed model. Likewise ${ }^{[11]}$, considered the stochastic model of some selected assets in the Nigerian Stock Exchange (NSE), this study deduced the coefficients of the drift and volatility for the stochastic differential equations ${ }^{[12]}$, built the geometric Brownian Motion and studied the accuracy of the model with detailed analysis of simulated data ${ }^{[13]}$. Worked on the solution of differential equations and stochastic differential equations of time-varying investment returns; where specific conditions were gotten which governs stock return rates by way of multiplicative effect and multiplicative inverse trends series.
This paper, we considered empirically four stochastic investment systems where we assumed stock return to follow multiplicative , multiplicative inverse, additive and additive inverse effects was used extensively in the model. These equations were solved and four different solutions obtained. So, applying initial stock price and other parameter values into the solutions; the effect of relevant parameters and value of asset prices were obtained. However, as far as we are aware, this is the first study that has studied empirically the stock asset value function which was not considered by the previous efforts.
Therefore, this paper is arranged as follows: Section 2.1 presents formulation of the problem, Results and Discussion are seen in Section 3.1, while the paper is concluded in Section 4.1.

## 2. Formulation of the Problem

we assumed the company's rate of returns to follow multiplicative inverse effects, additive effects, additive inverse effects and multiplicative effects; all having quadratic functions over time. Hence, this rate of return is defined in the system of stochastic differential equations of $(2-5)$ below:
Therefore, the stochastic process illustrating the process is of the form:
$d S(t)=\theta d t+\sigma d Z^{(1)}(t)$
Where $\theta$ is the predicted rate of returns on asset, $\sigma$ is the volatility of the asset, $d t$ is the relative variation in the price during the period of time and $Z^{(1)}$ is a Wiener process.
$d S_{K_{1}}(t)=\left\{\left(\theta_{1} \theta_{2}\right)^{-1}\right\}^{2} S_{K_{1}}(t) d t+\sigma S_{K_{1}}(t) d Z^{(1)}(t)$
$d S_{K_{2}}(t)=\left(\theta_{1}+\theta_{2}\right)^{2} S_{K_{2}}(t) d t+\sigma S_{K_{2}}(t) d Z^{(2)}(t)$
$d S_{K_{3}}(t)=\left\{\left(\theta_{1}+\theta_{2}\right)^{-1}\right\}^{2} S_{K_{3}}(t) d t+\sigma S_{K_{3}}(t) d Z^{(3)}(t)$
$d S_{K_{4}}(t)=\left(\theta_{1} \theta_{2}\right)^{2} S_{K_{4}}(t) d t+\beta S_{K_{4}}(t) d Z^{(4)}(t)$
Where $S_{K_{1}}(t), S_{K_{2}}(t), S_{K_{3}}(t)$ and $S_{K_{4}}(t)$ are basic assets with initial conditions as follows:
$S_{K_{1}}(0)=S_{0\left(K_{1}\right)}, t>0$
$S_{K_{2}}(0)=S_{0\left(K_{2}\right)}, t>0$
$S_{K_{3}}(0)=S_{0_{\left(K_{3}\right)}}, t>0$
$\left.S_{K_{4}}(0)=S_{0\left(K_{4}\right)}\right), t>0$

All the same, the price evolution of equity capitals are generally modeled as the trajectory of a risky assets that are typically of a diffusion process defined on some underlying probability space, with the geometric Brownian motion, the main tool used as the established reference model ${ }^{[13]}$.

Theorem 1.1: (Ito's formula) Let $(\Omega, \beta, \mu, F(\beta)$ be a filtered probability space $X=\{X, t \geq 0\}$ be an adaptive stochastic process on ( $\Omega, \beta, \alpha, F(\beta)$ processing a quadratic variation $(X)$ with $\operatorname{SDE}$ defined as:
$d X(t)=g(t, X(t)) d t+f(t, X(t)) d W(t)$
$t \in \mathfrak{R}$ and for $u=u\left(t, X(t) \in C^{1 \times 2}(\Pi \times \mathbb{R})\right)$
$d u(t, X(t))=\left\{\frac{\partial u}{\partial t}+g \frac{\partial u}{\partial x}+\frac{1}{2} f^{2} \frac{\partial^{2} u}{\partial x^{2}}\right\} d \tau+f \frac{\partial u}{\partial x} d W(t)$

## Method of Solution

The investment equation (2)-(5), comprise of a system of stochastic differential equations and whose solutions are not trivial. We applying Theorem 1.1 solving for $S_{K_{1}}(t), S_{K_{2}}(t), S_{K_{3}}(t)$, and $S_{K_{4}}(t)$ To grab this problem, we would like to point out that;
$S_{K_{1}}(t), S_{K_{2}}(t), S_{K_{3}}(t)$ and $\quad S_{K_{4}}(t)<\infty$ forall $S \in[0,1)$
From (2), Thus, generalizing and considering a function $f\left(S_{K_{1}}(t), t\right)$, hence, it relates to partial derivatives. Expansion of $f\left(S_{K_{1}}(t), d S_{K_{1}}(t), t+d t\right)$ in a Taylor series about $\left(S_{K_{1}}(t), t\right)$ gives
$d f=\frac{\partial f}{\partial S_{K_{1}}(t)} d S_{K_{1}}(t)+\frac{\partial f}{\partial t} d t+\frac{1}{2} \frac{\partial^{2} f}{\partial S_{K_{1}}{ }^{2}(t)} d S_{K_{1}}^{2}(t)+\ldots$
Substituting (2) in (10), gives
$d f=\frac{\partial f}{\partial S_{K_{1}}(t)}\left\{\sigma S_{K_{1}}(t) d Z^{(1)}(t)+\left\{\left(\theta_{1} \theta_{2}\right)^{-1}\right\}^{2} S_{K_{1}}(t) d t \quad\right\}+\frac{1}{2} \frac{\partial^{2} f}{\partial S_{K_{1}}{ }^{2}(t)} d S_{K_{1}}^{2}(t) d S_{K_{1}}^{2}(t)$
$d f=\sigma S_{K_{1}}(t) \frac{\partial f}{\partial S_{K_{1}}(t)} d Z^{(1)}(t)+\left(\left\{\left(\theta_{1} \theta_{2}\right)^{-1}\right\}^{2} S_{K_{1}}(t) \frac{\partial f}{\partial S_{K_{1}}(t)}+\frac{1}{2} \sigma^{2} S_{K_{1}}^{2}(t) \frac{\partial^{2} f}{\partial S_{K_{1}}{ }^{2}(t)} d S_{K_{1}}^{2}(t)\right) d t$

Taking into account the SDE in (2),
$\operatorname{Let} f\left(S_{K_{1}}(t)\right)=\ln S_{K_{1}}(t)$, the partialderivativesbecomes
$\frac{\partial f}{\partial S_{K_{1}}(t)}=\frac{1}{S_{K_{1}}(t)}, \frac{\partial^{2} f}{\partial S_{K_{1}}^{2}(t)}=-\frac{1}{S_{K_{1}}^{2}(t)}, \frac{\partial f}{\partial t}=0$
following theorem 1.1(Ito's), substituting (1.14) and simplifying gives
$d f=\left\{\left\{\left(\theta_{1} \theta_{2}\right)^{-1}\right\}^{2}-\frac{1}{2} \sigma^{2}\right\} d t+\sigma d Z^{(1)}$
Because the RHS of (12) is independent of $f\left(S_{K_{1}}(t)\right)$,the stochastic is determined as follows
$f\left(S_{K_{1}}(t)=f_{0}+\int_{0}^{t}\left\{\left\{\left(\theta_{1} \theta_{2}\right)^{-1}\right\}^{2}-\frac{1}{2} \sigma^{2} \quad\right\} d t+\int_{0}^{t} \sigma d Z^{(1)}(t)\right.$
$=f_{0}+\left\{\left\{\left(\theta_{1} \theta_{2}\right)^{-1}\right\}^{2}-\frac{1}{2} \sigma^{2}\right\} t+\sigma d Z^{(1)}(t)$, Since $f\left(S_{K_{1}}(t)\right)=\ln S_{K_{1}}(t)$ anestablishedsolutionfor $S_{K_{1}}(t) y i e l d s$
$\ln S_{K_{1}}(t)=\ln S_{o\left(K_{1}\right)}+\left\{\left\{\left(\theta_{1} \theta_{2}\right)^{-1}\right\}^{2}-\frac{1}{2} \sigma^{2}\right\} t+\sigma d Z^{(1)}(t), \ln S_{K_{1}}(t)-\ln S_{0\left(K_{1}\right)}=\left\{\left\{\left(\theta_{1} \theta_{2}\right)^{-1}\right\}^{2}-\frac{1}{2} \sigma^{2}\right\} t+\sigma d Z^{(1)}(t)$
$\ln \left(\frac{s_{K_{1}}(t)}{s_{0\left(K_{1}\right)}}\right)=\left\{\left\{\left(\theta_{1} \theta_{2}\right)^{-1}\right\}^{2}-\frac{1}{2} \sigma^{2}\right\} t+\sigma d Z^{(1)}(t)$
$S_{K_{1}}(t)=S_{0\left(K_{1}\right)} \exp \left\{\left(\left\{\left(\theta_{1} \theta_{2}\right)^{-1}\right\}^{2}-\frac{1}{2} \sigma^{2}\right) t+\sigma d Z^{(1)}(t)\right\}$
This is the complete solution of investment equation whose rate of returns and asset price valuation is multiplicative inverse effects with a quadratic function.

From (3), Thus, generalizing and considering a function $f\left(S_{K_{2}}(t), t\right)$, therefore, it relates with partial derivatives. Expansion of $f\left(S_{K_{2}}(t), d S_{K_{2}}(t), t+d t\right)$ in a Taylor series about $\left(S_{K_{2}}(t), t\right)$, yields;
$d f=\frac{\partial f}{\partial S_{K_{2}}(t)} d S_{K_{2}}(t)+\frac{\partial f}{\partial t} d t+\frac{1}{2} \frac{\partial^{2} f}{\partial S_{K_{2}}{ }^{2}(t)} d S_{K_{2}}^{2}(t)+\ldots$
Substituting (3) in (14) gives
$d f=\frac{\partial f}{\partial S_{K_{2}}(t)}\left\{\sigma S_{K_{2}}(t) d Z^{(2)}(t)+\left(\theta_{1}+\theta_{2}\right)^{2} S_{K_{2}}(t) d t \quad\right\}+\frac{1}{2} \frac{\partial^{2} f}{\partial S_{K_{2}}{ }^{2}(t)} d S_{K_{2}}^{2}(t) d S_{K_{2}}^{2}(t)$
$d f=\sigma S_{K_{2}}(t) \frac{\partial f}{\partial s_{K_{2}}(t)} d Z^{(2)}(t)+\left(\left(\theta_{1}+\theta_{2}\right)^{2} S_{K_{2}}(t) \frac{\partial f}{\partial S_{K_{2}}(t)}+\frac{1}{2} \sigma^{2} S_{K_{2}}^{2}(t) \frac{\partial^{2} f}{\partial s_{K_{1}}{ }^{2}(t)} d S_{K_{2}}^{2}(t)\right) d t$

Taking into account the SDE in (3),
$\operatorname{Let} f\left(S_{K_{2}}(t)\right)=\ln S_{K_{2}}(t)$, thepartialderivativesbecomes
$\frac{\partial f}{\partial S_{K_{2}}(t)}=\frac{1}{S_{K_{2}}(t)}, \frac{\partial^{2} f}{\partial S_{K_{2}}^{2}(t)}=-\frac{1}{S_{K_{2}}^{2}(t)}, \frac{\partial f}{\partial t}=0$
following theorem 1.1(Ito's), substituting (15) and simplifying, yields;
$d f=\left\{\left(\theta_{1}+\theta_{2}\right)^{2}-\frac{1}{2} \sigma^{2}\right\} d t+\sigma d Z^{(2)}$
Similarly, because the RHS of (16) is independent of $f\left(S_{K_{2}}(t)\right)$, the stochastic is determined as follows:
$f\left(S_{K_{2}}(t)=f_{0}+\int_{0}^{t}\left(\theta_{1}+\theta_{2}\right)^{2}-\frac{1}{2} \sigma^{2} \quad\right\} d t+\int_{0}^{t} \sigma d Z^{(2)}(t)$
$=f_{0}+\left\{\left(\theta_{1}+\theta_{2}\right)^{2}-\frac{1}{2} \sigma^{2}\right\} t+\sigma d Z^{(2)}(t)$,Since $f\left(S_{K_{2}}(t)\right)=\ln S_{K_{2}}(t)$ anestablishedsolutionfor $S_{K_{2}}(t) y i e l d s$
$\ln S_{K_{2}}(t)=\ln S_{o\left(K_{2}\right)}+\left\{\left(\theta_{1}+\theta_{2}\right)^{2}-\frac{1}{2} \sigma^{2}\right\} t+\sigma d Z^{(2)}(t), \ln S_{K_{2}}(t)-\ln S_{0\left(K_{2}\right)}=\left\{\left(\theta_{1}+\theta_{2}\right)^{2}-\frac{1}{2} \sigma^{2}\right\} t+\sigma d Z^{(2)}(t)$
$\ln \left(\frac{s_{K_{2}}(t)}{s_{\left.O_{\left(K_{2}\right)}\right)}}\right)=\left\{\left(\theta_{1}+\theta_{2}\right)^{2}-\frac{1}{2} \sigma^{2}\right\} t+\sigma d Z^{(2)}(t)$
$S_{K_{2}}(t)=S_{0\left(K_{2}\right)} \exp \left\{\left(\left(\theta_{1}+\theta_{2}\right)^{2}-\frac{1}{2} \sigma^{2}\right) t+\sigma d Z^{(2)}(t)\right\}$
This is the complete solution of investment equation whose rate of returns and asset price valuation is additive effects with a quadratic function.

From (4), Thus, generalizing and considering a function $f\left(S_{K_{3}}(t), t\right)$, thus, it relates to partial derivatives. Expansion of $f\left(S_{K_{3}}(t), d S_{K_{3}}(t), t+d t\right)$ in a Taylor series about $\left(S_{K_{3}}(t), t\right)$ yields
$d f=\frac{\partial f}{\partial S_{K_{3}}(t)} d S_{K_{2}}(t)+\frac{\partial f}{\partial t} d t+\frac{1}{2} \frac{\partial^{2} f}{\partial S_{K_{3}}{ }^{2}(t)} d S_{K_{3}}^{2}(t)+\ldots$
Substituting (4) in (18), yields;
$d f=\frac{\partial f}{\partial S_{K_{3}}(t)}\left\{\sigma S_{K_{3}}(t) d Z^{(3)}(t)+\left\{\left(\theta_{1}+\theta_{2}\right)^{-1}\right\}^{2} S_{K_{3}}(t) d t \quad\right\}+\frac{1}{2} \frac{\partial^{2} f}{\partial S_{K_{3}}{ }^{2}(t)} d S_{K_{3}}^{2}(t) d S_{K_{3}}^{2}(t)$
$d f=\sigma S_{K_{3}}(t) \frac{\partial f}{\partial S_{K_{3}}(t)} d Z^{(3)}(t)+\left(\left\{\left(\theta_{1}+\theta_{2}\right)^{-1}\right\}^{2} S_{K_{3}}(t) \frac{\partial f}{\partial S_{K_{3}}(t)}+\frac{1}{2} \sigma^{2} S_{K_{3}}^{2}(t) \frac{\partial^{2} f}{\partial S_{K_{3}}{ }^{2}(t)} d S_{K_{3}}^{2}(t)\right) d t$
Taking into account the SDE in(4)

Let $f\left(S_{K_{3}}(t)\right)=\ln S_{K_{3}}(t)$, the partialderivatives becomes
$\frac{\partial f}{\partial S_{K_{3}}(t)}=\frac{1}{s_{K_{3}}(t)}, \frac{\partial^{2} f}{\partial s_{K_{3}}^{2}(t)}=-\frac{1}{s_{K_{3}}^{2}(t)}, \frac{\partial f}{\partial t}=0$
following theorem 1.1(Ito's), substituting (19) and simplifying, yields;
$d f=\left\{\left\{\left(\theta_{1}+\theta_{2}\right)^{-1}\right\}^{2}-\frac{1}{2} \sigma^{2}\right\} d t+\sigma d Z^{(3)}$
Also, because the RHS of (20) is independent of $f\left(S_{K_{3}}(t)\right)$, the stochastic is determined as follows:

$$
\begin{aligned}
f\left(S_{K_{3}}(t)\right. & \left.=f_{0}+\int_{0}^{t}\left(\left(\theta_{1}+\theta_{2}\right)^{-1}\right)^{2}-\frac{1}{2} \sigma^{2} \quad\right\} d t+\int_{0}^{t} \sigma d Z^{(3)}(t) \\
& =f_{0}+\left\{\left(\left(\theta_{1}+\theta_{2}\right)^{-1}\right)^{2}-\frac{1}{2} \sigma^{2}\right\} t+\sigma d Z^{(2)}(t), \text { Since } f\left(S_{K_{3}}(t)\right)=\ln S_{K_{3}}(t) \text { afoundsolutionfor } S_{K_{3}}(t) \text { becomes }
\end{aligned}
$$

$\ln S_{K_{2}}(t)=\ln S_{o\left(K_{2}\right)}+\left\{\left(\left(\theta_{1}+\theta_{2}\right)^{-1}\right)^{2}-\frac{1}{2} \sigma^{2}\right\} t+\sigma d Z^{(2)}(t), \ln S_{K_{3}}(t)-\ln S_{0\left(K_{3}\right)}=\left\{\left(\left(\theta_{1}+\theta_{2}\right)^{-1}\right)^{2}-\frac{1}{2} \sigma^{2}\right\} t+\sigma d Z^{(3)}(t)$
$\ln \left(\frac{s_{K_{3}}(t)}{s_{0\left(K_{3}\right)}}\right)=\left\{\left(\left(\theta_{1}+\theta_{2}\right)^{-1}\right)^{2}-\frac{1}{2} \sigma^{2}\right\} t+\sigma d Z^{(3)}(t)$
$S_{K_{3}}(t)=S_{0_{\left(K_{3}\right)}} \exp \left\{\left(\left(\left(\theta_{1}+\theta_{2}\right)^{-1}\right)^{2}-\frac{1}{2} \sigma^{2}\right) t+\sigma d Z^{(3)}(t)\right\}$
This is the complete solution of investment equation whose rate of returns and asset price valuation is additive inverse effects with a quadratic function.

From (5), therefore, generalizing and considering a function $f\left(S_{K_{4}}(t), t\right)$, thus, it relates to partial derivatives. Expansion of $f\left(S_{K_{4}}(t), d S_{K_{4}}(t), t+d t\right)$ in a Taylor series about $\left(S_{K_{4}}(t), t\right)$ gives
$d f=\frac{\partial f}{\partial S_{K_{4}}(t)} d S_{K_{4}}(t)+\frac{\partial f}{\partial t} d t+\frac{1}{2} \frac{\partial^{2} f}{\partial S_{K_{4}}{ }^{2}(t)} d S_{K_{4}}^{2}(t)+\ldots$
Substituting (5) in (22) gives
$d f=\frac{\partial f}{\partial S_{K_{4}}(t)}\left\{\sigma S_{K_{4}}(t) d Z^{(1)}(t)+\left(\theta_{1} \theta_{2}\right)^{2} S_{K_{4}}(t) d t \quad\right\}+\frac{1}{2} \frac{\partial^{2} f}{\partial S_{K_{4}}{ }^{2}(t)} d S_{K_{4}}^{2}(t) d S_{K_{4}}^{2}(t)$
$d f=\sigma S_{K_{4}}(t) \frac{\partial f}{\partial S_{K_{4}}(t)} d Z^{(4)}(t)+\left(\left(\theta_{1} \theta_{2}\right)^{2} S_{K_{4}}(t) \frac{\partial f}{\partial S_{K_{4}}(t)}+\frac{1}{2} \sigma^{2} S_{K_{4}}^{2}(t) \frac{\partial^{2} f}{\partial S_{K_{4}}{ }^{2}(t)} d S_{K_{4}}^{2}(t)\right) d t$
Now considering the SDE in (5)

Let $f\left(S_{K_{4}}(t)\right)=\ln S_{K_{4}}(t)$, thepartialderivativesbecomes
$\frac{\partial f}{\partial S_{K_{4}}(t)}=\frac{1}{S_{K_{4}}(t)}, \frac{\partial^{2} f}{\partial S_{K_{4}}^{2}(t)}=-\frac{1}{S_{K_{4}}^{2}(t)}, \frac{\partial f}{\partial t}=0$
following theorem 1.1 (Ito's), substituting (23) and simplifying gives
$d f=\left\{\left(\theta_{1} \theta_{2}\right)^{2}-\frac{1}{2} \sigma^{2}\right\} d t+\sigma d Z^{(4)}$
Since the RHS of (24) is independent of $f\left(S_{K_{2}}(t)\right)$, the stochastic is computed as follows:
$f\left(S_{K_{4}}(t)=f_{0}+\int_{0}^{t}\left(\theta_{1} \theta_{2}\right)^{2}-\frac{1}{2} \sigma^{2} \quad\right\} d t+\int_{0}^{t} \sigma d Z^{(4)}(t)$
$=f_{0}+\left\{\left(\theta_{1} \theta_{2}\right)^{2}-\frac{1}{2} \sigma^{2}\right\} t+\sigma d Z^{(4)}(t)$, Since $f\left(S_{K_{4}}(t)\right)=\ln S_{K_{4}}(t)$ afoundsolutionfor $S_{K_{4}}(t)$ becomes
$\ln S_{K_{4}}(t)=\ln S_{o\left(K_{4}\right)}+\left\{\left(\theta_{1} \theta_{2}\right)^{2}-\frac{1}{2} \sigma^{2}\right\} t+\sigma d Z^{(4)}(t), \ln S_{K_{4}}(t)-\ln S_{0\left(K_{4}\right)}=\left\{\left(\theta_{1} \theta_{2}\right)^{2}-\frac{1}{2} \sigma^{2}\right\} t+\sigma d Z^{(4)}(t)$
$\ln \left(\frac{s_{K_{4}}(t)}{s_{O_{\left(K_{4}\right)}}}\right)=\left\{\left(\theta_{1} \theta_{2}\right)^{2}-\frac{1}{2} \sigma^{2}\right\} t+\sigma d Z^{(4)}(t)$
$S_{K_{4}}(t)=S_{\left.0_{\left(K_{4}\right)}\right)} \exp \left\{\left(\left(\theta_{1} \theta_{2}\right)^{2}-\frac{1}{2} \sigma^{2}\right) t+\sigma d Z^{(4)}(t)\right\}$
This is the complete solution of investment equation whose rate of returns and asset price valuation is multiplicative effects with a quadratic function.

## 3. Results and Discussion

This Section presents the graphical results for the problems in (2)-(5) whose solutions are in (13)-(25), Hence the following parameter values were used in the simulation study:

Table 1: A multiplicative inverse stock returns for assessing the value of assets with the following parameters through the solution below: $S_{0}=$ 12.32, $d z=1, t=1, S_{k_{1}}(t)=S_{0}\left(k_{1}\right) \exp \left\{\left(\theta_{1} \theta_{2}\right)^{-1}-\frac{1}{2} \sigma^{2}\right\} t+\sigma d z^{(1)}(t)$

| $\sigma$ | $\left(\theta_{1} \theta_{2}\right)^{-1}$ | $S_{k_{1}}(t)$ | $\left(\theta_{1} \theta_{2}\right)^{-1}$ | $S_{k_{1}}(t)$ | $\left(\theta_{1} \theta_{2}\right)^{-1}$ | $S_{k_{1}}(t)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.1 | 1.0000 | 36.8267 | 2.0000 | 100.1054 | 3.0000 | 272.1148 |
| 0.2 | 1.0000 | 40.0939 | 2.0000 | 108.9865 | 3.0000 | 296.2560 |
| 0.3 | 1.0000 | 43.2166 | 2.0000 | 117.4748 | 3.0000 | 319.3297 |
| 0.4 | 1.0000 | 46.1190 | 2.0000 | 125.3643 | 3.0000 | 340.7755 |
| 0.5 | 1.0000 | 48.7266 | 2.0000 | 132.4525 | 3.0000 | 360.0432 |
| 0.6 | 1.0000 | 50.9693 | 2.0000 | 138.5490 | 3.0000 | 376.6152 |
| 0.7 | 1.0000 | 52.7848 | 2.0000 | 143.4841 | 3.0000 | 390.0301 |
| 0.8 | 1.0000 | 54.1211 | 2.0000 | 147.1164 | 3.0000 | 399.9038 |
| 0.9 | 1.0000 | 54.9390 | 2.0000 | 149.3398 | 3.0000 | 405.9476 |
| 1.0 | 1.0000 | 55.2144 | 2.0000 | 150.0883 | 3.0000 | 407.9824 |

Table 2: An additive stock returns for assessing the value of assets with the following parameters through the solution below: $S_{0}=30.50, d Z=$ $1, t=1 S_{k_{2}}(t)=S_{0}\left(k_{2}\right) \exp \left\{\left(\theta_{1}+\theta_{2}\right)^{2}-\frac{1}{2} \sigma^{2}\right\} t+\sigma d z^{(2)}(t)$

| $\sigma$ | $\left(\theta_{1}+\theta_{2}\right)$ | $S_{k_{2}}(t)$ | $\left(\theta_{1}+\theta_{2}\right)$ | $S_{k_{2}}(t)$ | $\left(\theta_{1}+\theta_{2}\right)$ | $S_{k_{2}}(t)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.1 | 1.0000 | 91.1701 | 2.0000 | 1831.1998 | 3.0000 | 271774.1538 |
| 0.2 | 1.0000 | 99.2584 | 2.0000 | 1993.6585 | 3.0000 | 295885.1596 |
| 0.3 | 1.0000 | 106.9891 | 2.0000 | 2148.9329 | 3.0000 | 318929.924 |
| 0.4 | 1.0000 | 114.1744 | 2.0000 | 2293.2532 | 3.0000 | 340348.9465 |
| 0.5 | 1.0000 | 120.6298 | 2.0000 | 2422.9151 | 3.0000 | 359592.4352 |
| 0.6 | 1.0000 | 126.1822 | 2.0000 | 2534.4367 | 3.0000 | 376143.7577 |
| 0.7 | 1.0000 | 130.6768 | 2.0000 | 2624.7126 | 3.0000 | 389541.8888 |
| 0.8 | 1.0000 | 133.9848 | 2.0000 | 2691.1575 | 3.0000 | 3999403.1887 |
| 0.9 | 1.0000 | 136.0098 | 2.0000 | 2731.8291 | 3.0000 | 405439.3949 |
| 1.0 | 1.0000 | 136.6915 | 2.0000 | 2745.5225 | 3.0000 | 407447.6683 |

Table 3: An additive inverse stock returns for assessing the value of assets with the following parameters through the solution below: $S_{0}=$ 84.57, $d Z=1, t=1, S_{k_{3}}(t)=S_{0}\left(k_{3}\right) \exp \left\{\left(\left(\theta_{1}+\theta_{2}\right)^{-1}\right)^{2}-\frac{1}{2} \sigma^{2}\right\} t+\sigma d z^{(3)}(t)$

| $\boldsymbol{\sigma}$ | $\left(\boldsymbol{\theta}_{\mathbf{1}}+\boldsymbol{\theta}_{\mathbf{2}}\right)^{\mathbf{- 1}}$ | $\boldsymbol{S}_{\boldsymbol{k}_{\mathbf{3}}}(\boldsymbol{t})$ | $\left(\boldsymbol{\theta}_{\mathbf{1}}+\boldsymbol{\theta}_{\mathbf{2}}\right)^{-\mathbf{1}}$ | $\boldsymbol{S}_{\boldsymbol{k}_{\mathbf{3}}}(\boldsymbol{t})$ | $\left(\boldsymbol{\theta}_{\mathbf{1}}+\boldsymbol{\theta}_{\mathbf{2}}\right)^{-\mathbf{1}}$ | $\boldsymbol{S}_{\boldsymbol{k}_{\mathbf{3}}}(\boldsymbol{t})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.1 | 1.0000 | 252.7952 | 2.0000 | 5077.5269 | 3.0000 | 753571.8095 |
| 0.2 | 1.0000 | 275.2224 | 2.0000 | 5527.9902 | 3.0000 | 820426.49 |
| 0.3 | 1.0000 | 296.6579 | 2.0000 | 5958.5330 | 3.0000 | 884324.7105 |
| 0.4 | 1.0000 | 316.5812 | 2.0000 | 6358.7023 | 3.0000 | 943715.0953 |
| 0.5 | 1.0000 | 334.4808 | 2.0000 | 6718.2272 | 3.0000 | 997073.3269 |
| 0.6 | 1.0000 | 349.8763 | 2.0000 | 7027.4529 | 3.0000 | 1042966.478 |
| 0.7 | 1.0000 | 362.3388 | 2.0000 | 7277.7687 | 3.0000 | 1080116.641 |
| 0.8 | 1.0000 | 371.5114 | 2.0000 | 7462.0063 | 3.0000 | 1107459.924 |
| 0.9 | 1.0000 | 377.1261 | 2.0000 | 7574.7801 | 3.0000 | 1124197.037 |
| 1.0 | 1.0000 | 379.0165 | 2.0000 | 7612.7488 | 3.0000 | 1129832.098 |

Table 4: A multiplicative stock returns for assessing the value of assets with the following parameters through the solution below:

$$
S_{0}=19.21, d Z=1, t=1, S_{k_{4}}(t)=S_{0}\left(k_{4}\right) \exp \left\{\left(\theta_{1} \theta_{2}\right)^{2}-\frac{1}{2} \sigma^{2}\right\} t+\sigma d z^{(4)}(t)
$$

| $\boldsymbol{\sigma}$ | $\left(\boldsymbol{\theta}_{\mathbf{1}} \boldsymbol{\theta}_{\mathbf{2}}\right)$ | $\boldsymbol{S}_{\boldsymbol{k}_{\mathbf{4}}}(\boldsymbol{t})$ | $\left(\boldsymbol{\theta}_{\mathbf{1}} \boldsymbol{\theta}_{\mathbf{2}}\right)$ | $\boldsymbol{S}_{\boldsymbol{k}_{\mathbf{4}}(\boldsymbol{t})}$ | $\left(\boldsymbol{\theta}_{\mathbf{1}} \boldsymbol{\theta}_{\mathbf{2}}\right)$ | $\boldsymbol{S}_{\boldsymbol{k}_{\mathbf{4}}(\boldsymbol{t})}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.1 | 1.0000 | 57.4222 | 2.0000 | 1153.3557 | 3.0000 | 171173.1638 |
| 0.2 | 1.0000 | 62.5165 | 2.0000 | 1255.6780 | 3.0000 | 186359.1448 |
| 0.3 | 1.0000 | 67.3856 | 2.0000 | 1353.4755 | 3.0000 | 200873.5685 |
| 0.4 | 1.0000 | 71.9111 | 2.0000 | 1444.3733 | 3.0000 | 214364.0414 |
| 0.5 | 1.0000 | 75.9770 | 2.0000 | 1526.0393 | 3.0000 | 226484.3161 |
| 0.6 | 1.0000 | 79.4741 | 2.0000 | 1596.2796 | 3.0000 | 236908.9044 |
| 0.7 | 1.0000 | 82.3049 | 2.0000 | 1653.1387 | 3.0000 | 245347.5306 |
| 0.8 | 1.0000 | 84.3885 | 2.0000 | 1694.9881 | 3.0000 | 251558.5329 |
| 0.9 | 1.0000 | 85.6639 | 2.0000 | 1720.6045 | 3.0000 | 255360.3533 |
| 1.0 | 1.0000 | 86.0933 | 2.0000 | 1729.2291 | 3.0000 | 2 |

$\left(\boldsymbol{\theta}_{1} \boldsymbol{\theta}_{\mathbf{2}}\right)^{\mathbf{- 1}}$ : This is the multiplicative inverse parameter of the model and is used here in the first investment $S_{k_{1}}(t)$ to represent return rates of stock and its asset value which follows multiplication inverse effect in terms of trading, see Columns: 2,4 and 6 of Table 1 , when the stock rate of returns are made constants, using 1.0000, 2.0000 and 3.0000 ; there are positive responses to the values of asset changes as seen in Columns: 3,

5 and 7. Asset valuation yields the following means: 48.3011, 131.2961 and 356.8998 .
$\left(\theta_{1}+\theta_{2}\right)$ : This is the additive effect parameter of the model used here in the second investment $S_{k_{2}}(t)$ to demonstrate when rate of returns of stock and its value of assets follow additive effect, see columns 2,4 and 6 of Table 2. When the stock return rates are fixed at $1.0000,2.0000$ and 3.0000 changes in the
values of assets are seen, see Columns 3,5 and 7 . The mean of assets valuation of the second investment equation gives: $119.5767,2401.7618$ and 356453.0568.
$\left(\theta_{1}+\theta_{2}\right)^{-1}$ : This is the additive inverse effect parameters of the model used in the third investment $S_{k_{3}}(t)$ to display return rates of stock and its value of assets which follow additive inverse effect. See Columns: 2, 4 and 6 of Table 3. When the stock return rates are fixed at $1.0000,2.0000$ and 3.0000 resulted in changes in the value of assets as seen in Columns 3, 5 and 7 which gives the mean of assets valuation of the third investment equation as: $331.5607,6659.5736$ and 988368.361 . $\theta_{1} \theta_{2}$ : This is the multiplicative effect parameter effect of the model and is used in the fourth investment $S_{k_{4}}(t)$ to show stock return rates and value of assets which follow multiplicatively in terms of trading, see Columns: 2,4 and 6 of Table 4. When the stock return rates are made constant using 1.0000, 2.0000 and 3.0000 , there is a positive respond to the values of assets changes which are displayed in columns: 3,5 and 7 . The mean value of assets valuation yields the following: 75.3137, 1512.7162 and 224506.9908 .

Furthermore, it can be seen in Tables 1, 2, 3 and 4 that an increase in stock market prices, increases the assets values. This is quite encouraging to an investor whose vision and mission is to maximize profit over time. Secondly a multiplicative inverse stock return is the best in terms of precision and decision making. Thirdly a little increase in stock volatility also increases the value of asset. This is sure because volatility causes significant changes which inform an investor on how to properly take decision based on their level of investments. Finally, additive inverse effects have the highest rate of returns throughout the trading days which is categorized under high profit margin.

## 4. Conclusions

In nonexistence assessments of, trading business will certainly have a hard time crumbling which may not be beneficial to an investor. Therefore, this paper studied stochastic analysis on the valuation of asset returns; where multiplicative inverse effects, additive effects, additive inverse effects and multiplicative effects were used extensively in the model to obtain stock rate of returns. The analytical closed form solution showed: (i) increase in stock market prices, increases the assets values. (ii) Secondly a multiplicative inverse stock return is the best in terms of precision and decision making. (iii) a little increase in stock volatility also increases the value of asset.(iv) additive inverse effects has the highest rate of returns throughout the trading days.
However, studying the uniqueness of these asset pricing effects will be an interesting area to consider as it affects financial markets.

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