International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452
Maths 2023; SP-8(4): 262-265

https://www.mathsjournal.com
Received: 12-04-2023
Accepted: 17-05-2023

Anil Chandra Jha
Valmeeki Campus, Nepal
Sanskrit University, Nepal

Dinesh Panthi
Valmeeki Campus, Nepal
Sanskrit University, Nepal

Shailendra Kumar Mishra
Institute of Engineering,
Tribhuvan University, Nepal

Corresponding Author:
Anil Chandra Jha
Valmeeki Campus, Nepal
Sanskrit University, Nepal

Hindu mathematics in Vedic period

Anil Chandra Jha, Dinesh Panthi and Shailendra Kumar Mishra

DOI: https://doi.org/10.22271/maths.2023.v8.i4Sd.1079

Abstract
Hindu mathematics in the Vedic period (c.3000 B.C. or probably much earlier) is a rich and diverse field that encompasses a wide range of mathematical concepts and techniques. The Vedas are ancient religious texts that contain a vast collection of knowledge, including mathematics. The Śulbasūtras, which are a part of the Vedas, contain instructions for the construction of altars and the performance of sacrificial rituals. In this paper, we explore the mathematical concepts found in the Vedas and Śulbasūtras and their importance in Hindu mathematics. We also examine how these texts provided practical solutions to real-world issues like calculating the area of fields and building altars for religious ceremonies, as well as laying the groundwork for advanced mathematical concepts.

Keywords: Hindu mathematics, Vedic period, Saṁhitās, Brāhmaṇas, Śulbasūtras

Introduction

Gaṇita, which literally means "the science of calculation," is the Hindu name for mathematics. The term is an extremely old one and happens bountifully in Vedic writing (Datta & Singh, 1962, p. 7) [5]. The Vedic period saw the development of several important mathematical texts, including Saṁhitās, Brāhmaṇas, the Śulbasūtras and the Vedāṅga Jyotiṣa. The oldest written sources on Hindu mathematics that we are aware of are the Śulbasūtras which mean rules of cord, a literary subgenre of the kalpa vedāṅgas. These texts were composed to provide instructions for building altars for sacrifices. (Pingree, 1981, p.3, & Plofker, 2009, p.17) [11, 9]. In a similar vein, ancient Hindus also practised jyotiṣa, which is also one of the vedāṅgas. Its primary goal was to perform sacrifices and other rites on time and according to the prescribed calendar (Rao, 2004, p. 25) [13]. These calendars were created and the most favourable times were calculated using the planetary motions, constellations, and other natural phenomena. As a result, systematic research into mathematical and astronomical phenomena began. Astrology has also made a significant contribution to the development of the field. Over the course of history, Hindu astronomers and mathematicians wrote hundreds of scientific works, some are still in use today. The Vedāṅga Jyotiṣa, composed around 1,200 B.C., offers it the most elevated honour among the sciences that make up the Vedāṅga:

yathā śikhā mayūrāṇāṁ nāgānāṁ maṇayo yathā |
tadvadvedāṅga śāstrāṇāṁ gaṇitaṁ mūrdhani sthitam || (Vedāṅga Jyotiṣa, 4)

The meaning of this verse is: Like the crown of the peacock's head and the jewels on the head of the cobra, Gaṇita is at the pinnacle of all science known as Vedāṅga.
The early period of each civilization is little known and thus vague. In any case, in the event of the progress that thrived in South Asia, of which the holy Vedas and different sacred texts are proof, we need to notice and note else. The ruins of the Indus Valley and the mention of numbers in a variety of contexts scattered throughout the various disciplines of the Vedas, as well as in their supplementary works such as the Vedāṅgas, the Purāṇas, and so forth (Deka, 2005, p. 11) [6]. These demonstrate that this civilization had a well-marked footing in the roots of mathematical thought, particularly and typically the counting of numbers. According to Tilak (1893, p. v) [17], the beginning of the concept of counting precedes the Ṛg-veda (3000 B.C. or indeed before). Dasaguṇottara (Decimal system) was well known by that time.

(Datta, 1932, p. 31) [4]. According to Arab historian Abul Hasan Al-Masaundi (943 A.D.), “The nine figures were created by a congress of sages at the command of the Creator Brahmā, as well as the astronomy and other sciences of the Hindus.” (Puttaswamy, 2012, p. 3) [12]. His claim that Brahmā is the one who invented the numbers is crystal clear. The Vedas’ primary and essential requirements were spiritual and philosophical hymns; Rāśividyā (mathematics) did not hold a separate place. Even then, the scriptures provide us with information by counting numbers. That also, in one way, demonstrates that people at the time were used to the number system.

The Vedic period may be studied under two sub-topics: Saṁhitās and Brāhmaṇas; and Śulbasūtras.

Saṁhitās and Brāhmaṇas

The Vedas (c. 3000 B.C. or probably much earlier) provide evidence of an early Hindu mathematics culture (Datta & Singh, 1962, p. 1) [5]. According to Apte (1962) [2], “The Vedas are śruti, which sets them apart from other religious texts, which are called smṛti.” (p.887). The Vedas, which literally means “Vidyā” (knowledge), are considered by Hindus to be apauruṣeya, which means “not of a man, superhuman” (ibid.). The Vedas are the earliest works written by Hindus. There are four of them: Ṛg-veda, Yajur-veda, Sāma-veda, and Atharva-veda, the Ṛg-veda being the world’s oldest known literary work. The Saṁhitās, the Āraṇyakas, the Brāhmaṇas and the Upniṣadas are the four disciplines that are contained within each Veda. The Vedas contain a wealth of mathematical knowledge, which is evident from the use of numbers in rituals and astronomical observations. The decimal system, rational numbers, and whole numbers with powers of ten are frequently mentioned and used in these Hindu texts in a variety of contexts. That is, Hindus have used 10 as their standard numeral since the earliest known times.

The Ṛg-veda and the Atharva-veda, for example, contain hymns that describe the importance of numbers in the universe, such as the concept of decimal system. In Ṛg-veda (vii. 183), a historical conflict in which the Śu-dās of Uttara Pāncāla defeated the ten kings combined is mentioned. In accordance with the Ṛg-veda (vii. 46. 22), King Pṛithusrabā gave Ṛiṣi Vāśobya 7000 horses, 2,000 camels, and 10,000 cows. According to Atharva-veda (viii. 8. 7), Indra and his army killed thousands of daśyus. This system, which is based on decades and powers of ten and includes combined numbers involving both decades and units, is mentioned in some of the ancient Vedic hymns (Plofker, 2007, pp. 13-4) [9].

Puttaswamy (2012) [12] remarks “The Greeks had no terminology for denominations above the myria (10^4) and the Romans the Millie (10^3). The ancient Hindus used at least 18 denominations” (p. 4). Even in modern times, no other nation's numeral language is as scientific and perfect as the Hindus'. The Yajurveda describes the use of mathematics in performing sacrificial rituals. The Yajurveda contains a list of numbers known as the Saṅkalpa Sūtras, which are used to determine the duration of a ritual. These numbers are based on the lunar calendar and are used to calculate the length of a day, month, and year. In the Yajur-veda Saṁhitā (xvii. 2), the following numeral list is provided: Eka ($1=10^0$), daśa ($10=10^1$), sata ($100=10^2$), sahasra ($1000=10^3$), ayuta ($10{,}000=10^4$), niyuta ($100{,}000=10^5$), prayuta ($1{,}000{,}000=10^6$), arbuda ($10{,}000{,}000=10^7$), nyarbuda ($100{,}000{,}000=10^8$), samudra ($1{,}000{,}000{,}000=10^9$), madhya ($10{,}000{,}000{,}000=10^{10}$), anta ($100{,}000{,}000{,}000=10^{11}$), and parārdha ($1{,}000{,}000{,}000{,}000=10^{12}$). The same list occurs at two places (iv. 40. 11.4) and (vii. 2. 20. 1) in Taittirīya Saṁhitā (Datta & Singh, 1962, p. 9) [5]. The Maitrāyanī (ii. 8. 14) and the Kāṭhaka Saṁhitā (xvii. 10) share a list that has been slightly modified (ibid., p. 10). The Pañcaviṁśa Brāhmaṇa has the Yajur-veda Saṁhitā list up to nyarbuda comprehensive and afterwards follow nikharva, vādava, akṣiti, and so on. The series that follows nyarbuda includes nikharva, samudra, salila, and ayuta (10 billion) in the Sāṅkhyāyana Srauta Sūtra.

The post-Vedic era, which spanned several millennia, saw the creation of extensive literature, including the Vedāṅgas, Purāṇas, Upaniṣads, all of which served specific purposes and two great epics, namely the Rāmāyaṇa and the Mahābhārata. The Purāṇas are a significant branch of Hindu sacred literature. Vedāṅgas were written to preserve and comprehend the divine character in its entirety. The six Vedāṅgas, namely Śikṣā, Kalpa, Vyākaraṇa, Nirukta, Chandaḥ, and Jyotiṣa respectively deal with phonetics, rituals, grammar, etymology, metric, and astronomy (Deka, 2005, p. 16) [6]. The astronomical supplementary or appendages to the Vedas known as the Vedāṅga Jyotiṣa provide us with the astronomical findings and mathematical wisdom of the ancient people. The wisdom of sage Lagadha (circa 1180 B.C.) is the source of the work Vedāṅga Jyotiṣa. It contains two recentions, one of which is related to the Ṛg-veda and is known as Ārka Jyotiṣa and the other to the Yajur-veda and is known as Yajuṣa Jyotiṣa. Metaphorically astronomy is known as the eyes of the Veda (ibid.). According to Kline (1961) [7], “These were written in the Sūtra, or aphoristic style, which is a form of expression characterized by great precision, conciseness, and word economy that is uncommon in all of literature.” (p. 266). Chandaḥ Sūtra kept up with numeration in its synthesis. The number of 12 letters (Varṇas) in a Chanda determines its name; for instance, the Gāyatrī Chanda has 24 letters. Starting from Gāyatrī to Utkṛti, the different Chandaḥs have letters 24, 28, 32, 36, 40, 44, 48, 52, 56, 60, 64, 68, 72, 76, 80, 84, 88, 92, 96, 100, and 104 respectively which are in A.P. (Deka, 2005, p. 12) [6].

Puttaswamy (2012) [12] remarks: “In the Yudha Kāṇḍa of Vālmīki Rāmāyaṇa (6-28-33), a spy of Rāvaṇa narrates to his King the precise strength of the army of his enemy Rāma. The spy assessed the strength of Sri Rama to be:

$$10^{10} + 10^{12} + 10^{20} + 10^{24} + 10^{30} + 10^{34} + 10^{40} + 10^{44} + 10^{52} + 10^{57} + 10^{62} + 5.$$

Five people who were on Rāvaṇa’s side have left Rāvaṇa and joined Rāma’s Camp here.” (P. 5).

In the Mahābhārata war, a division of troopers was called Akṣauhiṇi comprising of 109350 feet, 6561 cavalry, 21870 charioteers and 21870 elephant riders (Deka, 2005) [6]. At the Kurūkṣetra battlefield, the Pāndavas had eight divisions facing each other, while the Kaurava had fourteen of these divisions. Moreover, the total number of troopers is

$$(10 \times 3^7) \times (1+3+5) = 10 \times 3^9.$$

As a result, we can say that the Vedic texts clearly demonstrate a long history of numerology, as well as a variety of ideas about quantities both finite and infinite and their significance in the universe.

Śulbasūtras

The Śulbasūtras, which are a part of the Vedas, are texts that contain many geometric concepts, including the use of the Pythagorean Theorem, which was discovered by the Hindus long before Pythagoras. The Śulbasūtras are the earliest known texts in Hinduism that directly address mathematics (Knudsen, 2010, p. 63) [8]. The geometric techniques needed for the construction of the Vedīs (altars) and Agnis (fireplaces), described by the Brāhmaṇas as for the obligatory (nitya) and optional (kāmya) rites, comprise the Vedic period's mathematics. This knowledge is contained in Śulbasūtras which are a part of the so-called Kalpa Sūtras (more specifically, the Śrauta Sūtras) that are affixed to the Vedas as Vedāṅgas (Sridharan, 2005, p. 3) [16]. Nowhere in the Śulbasūtras is the word "Śulba" found (Amma, 1999, p. 2) [1]. The word Śulba means 'To measure' and Sūtras means 'rajju' or 'rope' so the meaning of Śulbasūtras is 'measuring tape or a cord'. For the construction of altars and fireplaces, the Śulbasūtras were used for operating instructions. Hence Śulbasūtras basically contain geometry of construction. There are nine Śulbasūtras on record, but only four of them are in extant. They are: Baudhāyana (800-500 B.C.), Āpastamba, Mānava (650-300 B.C.), and Kātyāyana (300-100 B.C.). According to Seidenberg (1978 & 1983), and Datta (1932) [4] remark: The composers of the Brāhmaṇas and Saṁhitās of the early Vedic period were familiar with the mathematical knowledge described in these sutras.

The following are some notable contributions to mathematics in the Śulbasūtras: The units of measurements (Bhūmīparimān) described in the Śulbasūtra texts are as follows: 1 Aṅgula = 14 Anus = 34 Tilas, 1 small pāda = 10 Aṅgulas, 1 Pradeśa = l2 Aṅgulas (= 1 Vitasti), 1 Pāda = 15 Aṅgulas, 1 Isa =188 Aṅgulas, 1 Aksa = I04 Aṅgulas, 1 Yuga = 86 Aṅgulas, 1 Jānu = 32 Aṅgulas, 1 Samya =36 Aṅgulas, 1 Bāhu = 36 Aṅgulas, 1 Prakarma = 2 Pādas, 1 Aratni = 2 Pradeśa = 24 Aṅgulas, 1 Puruṣa = 5 Aratnis = 120 Aṅgulas, 1 Vyāma = 4 Aratnis (Plofker, 2007, Sen & Bag, 1983) [9]. The parts of the human body are the source of these units. In this period the aṅgula was the standard unit for measuring lengths of different vedis and fireplaces.

The Śulbasūtras provides detailed instructions on how to construct altars and fireplaces (Amma, 1999, p. 5) [1]. Drawing the east-west line (prācī) and the north-south line (udīcī), a perpendicular bisector of the east-west line, is the fundamental step in the construction of any altar. The following verses provide the rules for building prācī and udīcī:

Same śaṁkuṁ nikhāya śaṁkusammitayā rajvā maṇḍalaṁ parilikhya yatra
lekhayoḥ śaṁkvagracchāyā nipatati tatra śaṁkū nihanti sā prācī | (K. Sl. I. 2)
tadantaraṁ rajvābhyasya, pāśau kṛtvā, śaṅkvoḥ pāśau pratimucya,
dakṣiṇāyamya madhye śaṁkuṁ nihanti |
evamutarataḥ sodīcī | (K. Sl. I. 3)

The meaning of the above two verses is: The prācī is obtained by fixing a gnomon (or pin) on level ground and drawing a circle with a gnomon-measured cord, and then by fixing pins at points on the line (of the circumference) where the gnomon's shadow falls. Then, after doubling the cord and making ties, one fixes the ties on the pins, stretches the cord to the south, and strikes a pin at the middle of the cord; the same is repeated to the north. That is the north-south line.

The present-day technique for drawing the perpendicular bisector of a line is same as this method. To get two points that are at equal distances from the two ends of the line, use the line as a base and draw isosceles triangles with connected vertices to the sides of the line. This is preferable to drawing intersecting arcs.

In Baudhāyana Śulbasūtra, the so-called Pythagorean Theorem is presented in the following manner:

dīrghasyākṣaṇayārajjuḥ pārśvamānī, tiryaṅgmānī,
ca yat pṛthagbhūte kurūtastadubhayaṁ karoti |(B. Sl. I. 48)

The meaning of this verse is: The square of the diagonal of a rectangle makes both (the squares) that the vertical side and horizontal side make independently. This is the oldest theorem in Geometry for Bhuja- Koṭi-Karṇa-Nyāya and

caturasrasyākṣṇayārajjurdvistāvatiṁ bhūmi karoti |(Āp. Sl. I. 4)

The meaning of this verse is: The diagonal cord of a square makes double the area (Amma, 1999, p. 19) [1]. The Śulbasūtras demonstrate complete knowledge of the sides and diagonals of rectangles and squares.

There are several triplets mentioned in the Śulbasūtras. Some of them are:
15; 36; 39 (Āp. Sl. V. 2; B. Sl. I. 49)
3; 4; 5 (Āp. Sl. V. 3; B. Sl. I. 49)
5; 12; 13, 7; 24; 25, & 12; 35; 37 (Āp. Sl. V. 4; B. Sl. I. 49)
72; 96; 120, 40; 96; 104, $2\frac{1}{2}$; 6; $6\frac{1}{2}$, $7\frac{1}{2}$; 10; $12\,\frac{1}{2}$ (M. Sl.)

The most surprising assertions found in Baudhāyana, Āpastamba and Kātyāyana Śulbasūtras is a rational approximation to √2. The rough estimate is:

samasya dvikaraṇī | *pramāṇaṁ tṛtīyena vardhayet*
taccaturthernatma catusasṁsiśonena saviśeṣaḥ | (B. Sl. I. 62, Āp. Sl. I. 5& K. Sl. II. 13)

The meaning of this verse is: The diagonal of the square is the double-maker. The measure to be increased by its one-third and this one-third again by its own one-fourth less the thirty-fourth part of that fourth; this is the value of a square's diagonal, whose side serves as the measurement. This is the saviśeṣa.

The following is a numerical representation of the above verse:

$$\sqrt{2} \approx 1+\frac{1}{3}+\frac{1}{(3)(4)}-\frac{1}{(3)(4)(34)}=\frac{577}{408}\approx 1.414216.$$

This approximation is correct up to five decimal places.

The Śulbasūtras describes the ways to change one geometrical figure into another. In fact, there is no exact geometric method for transforming a circle into a square with the same area. The Śulbasūtras provides the approximate construction (B. Sl. I. 58, Āp. Sl. III. 2, & K. Sl. III.). If the side of the rectangle is s and the circle's radius is r, then

$$r=\frac{s}{2}.\frac{2+\sqrt{2}}{3}.$$

Using this value of r, the calculated approximate value of π is 3.088 (approx.) (Amma, 1999, p. 34, & Puttaswamy, 2012, p. 26) [1, 12], which is not a good representation of the true value of π.

Fractions and surds are dealt in the Śulbasūtras. Fractions like thri aṣṭama (3/8), dvisaptama (2/7), caturbhagoṇa (3/4), and so forth and surds of the form $\sqrt{2}, \sqrt{3}$ and so on are called karaṇīs. For instance, $\sqrt{2}$ is dvikaraṇī, $\sqrt{3}$ is trikaraṇī, $\sqrt{18}$ is aṣṭadaśa karaṇī, $\sqrt{1/3}$ is tṛtīya karaṇī etc. In the Śulbasūtras, it is found that the Asvamedhikī Vedikā has the shape of an isosceles trapezium with face, base, and altitude of $24\sqrt{2}$; $30\sqrt{2}$; and $36\sqrt{2}$ prakramas respectively, with a total area of 1944 square prakramas (Puttaswamy, 2012, p. 34) [12].

The Śulbasūtras also contain properties of the square, rectangle, rhombus, trapezium, triangle, and circle. Contingent upon this reality, a few fascinating outcomes seen in the Śulbasūtras are: A line segment can be divided into several equal segments; the diagonals of the rectangle bisect each other; they divide this rectangle into four parts, two of which are perpendicular to each other, and the two parts are equal in all respects; the diagonals of the rhombus make a right angle; an isosceles triangle passing through a vertex and divided into two halves by a line perpendicular to the base; a circle can be divided into several parts of the diameter; Constructions of a square with a given side, a rectangle with a given length and breadth, an isosceles trapezium with a given altitude, face, and base, a square that equals the sum and difference of two different squares, a square that is equivalent to a given isosceles triangle, a square that is equivalent to a given rectangle in area, a rhombus with a given area (Amma, 1999, Dani, 2010, & Puttaswamy, 2012) [1, 12, 3].

Conclusion

The mathematics found in the Vedas and Śulbasūtras played a significant role in the development of mathematics. It was used in astronomy, astrology, architecture, and other areas of life. The Vedic period (c. 3000 B.C. or probably much earlier) provides the use of numbers, including the decimal system, rational numbers, and whole numbers which are powers of ten in a variety of contexts, which are still used today. The Vedic texts (Saṁhitās and Brāhmaṇas) obviously show a long practice of decimal numeration and a profound association with the different ideas of both finite and infinite quantities and their importance in the universe. This system allowed for efficient calculations and paved the way for the development of advanced mathematical concepts. The earliest known Hindu texts that deal directly with mathematics are the Śulbasūtras which are available from the period 800 B.C. There are nine Śulbasūtras on record. Four of them, namely Baudhāyana, Āpastamba, Mānava and Kātyāyana, are the most significant in mathematics. Consistency and completeness of geometrical results and their application to actual construction are Śulbasūtras' most notable characteristics. Overall, Hindu mathematics from the Vedic period in the history of mathematics has had a profound impact on modern mathematics and continues to inspire new generations of mathematicians and scholars around the world.

Acknowledgement

The authors would like to thank Professor Dinesh G Sarvate for his valuable suggestions on this manuscript.

References

1. Amma TS. Geometry in Ancient and Medieval India. Second Revised Edition. Motilal Banarsidass Publishers Pvt Ltd. Delhi; c1999.
2. Apte VS. The Practical Sanskrit-English Dictionary. 4th Ed. Motilal Banarsidass. Delhi; c1965.
3. Dani SG. *Geometry* in the Śulbasūtras. In Studies in the History of Indian Mathematics CS Seshadri (Ed.). Hindustan Book Agency. New Delhi; c2010.
4. Datta BB. The Science of the Śulba: A Study in Early Hindu Mathematics. Calcutta University Press. Calcutta; c1932.
5. Datta B, Singh AN. History of Hindu Mathematics; Parts I and II. Single Volume Edition. Asian Publishing House. Bombay; c1962.
6. Deka RC. An Analytical Study of Continuous and Chronological Development of Mathematics. Pro Quest; c2005.
7. Kline M. Mathematics - A Cultural Approach. Addition Wesley Publishing Co. USA; c1961.
8. Knudsen TK. On the Application of Areas in the Śulbasūtras. In Ancient Indian Leaps into Mathematics. B. S. Yadav, & Man Mohan (Eds.). Birkhauser; c2010, p. 63-74.
9. Plofker K. Mathematics in India. Princeton University Press. Princeton, New Jersey; c2007.
10. Plofker K. Mathematics in India. Princeton University Press. Princeton, New Jersey; c2009.
11. Pingree D. Census of the Exact Sciences in Sanskrit: Series A, Philadelphia: American Philosophical Society; c1981. p. 4.
12. Puttaswamy TK. Mathematical Achievements of Pre-modern Indian Mathematicians. Elsevier; c2012.
13. Rao SB. Indian Mathematics and Astronomy: Some Landmarks. 3rd Ed. Jnana Deep Publication, Bangalore; c2004.
14. Seidenberg A. The Origin of Mathematics. In Archive for History of Exact Sciences. 1978;18(4);301-42. Springer.
15. Seidenberg A. Agni: The Vedic Ritual of the Fire Altar. 2nd ed. Motilal Banarsidass. Delhi; c1983.
16. Sridharan R. Mathematics in Ancient and Medieval India. Science in India: Past and Present. Nehru Centre, Mumbai; c2005, p. 1-29.
17. Tilak BG. The Orion or the Antiquity of the Vedas. 1st Ed. Mrs. Radhabai Atmaram Sagoon. Bombay; c1893.