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## Effect of slip velocity on double layered rough porous parabolic slider bearing

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### Abstract

This study is analyze the influence of Ferrofluid based double layer porous rough parabolic slider bearing with the effect of slip velocity. The slip model of Beavers and Joseph is considered to evaluate slip effect. The modified Reynolds equation for the double-layered bearing system is solved to compute a dimensionless pressure profile, load bearing capacity and frictional force. The numerical results for bearing characteristics such as pressure, load bearing capacity and frictional force are plotted graphically to study the effect of double layered porous facing with effect of slip velocity. It is observed that the combined effect of double layered porous and adverse effect of negatively skewed roughness and negative variance improve the already increased load bearing capacity owing to magnetization.

**Keywords:** Slip velocity, load bearing capacity, bearing design, ferrofluid

### 1. Introduction

Slider bearings are designed for supporting the transverse load in engineering systems. A variety of bearing are adopted by agro-industrial sectors (viz. hydrodynamic bearings, hydrostatic bearings, rolling element bearings etc.) having great influence on reliability, life and power consumption of agricultural machines, tools or equipment's dealing with friction, wear, heat generation & its dissipation. Bearing performance characteristics for various film shapes have been analyzed by a numerous author (Cameron, Pinkus and Sternlicht, Hamrock, Patel et al.) [4, 16, 10, 30]. In order to improve the lubricating performance, applying the couple stress fluid model, many investigations concerning the fluid film lubrication have been conducted. From the studies of squeeze film performance characteristics in finite plates by Ramanaish [19] in partial journal bearings by Lin [12], it was found that the use of couple stress fluids increased the load bearing capacity and lengthened the response time of squeeze film action. In a view of the discussions of journal bearings by various investigators (Mokhiamer et al. Lin, Patel et al.) [15, 11-13, 31] and slider bearings by Ramanaish [18] and Lin et al. [14], It has been concluded that the effects of couple stresses reduce the friction parameter and result in longer bearing life. Lin et al. [14] analyze the effects of couple stress on the steady state performance of wide parabolic shaped slider bearing in accordance with Stokes micro-continuum theory.

Agrawal [1] discussed the performance of a plane inclined slider bearing with a ferrofluid lubricant and established that its performance was comparatively better than the corresponding bearing with a conventional lubricant. Patel and Deheri [24] analyzed the performance of a magnetic fluid based double layered rough porous slider bearing considering the combined porous structures of Kozeny - Carman and Irmay. The Kozeny - Carman model was found to perform better.

It was established that the roughness of the bearing surfaces tends to retard the motion of the lubricant and hence affecting the bearing system adversely. From this point of view Christensen and Tonder [5, 6, 7] modified the approach of Tzeng and Saibel [20] to present a study on the effect of surface roughness on the performance of the bearing system. The following investigation made use of the modeling of surface roughness as given by Christensen and Tonder [5, 6, 7]. The performance of a transversely rough slider bearing with squeeze film by magnetic fluid was analyzed by Deheri et al. [8] by taking various shapes in to consideration. It was found that the magnetic fluid induced an increase in the load bearing capacity although,

the effect of transverse surface roughness was found to be adverse in general. This squeeze film performance in the case of longitudinal roughness was relatively better as compared to transversely rough slider bearing for a porous squeeze film formed by a magnetic fluid as investigated by Deheri et al. [9]. Gururajan and J. Prakash [27] investigated that the surface roughness considerably influences the bearing performance and its direction depends on the roughness type. Lin et al. [28] evaluated that the mean squeeze-film characteristics are considerably affected by the roughness pattern and the height of roughness also proved that the effect of circumferential roughness provides a reduction in the mean bearing eccentricity ratio as compared to the smooth-bearing case. However, the squeeze-film bearing with longitudinal roughness structure results in a reversed trend.

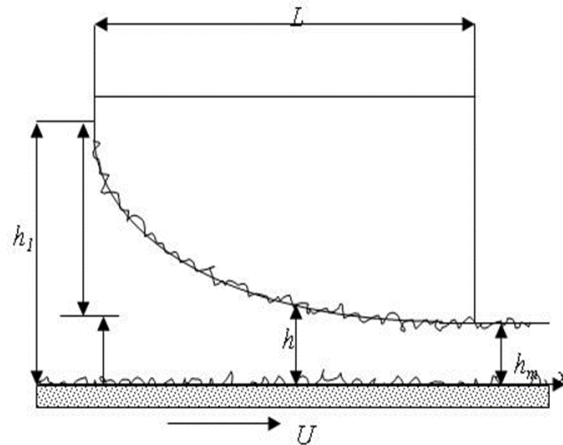
Srinivasan [21] investigated the load capacity and time height relationship for squeeze films between double layered porous plates, considering various geometries such as annular, circular, elliptic, rectangular etc. The comparison was made between conventional and double layered porous plates. The results showed that load capacity increases due to doubled layered porous plates. Verma [22] presented an investigation for a double layered porous journal bearing using short bearing approximations. The performance characteristics were found to be improved due to low permeability of the inner porous layer. Rao et al. [23] presented an analysis of journal bearing with double layered porous lubricant film using couple stress and Newtonian fluids. A double layered porous lubricant film configuration with a low permeability porous layer on top of a high permeability bearing adherent porous layer improved the bearing performance.

Patel et al. [25] pointed out that the different types of material combinations could possibly turn out to be a suitable option for the ferrofluid based journal bearing system and enhance the life period of the bearing system. Patel et al. [26] established that proper conditions due to the positive effect of non-Newtonian ferrofluid help to recompense the poor effect of roughness.

Naduvanamani et al. [29] found that the effect of permeability of the porous layer is to decrease the load bearing capacity as it gives an easy path for the lubricant to pass through. This adverse effect can be compensated by introducing double layered porous facing with different permeability. The presented results clearly found that the increase in load bearing capacity and decrease in co-efficient of friction for the double layered porous bearings as compared to that of single layered porous bearings.

**2. Materials and Methods**

The geometry and configuration of the bearing system is displayed in Figure 1, which consists of a stator lying along x-axis and having a porous matrix of uniform thickness  $H^*$  backed by a solid wall, and a parabolic shaped slider moving with a uniform velocity  $U$  in x-direction.



**Figure 1. Parabolic Shaped Slider Bearing**

The bearing has length  $L$  and breadth  $B$ , with  $L \ll B$ . The film thickness  $h$  is taken as

$$h = h_1 \left\{ 1 + (1 - m)(x^2 - 2x) \right\}$$

where  $h_1$  is the minimum value of  $h$ .

The applied magnetic field  $M$  is inclined and the inclination  $\phi$  can be determined as in the case of Bhat [3]. Following Bhat [3] and Prajapati [18], the magnitude of the magnetic field  $M$  is represented by

$$M^2 = Kx(L - x)$$

$K$  being a quantity chosen to suit the dimensions of both sides and the strength of the magnetic field, such a magnetic field was used in [17, 3].

With the usual assumptions of hydromagnetic lubrication, assuming that the z-components of velocities of fluid in the film and porous regions are continuous at the surface  $z = 0$ , the Reynolds type equation is obtained as [12, 13, 2].

$$\frac{d}{dx} \left\{ \left[ 12kH^* + \frac{g(h)(4+sh) - 3\rho\alpha^2\bar{\mu}ks h^2 M/\zeta}{(1+sh)[1 - \rho\alpha^2\bar{\mu}M/(2\zeta)]} \right] \frac{d}{dx} \left( P - \frac{1}{2}\mu_0\bar{\mu}HM^2 \right) \right\} = 6\zeta U \frac{d}{dx} \left( \frac{h(2+sh) - \rho\alpha^2\bar{\mu}ks M/\zeta}{1+sh} \right) \quad (3)$$

Where

$$g(h) = h^3 + 3\alpha h^2 + 3(\alpha^2 + \sigma^2)h + 3\sigma^2\alpha + \alpha^3 + \varepsilon$$

$k$  being the permeability of porous matrix. The use of equations (1), (2) and the dimensionless quantities

$$X = \frac{x}{L}, \quad \psi = \frac{kH^*}{h_0^3}, \quad \bar{h} = \frac{h}{h_0}, \quad \bar{s} = sh_0, \quad \beta^2 = \frac{\rho\alpha^2\bar{\mu}\sqrt{KL}}{2\zeta}, \quad P = \frac{h_0^2 p}{\zeta UL},$$

$$\mu' = \frac{\mu_0\bar{\mu}KLh_0^2}{\zeta U}, \quad \bar{\gamma} = \frac{6k}{h_0^2}, \quad \bar{\alpha} = \frac{\alpha}{h_0}, \quad \bar{\sigma} = \frac{\sigma}{h_0}, \quad \bar{\varepsilon} = \frac{\varepsilon}{h_0}, \quad W = \frac{h_0^2 w}{\zeta UL^2 B}, \quad F = \frac{h_0 f}{\zeta ULB},$$

in (3) leads to

$$\frac{d}{dX} \left\{ A' \frac{d}{dX} \left[ p - \frac{1}{2} \mu' X (1 - X) \right] \right\} = \frac{dB'}{dX}$$

Where

$$\bar{h} = \left\{ 1 + (1 - m) (X^2 - 2X) \right\}$$

$$g(\bar{h}) = \bar{h}^3 + 3\bar{\alpha}\bar{h}^2 + 3(\bar{\alpha}^2 + \bar{\sigma}^2)\bar{h} + 3\bar{\sigma}^2\bar{\alpha} + \bar{\alpha}^3 + \bar{\varepsilon}$$

$$A' = 12(\psi_1 + \psi_2) + \frac{g(\bar{h}) \left( 4 + 3 \left( \bar{\sigma}^2 + \bar{\alpha}^2 \right) \bar{s}\bar{h} \right)}{\left( 1 + 3 \left( \bar{\sigma}^2 + \bar{\alpha}^2 \right) \bar{s}\bar{h} \right)}$$

and

$$B' = \frac{18 \left( \bar{\sigma}^2 + \bar{\alpha}^2 \right) \bar{h} \left( 2 + 3 \left( \bar{\sigma}^2 + \bar{\alpha}^2 \right) \bar{s}\bar{h} \right)}{\left( 1 + 3 \left( \bar{\sigma}^2 + \bar{\alpha}^2 \right) \bar{s}\bar{h} \right)}$$

Since the pressure is negligible at the inlet and outlet of the bearing as compared to the inside pressure, one can resort to the boundary conditions  $P=0$  at  $x=0$ .

Solving equation (4) under the boundary conditions the expression for non-dimensional pressure distribution comes out to be

$$P = \frac{1}{2} \mu' X (1 - X) + \int_0^X \frac{B' - C'}{A'} dX$$

Where

$$C' = \frac{\int_0^X \frac{B'}{A'} dX}{\int_0^X \frac{1}{A'} dX}$$

The load bearing capacity  $W$  of the bearing and friction force  $F$  on the slider can be expressed in dimensionless form as

$$W = \frac{h_0^2 W}{\zeta UL^2 B} = \frac{\mu'}{12} - \int_0^X X \frac{B' - C'}{A'} dX \tag{6}$$

and

$$F = \frac{h_0 F}{\zeta UL B} = \frac{1}{2} \int_0^1 (m-1)(X-1) \bar{p} dX - \frac{1}{2[m(m-1)]^{1/2}} \ln \left| \frac{(m-1) - [m(m-1)]^{1/2}}{(m-1) + [m(m-1)]^{1/2}} \right| \tag{7}$$

### 3. Results and discussion

The variation of load bearing capacity with respect to the magnetization parameter  $\mu'$  presented in Figures 4-7 indicates

that magnetization increases the load bearing capacity. Further, the effect of higher values of variance on the load bearing capacity is negligible with respect to the magnetization parameter. It is found that the rate of increase in load bearing capacity is relatively more in the case of a slip coefficient. (4)

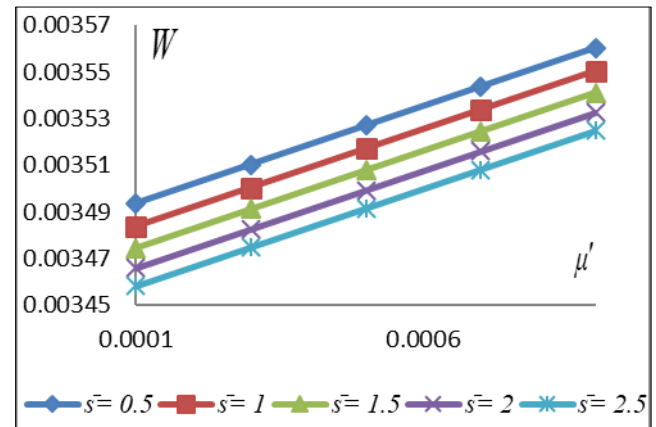


Fig 2: Variation of load bearing capacity with respect to  $\mu'$  and  $\bar{s}$

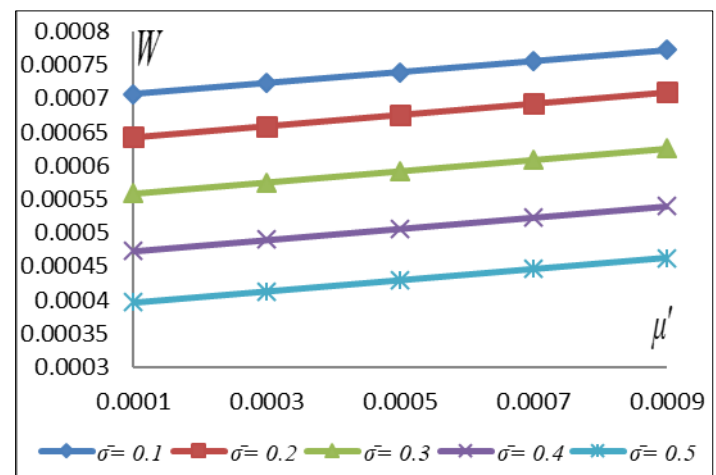


Fig 3: Variation of load bearing capacity with respect to  $\mu'$  and  $\bar{\sigma}$

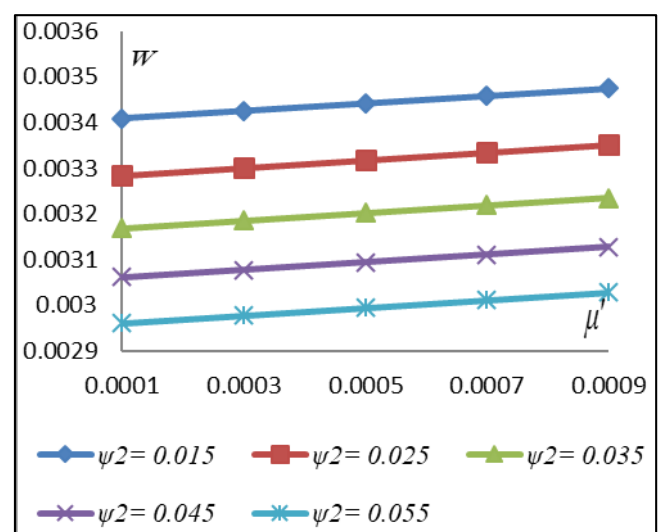


Fig 4: Variation of load bearing capacity with respect to  $\mu'$  and  $\psi_2$

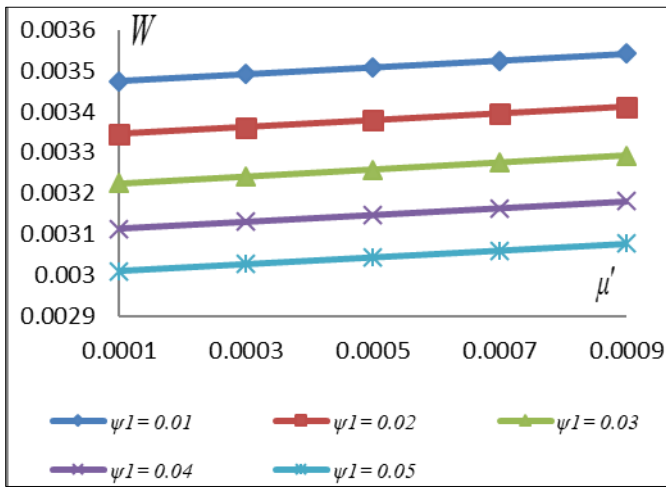


Fig 5: Variation of load bearing capacity with respect to  $\mu'$  and  $\psi_1$

Figures 8-11 present the effect of slip parameter on the distribution of load bearing capacity. It is clearly seen that the load bearing capacity decreases due to slip velocity. Also, it is seen that the effect of higher values of  $\alpha$  on the distribution of load bearing capacity with respect to slip parameter is negligible.

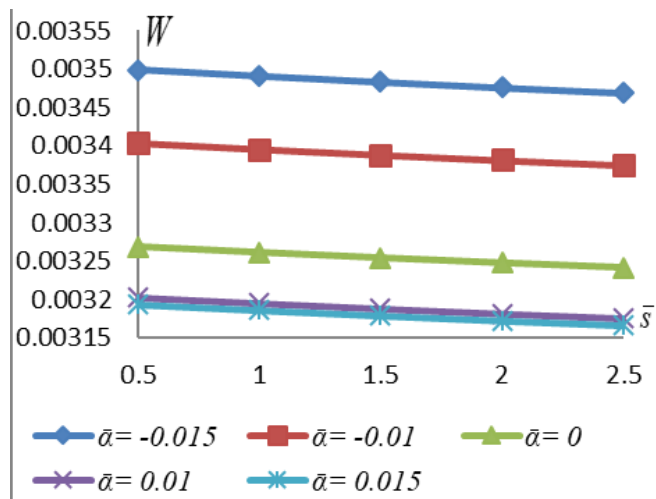


Fig 6: Variation of load bearing capacity with respect to  $\bar{s}$  and  $\bar{\alpha}$

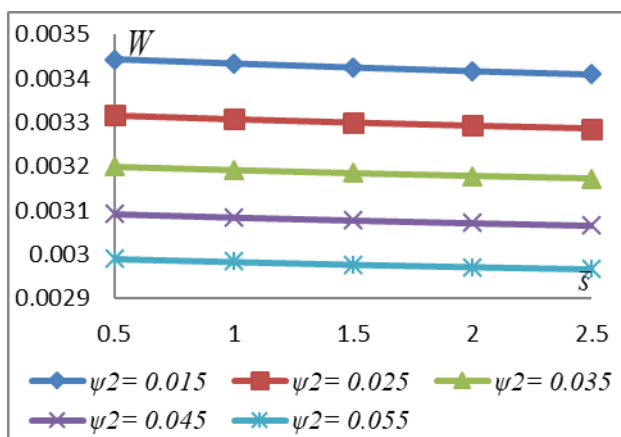


Fig 7: Variation of load bearing capacity with respect to  $\bar{s}$  and  $\bar{\sigma}$

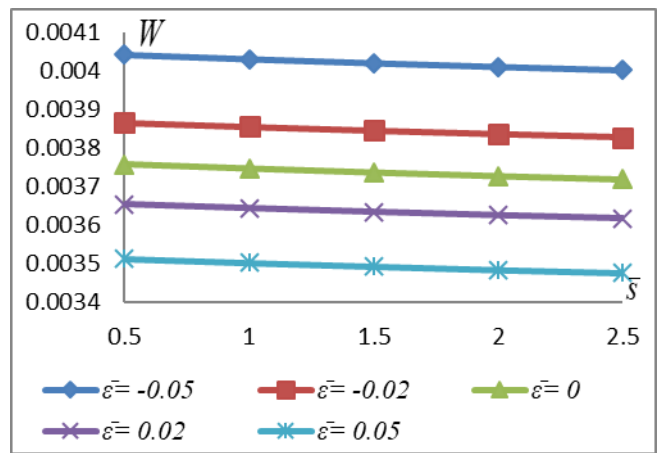


Fig 8: Variation of load bearing capacity with respect to  $\bar{s}$  and  $\bar{\epsilon}$

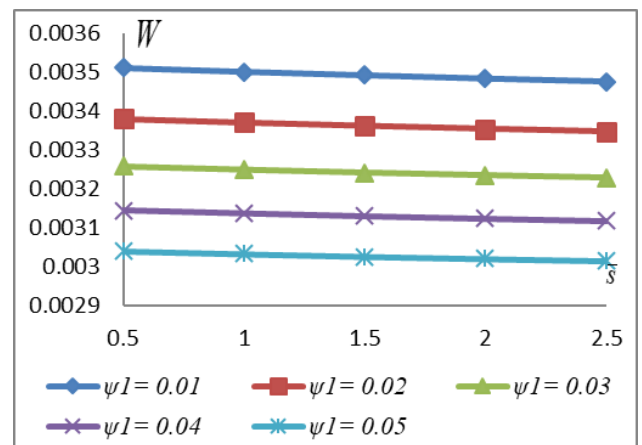


Fig 9: Variation of load bearing capacity with respect to  $\bar{s}$  and  $\psi_1$

The effect of roughness parameters are presented in Figures 12-15. It is clearly observed that the load bearing capacity decreases sharply due to the standard deviation. Besides, skewness (+ve) decreases the load bearing capacity, while the load bearing capacity gets enhanced by the negatively skewed roughness. The trends of the load bearing capacity with respect to variance follow almost the paths of the skewness. Thus, the combined effect of negatively skewed roughness and variance (-ve) is significantly positive.

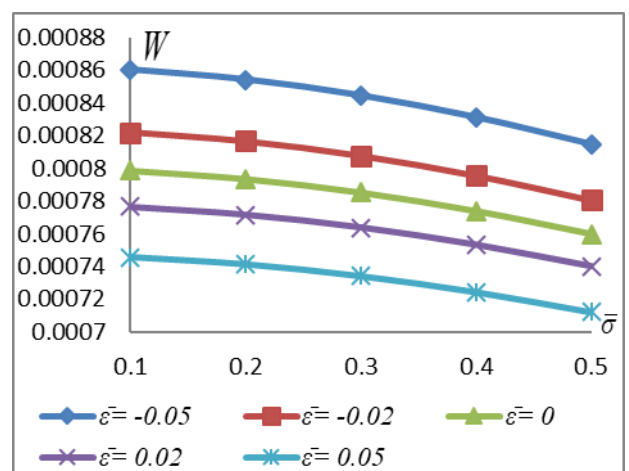
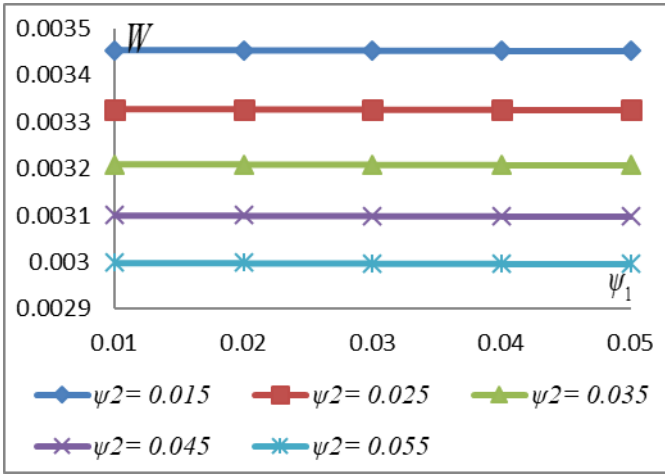
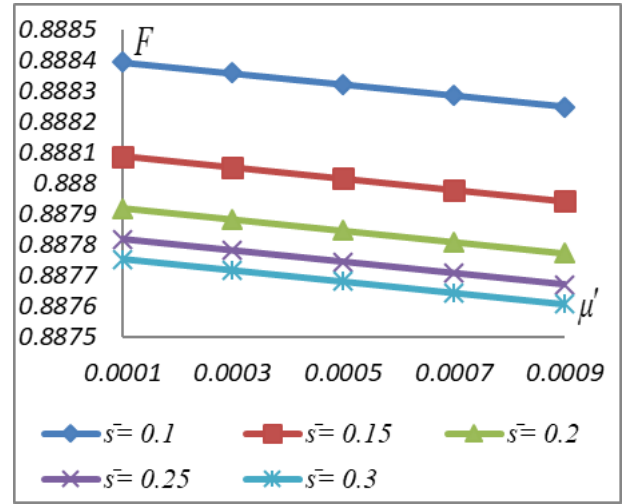


Fig 10: Variation of load bearing capacity with respect to  $\bar{\sigma}$  and  $\bar{\epsilon}$

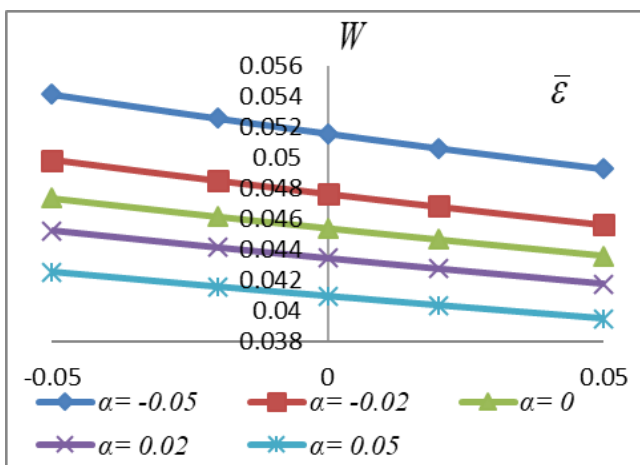




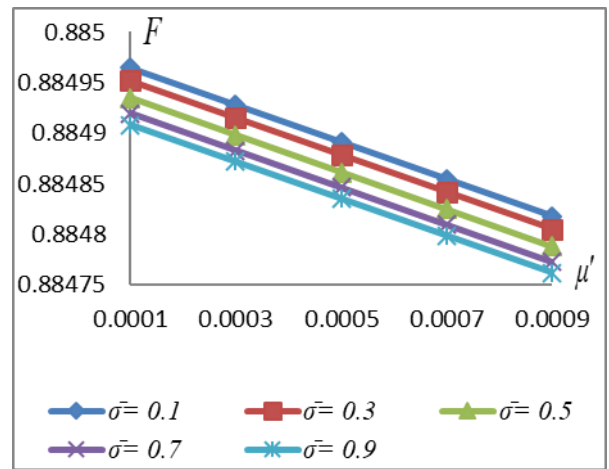
**Fig 11:** Variation of load bearing capacity with respect to  $\psi_1$  and  $\psi_2$



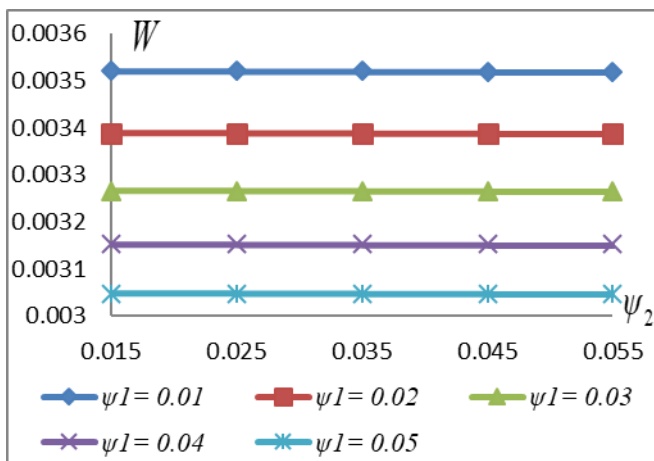
**Fig 14:** Variation of friction with respect to  $\mu'$  and  $\bar{\sigma}$



**Fig 12:** Variation of load bearing capacity with respect to  $\bar{\alpha}$  and  $\bar{\sigma}$

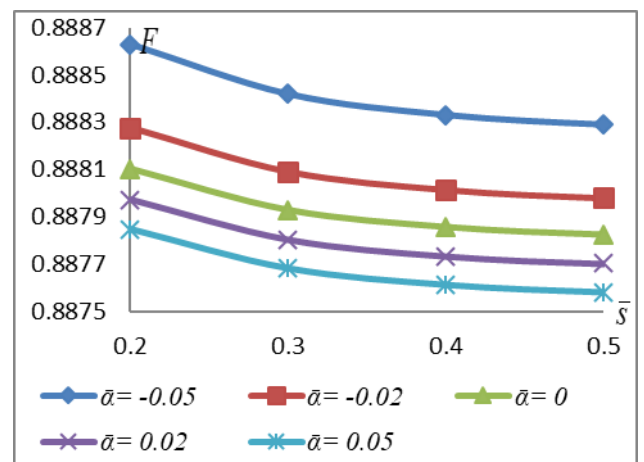


**Fig 15:** Variation of friction with respect to  $\mu'$  and  $\bar{\sigma}$



**Fig 13:** Variation of load bearing capacity with respect to  $\psi_2$  and  $\psi_1$

The effect of slip parameter described in Figures 18-19 establishes that the friction gets reduced significantly due to the slip parameter and this decrease in friction is relatively less in the case of standard deviation.



**Fig 16:** Variation of friction with respect to  $\bar{\sigma}$  and  $\bar{\alpha}$

The profiles of the variation of friction presented in Figures 16-17 make it clear that the magnetization reduces the friction considerably.

Figures 20-21 suggest that the friction reduces due to the standard deviation. Further, negatively skewed roughness increases the friction while the friction decreases with respect

to the positively skewed roughness. The variance follows the trends of skewness.

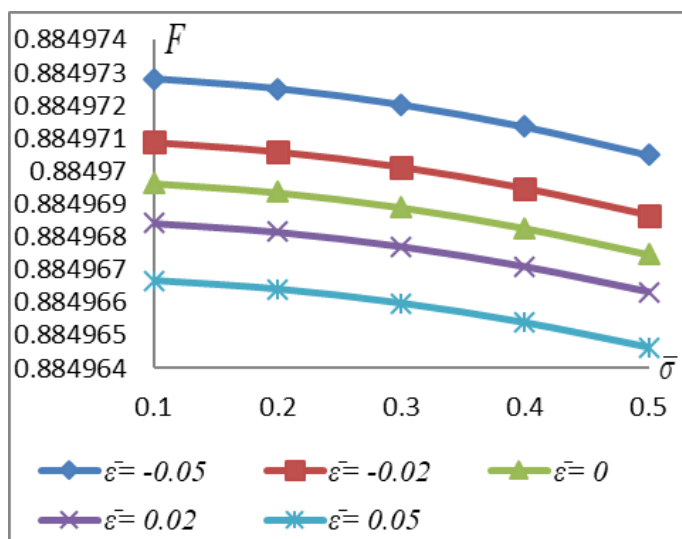


Fig 17: Variation of friction with respect to  $\bar{\sigma}$  and  $\bar{\epsilon}$

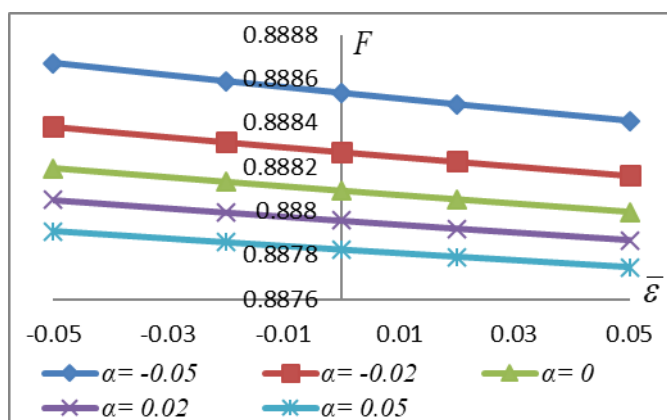


Fig 18: Variation of friction with respect to  $\bar{\epsilon}$  and  $\bar{\alpha}$

**4. Conclusion**

It is observed that this type of bearing system dealing with combined effect of double layered different porous structures may provide a better bearing design. The use of magnetic fluid lubricant not only improves the performance of the bearing system but also results in longer bearing life period. This analysis confirms that the roles of variance and skewness are equally crucial from bearing design point of view. Besides, this study establishes that the roughness should be accounted for while designing the bearing system even if, the magnetic field strength is suitably chosen.

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