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Estimation of HIV/TB Co-infection and HIV/AIDS in India: Application of truncated distributions

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Abstract
This paper intends to study the general trend and distribution of deaths in the working-age population due to HIV-TB co-infection and HIV/AIDS. The data is collected from Institute for Health Matrix and Evaluation from 1991 to 2015 (GBD 2015). An attempt is made to fit the Doubly Truncated Normal Distribution, Doubly Truncated Lognormal Distribution and Doubly Truncated Weibull Distribution to the working-age population of five year block periods. Doubly Truncated Lognormal Distribution well fitted as compare to other distributions and its mean increased from one block period to next block periods in both the diseases. Lognormal distribution is consistent as compare to others and its probability curve indicates improvement in the life span of the people who live with HIV.

Keywords: HIV/AIDS, TB, Co-infection, doubly truncated probability distributions, MLE, measures of dispersion

1. Introduction

HIV and TB can individually be the major causes for public health threats and the combination of the two has proven to have a far greater impact on the epidemiologic progression and consequently on the global health scene. Due to this relationship there has been a dramatic increase in the incidence of tuberculosis in countries with a high prevalence of HIV and TB. TB is one of the most common causes of death from infectious disease is the world being second only to HIV/AIDS. Most deaths occur in developing countries, affect the young people in their productive years (Pande, J. N., 2004) [11].

Human Immunodeficiency Virus (HIV) infection has become a pandemic far more extensive than what was predicted even a decade ago. The global spread has been so swift that no country has been spared and the pace of the epidemic is increasing in India (Kumaraswamy, N, *et al*., 2003) [7]. Tuberculosis remains the most common opportunistic infection and is the commonest cause of death in HIV infected patients. Clinical presentation of TB in early HIV infection resembles that observed in immune-competent persons but in a later stage, the clinical presentation of TB can be atypical. Co-infection with HIV leads to challenges in both the diagnosis and treatment of tuberculosis. Diagnosis of TB in HIV infected patients may be delayed because of atypical clinical presentation and involvement of inaccessible sites and low sputum smear positivity (Sharma, S. K., & Mohan, A., 2004) [12].

TB is one of the symptoms of AIDS in more than 50% of cases in developing countries. TB reduces the survival of patients suffering from HIV infection, speeding HIV progression and causing death in one-third of AIDS worldwide. Due to the increase in viral replication of Mycobacterium Tuberculosis, the highest deaths have led to AIDS progression rather than TB (Swaminathan, S., & Narendran, G., 2008) [13]. According to the UNAIDS 2009 estimate, 33.4 million people living with HIV / AIDS, one-third of which also infect TB. HIV seropositivity has a vast difference in the TB patients in India, which is about 30% in Mumbai from 9.4% in Delhi (Narain, J. P., & Lo, Y. R., 2004) [9].

Co-infection of HIV and TB is severely associated with malnutrition, drug abuse, alcoholism, unemployment, refugee, poverty and illiteracy (Ghiya, R. *et al*., 2009) [4]. Among the risk factors for co-infection, heterosexual sexuality and occasional sexuality were seen by some Indian observers as the most important; others noted that most people were abusers of intravenous drugs (Bhagyabati, D. S. *et al*., 2005) [2].

In the recent decade strategies implemented by collaborative HIV and TB control program, the mortality of co-infection has shown a quite declining trend. Since 2004, TB-related deaths among people living with HIV have declined by 36% worldwide at the end of 2012. WHO estimates that scaling up collaborative HIV and TB activities prevented about 1.3 million people from dying from 2005 to 2012. More people with TB is now receiving antiretroviral therapy (Van der Werf, M. J., *et al*., 2016) [15]. In India approximately 36% of infected adults (18-45 years) are receiving the therapy according to UNAIDS the Gap report 2014. As there is an increased burden worldwide and all over India, this study is taken up to know the current trends of these diseases through Doubly Truncated Normal Distribution (DTND), Doubly Truncated Lognormal Distribution (DTLND) and Doubly Truncated Weibull Distribution (DTWD). Nagamani *et al*. considered the estimation of common dispersion parameter λ of two inverse Gaussian populations when the location (mean) parameters μ_1& μ_2 and all the proposed estimators are compared in terms of their bias and mean squared errors (MSEs) through simulation and demonstrated the potential application of their model (N. Nagamani & Tripathy M. R., 2018) [10]. The rest of the paper is structured as follows: First section briefs about the introduction and significance of HIV/TB co-infection, section 2 discusses the Methods and Materials used, section 3 focuses on results and discussion and the last section concludes the study.

2. Methods and Materials

This paper focuses to study the general trend and distribution of deaths in the working-age group i.e. 15-59 years due to HIV-TB co-infection and HIV/AIDS. Data on deaths in the working-age group due to HIV-TB co-infection have been collected from Institute for Health Matrix and Evaluation (IHME) from 1991 to 2015 (GBD, 2015). The classification of five years block periods is done accordingly 1991-1995, 1996-2000, 2001-2005, 2006-2010 and 2011-2015. The study is attempted to fit the Doubly Truncated Normal Distribution (DTND), Doubly Truncated Lognormal Distribution (DTLND) and Doubly Truncated Weibull Distribution (DTWD) to the working-age group of each block periods of deaths occurred under the age of 15 years and over 59 years by neglecting non-working age groups. Here, age at death is considered as a random variable.

Maximum Likelihood Estimation method has been used to estimates the parameters of the DTND, DTLND and DTWD but due to implicit form of likelihood equations, the parameters of the distribution are estimated using numerical method *viz* Newton's Method [3]. Akaike's Information Criterion (AIC) and Bayesian Information Criterion (BIC) is calculated for understanding better fitting of the model.

2.1 Doubly Truncated Distribution

Doubly truncated distribution is applicable when observations have bounded in specified interval $[T_1, T_2]$ i.e. observation is taken into consideration only when they exceed a lower bond and smaller than some upper bond. Hence, doubly truncated distributions can be applicable to the working age group deaths due to HIV-TB co-infection data. Considering first truncated point is $T_1 = 14.5$ and second truncated point $T_2 = 59.5$ i.e. ignoring the deaths occurred under 15 and over 59 years.

2.1.1 Doubly Truncated Normal Distribution

If random variable x follows normal distribution and cdf of X is $\Phi\left(\frac{x-\mu}{\sigma}\right)$. Then pdf of doubly truncated normal distribution is

$$f_{DT}(x,\mu,\sigma,T_1,T_2) = \frac{\frac{1}{\sigma\sqrt{2\pi}} e^{-\left(\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right)}}{F(T_2)-F(T_1)}, T_1 < x < T_2, -\infty < \mu < \infty, \sigma > 0 \quad (1)$$

In standard units of the complete distribution, the $T_1 < x < T_2$ truncation points are $\xi_1 = \frac{T_1-\mu}{\sigma}$ and $\xi_2 = \frac{T_2-\mu}{\sigma}$. It follows that

$F(T_1) = \Phi(\xi_1)$ and $F(T_2) = \Phi(\xi_2)$.

The cumulative distribution function of the doubly truncated lognormal distribution is

$$F_{DTND}(x) = \frac{\Phi\left(\frac{x-\mu}{\sigma}\right) - \Phi(\xi_1)}{\Phi(\xi_2) - \Phi(\xi_1)}$$

Where, $\Phi(.)$ is cumulative distribution function of normal distribution. μ is location parameter and σ is scale parameter.

2.1.2 Doubly Truncated Lognormal Distribution

If random variable x follows lognormal distribution and cdf of X is $\Phi\left(\frac{\ln(x)-\mu}{\sigma}\right)$. Then pdf of doubly truncated lognormal distribution is

$$f_{DT}(x,\mu,\sigma,T_1,T_2) = \frac{\frac{1}{x}\frac{1}{\sigma\sqrt{2\pi}} e^{-\left(\frac{1}{2}\left(\frac{\ln(x)-\mu}{\sigma}\right)^2\right)}}{F(T_2)-F(T_1)}, T_1 < x < T_2, -\infty < \mu < \infty, \sigma > 0 \ldots \quad (2)$$

Where

$$F(T_1) = \Phi\left(\frac{\ln(T_1)-\mu}{\sigma}\right) \text{ and } F(T_2) = \Phi\left(\frac{\ln(T_2)-\mu}{\sigma}\right)$$

The cumulative distribution function of the doubly truncated lognormal distribution is

$$F_{DTLND}(x) = \frac{\Phi\left(\frac{\ln(x)-\mu}{\sigma}\right) - \Phi\left(\frac{\ln(T_1)-\mu}{\sigma}\right)}{\Phi\left(\frac{\ln(T_2)-\mu}{\sigma}\right) - \Phi\left(\frac{\ln(T_1)-\mu}{\sigma}\right)}$$

Where, $\Phi(.)$ is cumulative distribution function of normal distribution. μ is location parameter and σ is scale parameter.

2.1.3 Doubly Truncated Weibull Distribution:

If random variable x follows weibull distribution and cdf of X is $\left(1 - e^{-\left(x/k\right)^p}\right)$. Then pdf of doubly truncated weibull distribution is

$$f_{DT}(x, p, k, T_1, T_2) = \frac{\frac{p}{k}\left(\frac{x}{k}\right)^{p-1} e^{-\left(x/k\right)^p}}{e^{-\left(T_1/k\right)^p} - e^{-\left(T_2/k\right)^p}}, T_1 \leq x \leq T_2, p > 0, k > 0 \ ... \quad (3)$$

The cumulative distribution function of the doubly truncated weibull distribution is

$$F_{DTWD}(x) = \frac{e^{-\left(T_1/k\right)^p} - e^{-\left(x/k\right)^p}}{e^{-\left(T_1/k\right)^p} - e^{-\left(T_2/k\right)^p}}$$

Where k is scale parameter and p is shape parameter. Truncation points T_1 and T_2 are usually assumed to be known.

2.2 Method of parameters estimation

We have used maximum likelihood estimation method to estimates the parameters DTWD, DTLND and DTND with known truncation points T_1 and T_2.

2.2.1 The likelihood function of DTND

$$L = -N\ln\left(\sigma\sqrt{2\pi}\right) - 0.5\sum_{i=1}^{n} f_i \left(\frac{x_i-\mu}{\sigma}\right)^2 - N\ln\left(\Phi(\xi_2) - \Phi(\xi_1)\right) ... \quad (4)$$

Estimated the parameters μ and σ which are maximize the equation (4) with known values of T_1 and T_2. Obtained the parameters by solving the simultaneous solution of the following equations

$$\frac{\partial L}{\partial \mu} = \frac{N}{\sigma^2}\left(\bar{x} - \mu - \sigma(\bar{P_1} - \bar{P_2})\right) = 0 \ ... \quad (5)$$

$$\frac{\partial L}{\partial \sigma} = \frac{N}{\sigma^3}\left(s^2 + (\bar{x} - \mu) - \frac{N}{\sigma}(1 + \xi_1\bar{P_1} - \xi_2\bar{P_2})\right) = 0 \ ... \quad (6)$$

Where $\bar{P_1} = \frac{\phi_1}{\Phi_2-\Phi_1}$, $\bar{P_2} = \frac{\phi_2}{\Phi_2-\Phi_1}$. $\Phi = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$ and $\Phi = \int_{-\infty}^{z} \phi(t)dt$

These explicit forms of likelihood equations are solved in order to estimate the parameters μ and σ using Newton's Method (Cohen. 1991) [3].

2.2.2 The likelihood function of DTLND

$$L = -N\ln\left(\sigma\sqrt{2\pi}\right) - \sum_{i=1}^{n} f_i\ln(x_i) - 0.5\sum_{i=1}^{n} f_i \left(\frac{\ln(x_i)-\mu}{\sigma}\right)^2 - N\ln\left(\Phi(\xi_{22}) - \Phi(\xi_{11})\right) ... \quad (7)$$

Where $\xi_{11} = \frac{\ln(T_1)-\mu}{\sigma}$ and $\xi_{22} = \frac{\ln(T_2)-\mu}{\sigma}$

We have estimated the parameters μ and σ which are maximize the equation (7) with known values of T_1 and T_2. We have obtained the parameters by solving the simultaneous solution of the following equations

$$\frac{\partial L}{\partial \mu} = \frac{1}{\sigma}\sum_{i=1}^{n} f_i \left(\frac{\ln(x_i)-\mu}{\sigma}\right) + \frac{N}{\sigma}(\bar{P_1} - \bar{P_2}) = 0 \ ... \quad (8)$$

$$\frac{\partial L}{\partial \sigma} = -\frac{N}{\sigma} + \frac{1}{\sigma}\sum_{i=1}^{n} f_i \left(\frac{\ln(x_i)-\mu}{\sigma}\right)^2 - \frac{N}{\sigma}(\xi_{11}\bar{P_1} - \xi_{22}\bar{P_2}) = 0 \ ... \quad (4)$$

Where $\overline{P_1} = \frac{\phi(\xi_{11})}{\Phi(\xi_{22})-\Phi(\xi_{11})}$ and $\overline{P_2} = \frac{\phi(\xi_{22})}{\Phi(\xi_{22})-\Phi(\xi_{11})}$

Since x is random variable following the lognormal distribution, then ln(x) follows normal distribution. To estimate the parameters of the lognormal distribution we have taken the log transformation of the untruncated observation to convert into the normal distribution. The estimation of parameters of truncated lognormal distribution is given in (Cohen, 1991) [3].

2.2.3 The log-likelihood function of DTWD

$$L = N(\ln(p) - \ln(k)) + (p-1)\sum_{i=1}^{n} f_i \ln\left(\frac{x_i}{k}\right) - \sum_{i=1}^{n} f_i \left(\frac{x_i}{k}\right)^p - N\ln\left(e^{-\left(T_1/k\right)^p} - e^{-\left(T_2/k\right)^p}\right) \quad \ldots (10)$$

For the known values of T_1 and T_2, we need to estimates the unknown parameters p and k which can maximize the log-likelihood function of the DTWD, the estimates can be obtained by solving simultaneous solution of the following equations.

$$\frac{\partial L}{\partial p} = \frac{N}{p} + \sum_{i=1}^{n} f_i \ln\left(\frac{x_i}{k}\right) - \sum_{i=1}^{n} f_i \left(\frac{x_i}{k}\right)^p \ln\left(\frac{x_i}{k}\right) + \frac{N\left(e^{-\left(T_1/k\right)^p} \ln\left(\frac{T_1}{k}\right)\left(T_1/k\right)^p - e^{-\left(T_2/k\right)^p} \ln\left(\frac{T_2}{k}\right)\left(T_2/k\right)^p\right)}{e^{-\left(T_1/k\right)^p} - e^{-\left(T_2/k\right)^p}} = 0 \quad \ldots \quad (11)$$

$$\frac{\partial L}{\partial p} = -\frac{Np}{k} + \frac{p}{k}\sum_{i=1}^{n} f_i \left(\frac{x_i}{k}\right)^p + \frac{N\left(e^{-\left(T_1/k\right)^p}\left(T_1/k\right)^p - e^{-\left(T_2/k\right)^p}\left(T_2/k\right)^p\right)\left(\frac{p}{k}\right)}{e^{-\left(T_1/k\right)^p} - e^{-\left(T_2/k\right)^p}} = 0 \quad \ldots \quad (12)$$

Since likelihood equations (11) and (12) are in the form of implicit form thus iteration method has been applied to estimate the parameters. We have used the well-known iteration method i.e. Newton Raphson Method for multi-parameter situation using the second order partial derivatives.

2.3 Estimation of Mean and Measures of dispersion of Doubly Truncated Distributions

We have estimated mean and variance of DTND, DTLND and DTWD using the r^{th} moment of origin and also derived measures of dispersion *viz*. Coefficient of Variation, Coefficient of Skweness and Coefficient of Kurtosis.

2.3.1 Estimation of Mean and Measures of Dispersion of DTND

The r^{th} moment of origin of the DTND can be obtained by the moment generating function which is given below

$$M_X(t) = \frac{e^{\hat{\mu}t + \hat{\sigma}^2 t^2}}{c_1}\left(\Phi(\xi_2 - \hat{\sigma}t) - \Phi(\xi_1 - \hat{\sigma}t)\right) \quad (13)$$

Where $c_1 = \Phi(\xi_2) - \Phi(\xi_1)$

Differentiating equation (13) with respect to t four times and putting $t = 0$ to get the first four moments about the origin. Thus expected mean and variance are given below.

$$E(X) = \mu_1' = \hat{\mu} + \frac{\hat{\sigma}A}{c_1}$$

$$V(X) = \mu_2' - (\mu_1')^2 = \hat{\sigma}^2\left\{1 + \frac{B}{c_1} - \left(\frac{A}{c_1}\right)^2\right\}$$

Where $A = \phi(\xi_1) - \phi(\xi_2)$ and $B = \xi_1\phi(\xi_1) - \xi_2\phi(\xi_2)$

Again we derived some useful measures like coefficient of variation (CV), coefficient of skewness (β_1) and coefficient kurtosis (β_2) from equation (15) using the relationship between moments of origin and moments about mean. The expressions of measures are mentioned above

$$CV = \frac{\sqrt{V(X)}}{E(X)} = \frac{\hat{\sigma}\sqrt{\left\{1 + \frac{B}{c_1} - \left(\frac{A}{c_1}\right)^2\right\}}}{\hat{\mu} + \frac{\hat{\sigma}A}{c_1}}$$

$$\beta_1 = \frac{\mu_3}{\mu_2^{3/2}} = \frac{\frac{C}{c_1} - \frac{3AB}{{c_1}^2} + 2\left(\frac{A}{c_1}\right)^3}{\left\{1 + \frac{B}{c_1} - \left(\frac{A}{c_1}\right)^2\right\}^{3/2}}$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{3 + \frac{D}{c_1} + \frac{6B}{c_1} - \frac{4AC}{{c_1}^2} - \frac{6A^2}{{c_1}^2} + \frac{6A^2B}{{c_1}^3} - \frac{3A^4}{{c_1}^4}}{\left\{1 + \frac{B}{c_1} - \left(\frac{A}{c_1}\right)^2\right\}^2}$$

Where $C = \phi(\xi_1)({\xi_1}^2 - 1) - \phi(\xi_2)({\xi_2}^2 - 1)$ and $D = \xi_1\phi(\xi_1)({\xi_1}^2 - 3) - \xi_2\phi(\xi_2)({\xi_2}^2 - 3)$

2.3.2 Estimation of Mean and Measures of Dispersion of DTLND

The r[th] moment about the origin of the DTLND is given below

$$\mu'_r = \frac{e^{\hat{\mu}r + \hat{\sigma}^2 r^2}}{c_2}\left(\Phi(\xi_{22} - \hat{\sigma}r) - \Phi(\xi_{11} - \hat{\sigma}r)\right) \ldots \quad (14)$$

Where $c_2 = \Phi(\xi_{22}) - \Phi(\xi_{11})$

The expected mean and variance of the DTLND using equation (14) are mentioned below

$$E(X) = \frac{e^{\hat{\mu} + \frac{\hat{\sigma}^2}{2}}}{c_2} E$$

$$V(X) = \frac{e^{2\hat{\mu} + \hat{\sigma}^2}}{c_2^2}\left[c_2 e^{\hat{\sigma}^2} F - E^2\right]$$

We derived some useful measures of DTLND like CV, β_1 and β_2 from equation (14) using the relationship between moments of origin and moments about mean. The expressions of measures are mentioned below.

$$CV = \frac{\sqrt{V(X)}}{E(X)} = \frac{\sqrt{c_2 e^{\hat{\sigma}^2} F - E^2}}{E}$$

$$\beta_1 = \frac{\mu_3}{\mu_2^{3/2}} = \frac{c_2^2 e^{3\hat{\sigma}^2} G - 3c_2 e^{\hat{\sigma}^2} EF + 2E^3}{\left(c_2 e^{\hat{\sigma}^2} F - E^2\right)^{3/2}}$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{c_2^3 e^{6\hat{\sigma}^2} H - 4c_2^2 e^{3\hat{\sigma}^2} GE + 6c_2 e^{\hat{\sigma}^2} FE^2 - 3E^4}{\left(c_2 e^{\hat{\sigma}^2} F - E^2\right)^2}$$

Where $E = \Phi(\xi_{22} - \hat{\sigma}) - \Phi(\xi_{11} - \hat{\sigma})$, $F = \Phi(\xi_{22} - 2\hat{\sigma}) - \Phi(\xi_{11} - 2\hat{\sigma})$

$G = \Phi(\xi_{22} - 3\hat{\sigma}) - \Phi(\xi_{11} - 3\hat{\sigma})$, $H = \Phi(\xi_{22} - 4\hat{\sigma}) - \Phi(\xi_{11} - 4\hat{\sigma})$

1.3.3 Estimation of Mean and Measures of Dispersion of DTWD

The r[th] moment about the origin of the DTWD is given below

$$\mu'_r = \frac{k^r}{c_3}\left[\gamma\left(\frac{r}{p} + 1, \left(\frac{T_2}{k}\right)^p\right) - \gamma\left(\frac{r}{p} + 1, \left(\frac{T_1}{k}\right)^p\right)\right] \ldots \quad (15)$$

Where $c_3 = e^{-\left(\frac{T_1}{k}\right)^p} - e^{-\left(\frac{T_2}{k}\right)^p}$

The expected mean and variance of the DTWD using equation (15) are mentioned below

$$E(X) = \frac{kI}{c_3} \; and \; V(X) = \frac{k^2}{{c_3}^2}\left[J - c_3 I^2\right]$$

We have studied the dispersion of age at death of DTWD like CV, β_1 and β_2 from equation (15) using the relationship between moments of origin and moments about mean. The expressions of measures are mentioned above

$$CV = \frac{\sqrt{V(X)}}{E(X)} = \sqrt{\frac{J}{I^2} - c_3}$$

$$\beta_1 = \frac{\mu_3}{\mu_2^{3/2}} = \frac{c_3{}^2 K - 3c_3 JI + 2I^3}{[J - c_3 I^2]^{3/2}} \text{ and } \beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{c_3{}^3 L - 4c_3{}^2 KI + 6c_3 JI^2 - 3I^4}{[J - c_3 I^2]^2}$$

Where $I = \gamma\left(\frac{1}{p}+1, \left(\frac{T_2}{k}\right)^p\right) - \gamma\left(\frac{1}{p}+1, \left(\frac{T_1}{k}\right)^p\right)$, $J = \gamma\left(\frac{2}{p}+1, \left(\frac{T_2}{k}\right)^p\right) - \gamma\left(\frac{2}{p}+1, \left(\frac{T_1}{k}\right)^p\right)$

$K = \gamma\left(\frac{3}{p}+1, \left(\frac{T_2}{k}\right)^p\right) - \gamma\left(\frac{3}{p}+1, \left(\frac{T_1}{k}\right)^p\right)$ and $L = \gamma\left(\frac{4}{p}+1, \left(\frac{T_2}{k}\right)^p\right) - \gamma\left(\frac{4}{p}+1, \left(\frac{T_1}{k}\right)^p\right)$.

3. Results and Discussions

The doubly truncated normal distribution, lognormal distribution and Weibull distribution are fitted to the deaths due to HIV/TB co-infection and HIV/AIDS in the working-age group for five block periods. The Maximum Likelihood Estimation (MLE) method is used to estimate the parameters of the probability distributions mentioned above and considered two different criteria *viz* AIC and BIC for the best distribution fit are to conclude according to the lowest values of the AIC and BIC.

3.1 HIV/TB Co-infection and HIV/AIDS

The estimation of the parameters of the DTND, DTLND, and DTWD using MLE for all five-block periods for deaths due to HIV/TB co-infection and HIV/AIDS is given in Table 1. In the first block period 1991-1995 for HIV/TB data, the scale parameter i.e. logarithmic mean of DTLND is 3.58 and shape parameter i.e. logarithmic standard deviation of DTLND is 0.28.

Table 1: Estimation of parameters of the DTND, DTLND and DTWD.

Years	HIV/TB						HIV/AIDS					
	DTLND		DTND		DTWD		DTLND		DTND		DTWD	
	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\mu}$	$\hat{\sigma}$	$\hat{p}$	$\hat{k}$	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\mu}$	$\hat{\sigma}$	$\hat{p}$	$\hat{k}$
1991-1995	3.58	0.28	36.27	9.89	3.95	39.46	3.58	0.29	36.09	9.98	3.89	39.30
1996-2000	3.60	0.29	36.67	10.12	3.93	39.77	3.59	0.29	36.45	10.13	3.89	39.59
2001-2005	3.64	0.29	38.06	10.21	4.07	40.85	3.63	0.29	37.63	10.10	4.05	40.54
2006-2010	3.68	0.30	39.09	10.36	4.17	41.62	3.66	0.29	38.55	10.15	4.17	41.25
2011-2015	3.71	0.33	39.73	10.91	4.11	41.94	3.69	0.32	39.25	10.67	4.13	41.67
1991-2015	3.65	0.30	38.20	10.39	4.04	40.91	3.64	0.30	37.98	10.28	4.05	40.78

***Source*:** Authors' Computation

It is observed that scale parameter has slightly increased from the first block period to last block period indicating the logarithmic age distributed deaths which are stretching. At the same time shape parameter has also increased indicating the little changes in the shape of the distribution. The Location parameter of DTND is 36.27 and the scale parameter is 9.89. It is observed that location is 36 years age at death in the first block period changes to 40 years in the last block period. The shape and scale parameter of DTWD is 3.94 and 39.46 respectively and is increasing along with time.

HIV/AIDS data has also similar pattern as HIV/TB co-infection that logarithmic age distributed deaths are stretching and little changes can be observed in the shape of the distribution. The location parameter value of DTND is 36.08 and that of scale parameter is 9.97. In the first block period, the location of the distribution is 36 years of age that can be shifted to 39 years of age in the last block period 2011-2015. Shape and scale parameter of DTWD is 3.89 and 39.30 respectively and are increasing along with the time (See Table 1).

Table 2 gives the AIC and BIC for best fit of the distribution for HIV/TB and HIV/AIDS. Analysis shows, the AIC and BIC values are least for all block periods for lognormal distribution as compared to the other two distributions. This indicates the lognormal distribution is best fitted to the deaths of HIV/TB and HIV/AIDS as compared to the other two distributions and the poor fit is observed for Weibull distribution (See Fig 1a to 1f and Fig 2a to 2f).

Table 2: AIC and BIC values for DTLND, DTND and DTWD: HIV/TB Co-infections and HIV/AIDS.

HIV/TB						
Years	DTND		DTLND		DTWD	
	AIC	BIC	AIC	BIC	AIC	BIC
1991-1995	400179	400197	398047	398065	443756	443774
1996-2000	942413	942433	875907	875926	975369	975389
2001-2005	1456222	1456243	1347693	1347714	1499793	1499813
2006-2010	1343930	1343950	1242622	1242642	1379493	1379513
2011-2015	930654	930674	851010	851029	939206	939225
1991-2015	5185502	5185525	4732612	4732635	5244795	5244818
HIV/AIDS						
Years	DTND		DTLND		DTWD	
	AIC	BIC	AIC	BIC	AIC	BIC

1991-1995	1065797	1065817	1059637	1059657	1059637	1059657
1996-2000	2870723	2870745	2669814	2669836	2669814	2669836
2001-2005	5181261	5181284	4795597	4795620	4795597	4795620
2006-2010	5564046	5564069	5143238	5143261	5143238	5143261
2011-2015	4255830	4255852	3964103	3964126	3964103	3964126
1991-2015	19390499	19390524	17657319	17657345	17657319	17657345

***Source*: Authors' Computation**

In the first block period (1991-1995), HIV/TB co-infected individuals died in large numbers and it got continued up to the block period 2001-2005, but in later periods deaths due to this co-infection got gradually decreased. This may be the reason that the World Health Organization, Global Tuberculosis programme has promoted the TB management strategy is called Directly Observed Treatment Short-course (DOTS) strategy (Pande, J. N. 2004) [11]. This aims that identifying the TB infected individual and diagnosing them until they are cured of TB. The HIV infected individuals have responded well to the standard anti-tuberculosis treatment using the DOTS strategy. So by 2005, it is seen that the decline in the deaths due to this co-infection is noticeable one.

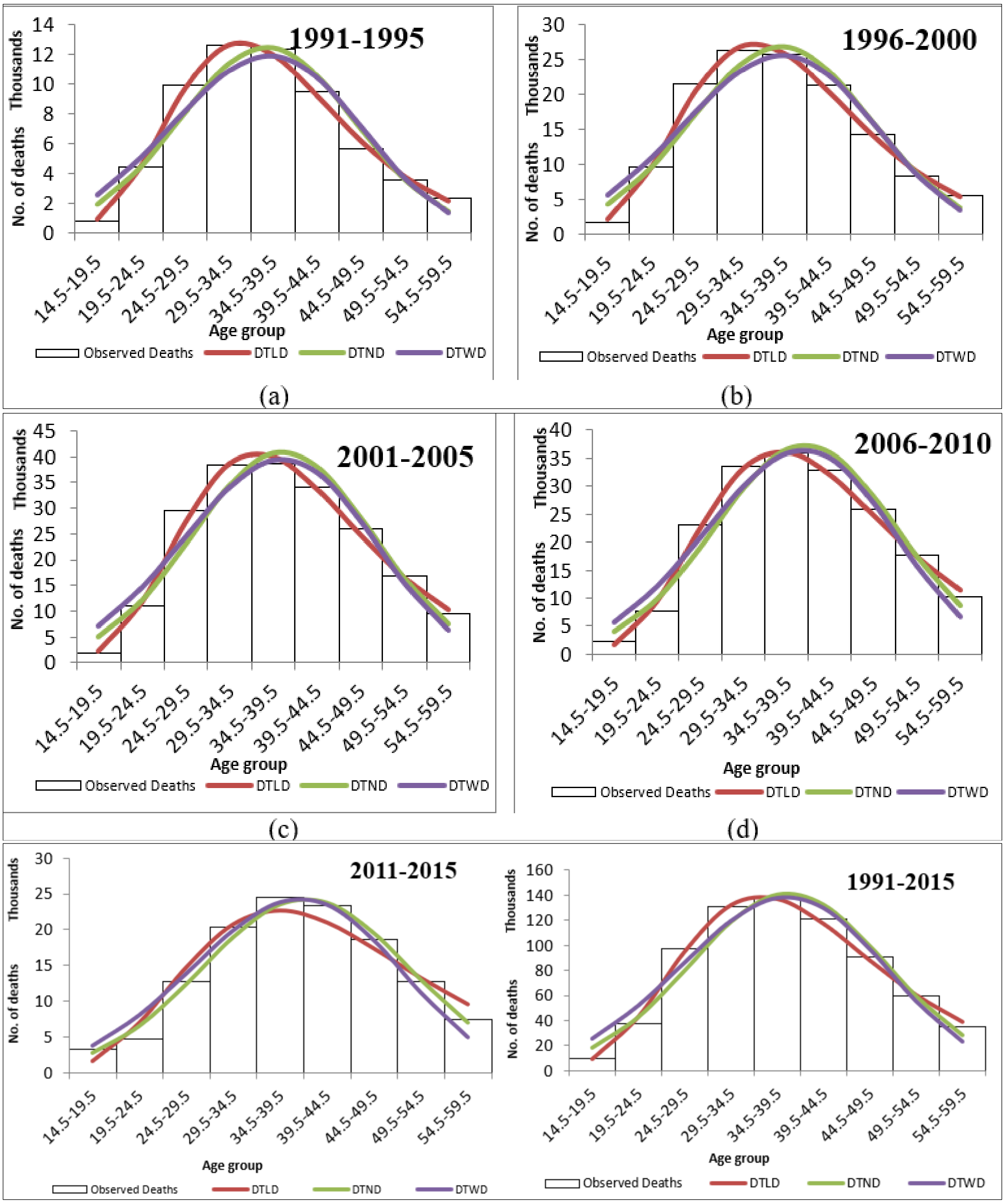

Fig 1a to 1f: Doubly Truncated probability distributions fitting to the age wise deaths due to the HIV-TB co-infection.

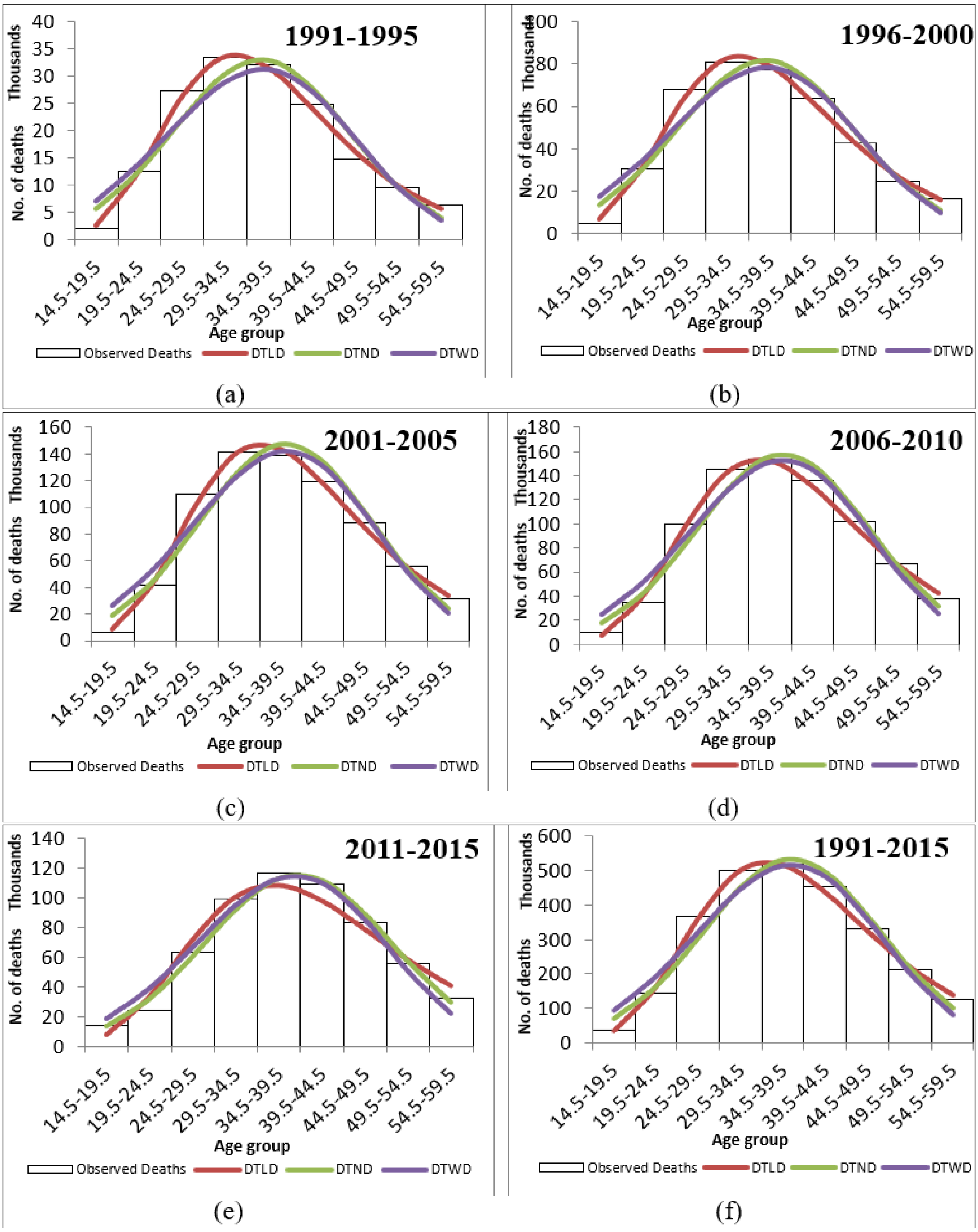

Fig 2a to 2f: Doubly Truncated probability distributions fitting to the Age wise deaths due to HIV/AIDS.

UNAIDS stated that by 2004, the number of TB- related deaths among the people living with HIV has declined. Since, there is an increase in the number of ART treatment receivers who have both HIV and TB. The average age at deaths due to HIV/TB co-infection and HIV/AIDS in the first block period is approximately 36 years by the above-mentioned truncated distribution. In the later period some variations can be seen in the mean of the three distributions for different block periods (See Table 3).

Table 3: Mean and Variance for HIV/TB co-infection and HIV/AIDS.

Years	HIV/TB						HIV/AIDS					
	DTLND		DTND		DTWD		DTLND		DTND		DTWD	
	Mean	Var	Mean	Var	Mean	Var	Mean	Var	Mean	Var	Mean	Var
1991-1995	36.37	83.88	36.37	84.06	36.04	89.43	36.22	84.49	36.22	84.98	35.89	90.31
1996-2000	36.72	86.51	36.72	86.65	36.27	91.14	36.54	86.34	36.54	86.74	36.12	91.34
2001-2005	37.89	87.51	37.89	87.40	37.19	91.10	37.54	86.31	37.54	86.33	36.94	90.47
2006-2010	38.74	90.00	38.74	88.37	37.84	91.08	38.32	87.96	38.31	86.51	37.56	89.92
2011-2015	39.19	98.79	39.16	93.34	38.04	93.26	38.85	96.21	38.82	91.38	37.85	92.20
1991-2015	38.00	90.50	37.99	89.21	37.22	92.17	37.82	89.55	37.82	88.21	37.12	91.45

***Source*:** Authors' Computation

But in the recent block period (2011-2015) it got increased by 3 years i.e. 39 years is the age at deaths due to HIV/TB co-infection and HIV/AIDS when DTLND and DTND are considered. The average age at death is increased by 2 years i.e. 38 years for DTWD for both the diseases (see Table 3). It is observed that the average age at death is slightly high when DTLND considered than the other two truncated distribution and minimum variance is also observed in the DTLND. The mean and variance of the whole period is in between third and fourth block periods.

As shown in the Fig. 3, Weibull; approximately 60 to 70 per cent of the deaths occurred in the middle adults i.e., 25- 44 years old. The higher probability of dying due to both the diseases *viz*. HIV/TB co-infection and HIV/AIDS can be seen in the 30-34 age groups in the first block period, and it is continued to the next block period also in the same age group. But in the third and fourth block periods, the probability is high in the 35-39 age groups and the fifth; 40-44 age group has a high probability.

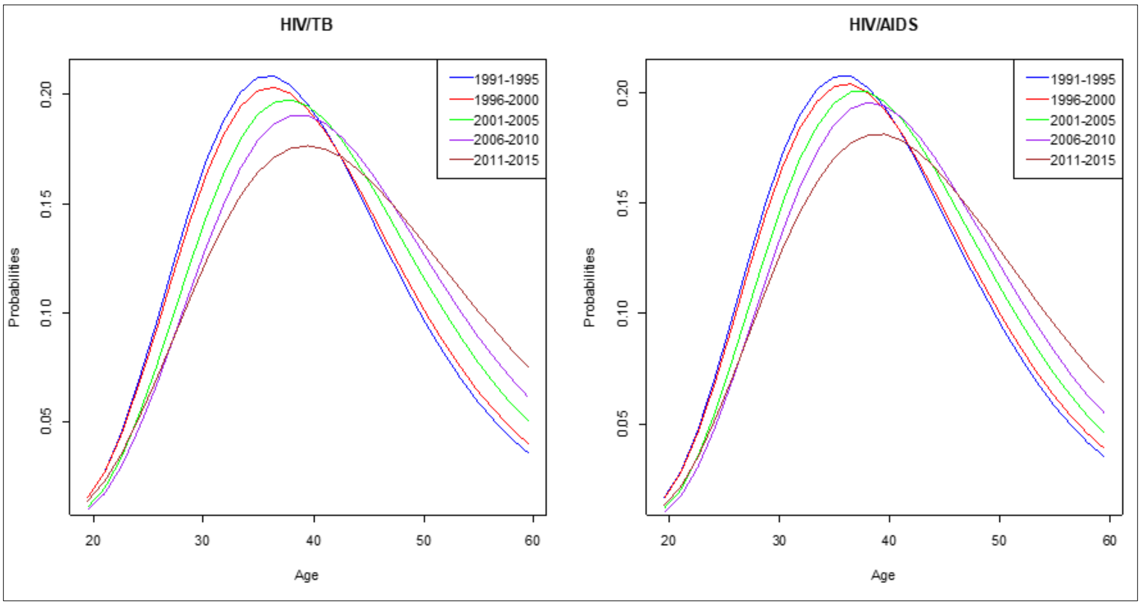

Fig 4: Probability density curves of DTLND, HIV/TB co-infection and HIV/AIDS.

In some of the studies the most affected age by co-infection was 36-45 years old followed by 26-35 years old (Balla, F., *et al*., 2016) [1], 25-44 years (Van der Werf, M. J., *et al*., 2016) [15] and 30-42 years (Kantipong, P., *et al*., 2012) [6], it may be due to behavioral variability in terms of exposure to the co-infection and various socio-economic practices. It can be observed that age distributed deaths curves have become flatter and flatter from the first block period to last block period and the peak of the curve is also shifting to the right. This indicates the improvement in the life span of the people living with HIV.

For doubly truncated normal and lognormal distribution the value of CV is observed that is approximately 25% of the variation in the deaths due to HIV/TB co-infection data in all the block periods, and DTWD gives the CV as 26%.

Table 4: Measures of dispersion for the DTND, DTLND and DTWD for HIV/TB co-infection.

	DTND			DTLND			DTWD		
Year	**CV (%)**	β_1	β_2	**CV**	β_1	β_2	**CV**	β_1	β_2
1991-1995	25.21	0.0333	3.6538	25.17	0.3033	2.5127	26.24	-0.0015	2.4187
1996-2000	25.35	0.0156	3.7755	25.33	0.2713	2.4582	26.31	-0.0094	2.4053
2001-2005	24.67	-0.0501	3.8511	24.68	0.1918	2.3757	25.66	-0.0680	2.4162
2006-2010	24.27	-0.1002	4.0206	24.48	0.1203	2.3086	25.22	-0.1083	2.4268
2011-2015	24.67	-0.1342	4.5088	25.36	0.0401	2.2235	25.38	-0.1154	2.4079
1991-2015	24.86	-0.0574	3.9736	25.03	0.1697	2.3404	25.80	-0.0661	2.4059

Note: CV- Coefficient of variance, β_1 -Coefficient of Skewness, β_2 -Coefficient of Kurtosis.
***Source*:** Author's computation

The conclusion can be made as DTLND explains better than other truncated distribution about the age distributed deaths due to HIV/TB co-infection. The co-efficient of skewness shows that the density curve of the age distributed deaths is shifting to the right over a period of time. This scenario can be observed in the all considered truncated distribution. This may indicate that the maximum number of deaths are happening in the right part of the curve i.e. may be greater than 35 years of age. In the meantime, by co-efficient of kurtosis, the curve seems to be flatter than the normal curve which indicates that number of the deaths is decreasing slightly over period of time when lognormal distribution assumption is made (See Table 4).

Table 5: Measures of dispersion of the DTND, DTLND and DTWD for HIV/AIDS.

	DTND			DTLND			DTWD		
Year	**CV**	β_1	β_2	**CV**	β_1	β_2	**CV**	β_1	β_2
1991-1995	25.45	0.0423	3.7090	25.38	0.3101	2.5161	26.47	0.0118	2.4100
1996-2000	25.49	0.0257	3.7867	25.47	0.2871	2.4767	26.46	0.0011	2.4024
2001-2005	24.75	-0.0297	3.7675	24.75	0.2207	2.4087	25.75	-0.0544	2.4188
2006-2010	24.28	-0.0731	3.8417	24.47	0.1606	2.3478	25.24	-0.0949	2.4339
2011-2015	24.62	-0.1096	4.2576	25.25	0.0799	2.2533	25.37	-0.1059	2.4154
1991-2015	24.83	-0.0465	3.8944	25.02	0.1862	2.3590	25.76	-0.0624	2.4117

Note: CV- Coefficient of variance, β_1 -Coefficient of Skewness, β_2 -Coefficient of Kurtosis**.**
Source: Author's computation

Table 5 interpretation is similar to Table 4. Age distributed deaths due to HIV/AIDS can be explained better by DTLND than the other two truncated distribution because of minimum CV. The pattern of the density curve of the age distributed deaths due to HIV/AIDS is similar as HIV/TB co-infection. It is also shifting to the right and in the meantime peakedness of the curve is also decreasing. Decrease in the deaths due to HIV/AIDS may be because of advancement and affordability of medical facilities and more importantly wide and effective implementation of ART in large scale. It is estimated that the scale-up of free ART since 2004 has saved cumulatively around 4.5 lakhs lives in India until 2014. Approximately 12 lakhs adults aged 15+ got free ART in the year 2015 (NACO, 2015) [8].

4. Conclusions

HIV/TB co-infection is growing slowly in developing countries like India. TB accounts for about one-third of deaths among patients with AIDS. The Doubly Truncated Lognormal Distribution is well fitted to the data of both the diseases for the working-age group in all block periods that can be concluded by the minimum values of AIC and BIC, and also a consistency of deaths is checked by the lesser CV value than the other two truncated distributions. The logarithmic mean of age at deaths has increased from one block period to the next block periods in both the diseases. The higher probability of dying due to HIV/TB co-infection and HIV/AIDS can be seen in the 30-34 age groups in the first block period**.** The observation is that age distributed deaths curve has become flatter and flatter from first block period to last block period i.e., kurtosis is increasing from first block period to last block period and a peak of the curve is also shifting to the right indicating that improvement in the life span of the people living with the HIV.

The following are a few recommendations

- Early diagnosis of these individual infections and screening for detection of the co-infection is a crucial step in arresting the progress of these deadly dual infections by the initiation of appropriate treatment.
- A sustained effort by the, people & Govt. including N.G.O's can bring down the rates further down to achieve the goals of the govt. of India & WHO.
- The impact of dual infection of HIV & TB on the economy and public health is enormous with increased morbidity & mortality. Screening of all T.B patients for HIV & vice versa will help in early detection and initiation of appropriate treatment at an early stage thus, reducing the mortality rate.

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