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Meteorological drought analysis using multi-scalar SPEI and Copula theory for Jaisalmer district of Rajasthan, India

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Abstract
Drought is a natural disaster of creeping phenomenon which is difficulty to understand. This study utilizes multi-scalar SPEI and copula theory to analyze the meteorological drought in Jaisalmer district of Rajasthan, India. Several marginal distributions are fitted to drought duration and severity. Archimedean and Metaelliptical copulas are used to construct the bivariate probability distributions. The univariate and bivariate return periods are calculated to identify the associate drought risks. The results of this study can provide effective information for the study area to assess drought risk and contribute to the decision-making process to reduce the associated risks. This can result in optimizing the allocation of water resources and reduce the impact of drought on the Jaisalmer district in the future.

Keywords: Copulas, drought, SPEI, Jaisalmer, Rajasthan

1. Introduction
Drought, an often-occurring natural disaster, has a significant impact on the ecology, social economy, and agriculture. It has been observed that the alteration of drought may emerge more swiftly compared to the average climate change in the context of global warming (Dai *et al.*, 2004) [2]. The magnitude of the matter becomes more severe due to the steady growth of industry and agriculture, accompanied by the expansion of social and economic development, the rapid escalation of the world's population, and the amplification of global warming. These developments have led to an upsurge in the demand for water, leading to an acute scarcity of water resources. Consequently, the global trend of drought is becoming increasingly conspicuous. Drought, a phenomenon characterized by a deficiency in water supply, is a multi-faceted concept that can be categorized into various types based on the field of study. These categories include hydrological drought, meteorological drought, agricultural drought, and socioeconomic drought. Meteorological drought, in particular, is triggered when the amount of precipitation falls below the average level for a prolonged period of time, thereby negatively influencing the other types of droughts (Duan *et al.*, 2014) [3]. It is imperative to note that the evaluation of meteorological drought holds significant value in understanding and assessing the overall impact and implications of drought. There are several drought indices available to assess the meteorological drought (Pei *et al.*, 2020) [7]. This study used Standardized Precipitation Evapotranspiration Index (SPEI) to assess the meteorological drought. The SPEI is a climatological tool that is akin to the Standardized Precipitation Index (SPI). Nonetheless, unlike the SPI, the SPEI takes into account the surface evapotranspiration that arises from the backdrop of global warming. Additionally, it substitutes precipitation with the contrast between monthly precipitation and the potential evapotranspiration (Gao *et al.*, 2017) [4]. The drought characteristics namely severity and duration are extracted with the SPEI values. The analysis of drought characteristics can be executed using either a univariate method or a multivariate method (Adarsh *et al.*, 2018) [1]. The univariate method is a conventional drought frequency analysis approach. However, due to the strong correlation existing between drought characteristics, multivariate analysis is capable of characterizing the drought situation more comprehensively. The Copula method is an exceptional technique for the evaluation of the joint probability distribution of multiple variables.

Its most significant advantage is that it does not necessitate the utilization of the idea that the marginal distribution of a univariate is independent (Zhang and Singh, 2007) [15]. Rajasthan is a state in India which is prone to frequent droughts (Rathore, 2005) [8]. Thus, this study aims to analyse the meteorological drought in a western district (Jaisalmer) of Rajasthan by employing SPEI and copula theory.

2. Materials and Methods

2.1 Study area and Data used

The study was conducted for Jaisalmer district in Rajasthan state of India. It is located between 26.913°N latitude and 70.915°E longitude in the west of the state. It is the largest district of the state with an area of about 38,401 km^2. Jaisalmer, being a city located in the desert, endures notably severe weather conditions, with summers being exceedingly hot and winters equally cold. In particular, the summer season tends to stretch from March to August and is characterized by an almost unbearable heat, with temperatures oscillating between 22 °C to 4 °C. On the other hand, the monsoon season in Jaisalmer typically spans from July to September and accounts for almost 70% of the total annual rainfall, with July being the rainiest month. Nevertheless, the monsoon season in the district is almost negligible due to its presence in the Thar Desert, which results in dry conditions for most of the year. As a matter of fact, western disturbances bring around 15 cm of rainfall every year. The winter season usually starts around mid-November and lasts until the end of February. The monthly rainfall data spanning for 50 years (1971-2020) was obtained from the Department of Water Resources, Govt. of Rajasthan. The maximum and minimum monthly gridded temperature data was obtained from the India Meteorological Department (Srivastava *et al.*, 2009) [11].

2.2 Standardized Precipitation Evapotranspiration Index (SPEI)

The Standardized Precipitation Evapotranspiration Index, commonly referred to as SPEI, serves as a replacement for the traditional monthly rainfall measurement utilized in the Standardized Precipitation Index (SPI). However, the SPEI distinguishes itself from the SPI by incorporating the difference between monthly rainfall and monthly potential evapotranspiration, as well as considering the temperature factor (Vicente-Serrano *et al.*, 2010) [12]. Additionally, the SPEI introduces the influence of surface evaporation changes, which proves to be more sensitive to the drought reaction caused by global temperature rise. Therefore, the SPEI represents a more comprehensive and accurate method for assessing drought conditions. The difference between monthly rainfall and monthly potential evapotranspiration i.e., the water balance was calculated using the Hargreaves method (Hargreaves and Samani, 1985) [5] using the SPEI package in R environment (Vicente-Serrano *et al.*, 2010) [12].

The SPEI is calculated by normalizing the water balance as log-logistic probability distribution. The pdf of the log-logistic distribution is given as follows:

$$f(x) = \frac{\beta}{\alpha}\left(\frac{x-\gamma}{\alpha}\right)\left[1 + \left(\frac{x-\gamma}{\alpha}\right)\right]^{-2} \quad (1)$$

where, α, β, and γ are the scale, shape, and origin parameters, respectively. The cumulative distribution function is given as follows:

$$F(x) = \left[1 + \left(\frac{\alpha}{x-\gamma}\right)^{\beta}\right]^{-1} \quad (2)$$

Finally, the SPEI is calculated as follows:

$$SPEI = W - \frac{C_0 + C_1W + C_2W^2}{1 + d_1W + d_2W^2 + d_3W^3} \quad (3)$$

When $P \le 0.5, W = \sqrt{-2\,ln(P)}$ and when $P > 0.5, W = \sqrt{-2\,ln(1-P)}$, $P = 1 - F(x)$, $C_0 = 2.5155$, $C_1 = 0.8028$, $C_2 = 0.0203$, $d_1 = 1.4327$, $d_2 = 0.1892$, and $d_3 = 0.0013$. The drought was classified into several categories based on SPEI values (Wang *et al.*, 2020) [13] as shown in Table 1.

Table 1: Classification of drought based on SPEI values

Category	SPEI value
Extremely Wet	SPEI > 2
Severely Wet	1.5 < SPEI ≤ 2
Moderately Wet	1 < SPEI ≤ 1.5
Normal	-1 < SPEI ≤ 1
Moderate drought	-1.5 < SPEI ≤ -1
Severe drought	-2 < SPEI ≤ -1.5
Extreme drought	SPEI ≤ -2

The SPEI was calculated for 1-, 3-, 6-, 9-, and 12-month time scales. The drought was defined at an SPEI threshold of -1. The drought characteristics namely the duration and severity were extracted from the multi-scalar SPEI values by employing the Run theory proposed by Yevjevich (1967) [14].

2.3 Marginal distributions

Several marginal distributions were fitted to drought duration and severity values in order to establish their joint probability distributions. Eight probability distributions namely Normal, GEV, Weibull, Log-Normal, Gamma, Exponential, Logistic, and Log-Logistic were used to fit drought severity. Except the GEV, other seven distribution were used to fit for drought duration. The parameters of the marginal distributions were estimated using the Maximum Likelihood Estimation (MLE) method. The goodness of fit of probability distribution was assessed using the AIC, BIC, and Kolmogorov-Smirnov test statistic values. The dependence between the drought characteristics was identified by the Kendall's tau.

2.4 Copula theory

The copula theory results from the Sklar's theorem (Sklar, 1959) [10]. If x and y are two random variables with joint probability distribution function as $F_{X,Y}(x,y)$ and marginal probability distribution functions as $F_X(x)$ and $F_Y(y)$, then according to Sklar's theorem there exists a copula $C(x,y)$ such that $F_{X,Y}(x,y) = C\big(F_X(x), F_Y(y)\big)$. In this study, three Archimedean copulas (Clayton, Frank, and Gumbel) and a Metaelliptical (Gaussian) copula were used to model the joint distribution of drought duration and severity. The Kendalls's tau was used to estimate the copula parameters. The RMSE, AIC, and BIC values were calculated to identify the best fit copula model. The copula expressions, parameter range and relationship with Kendall's tau are given in Table 2.

Table 2: Mathematical expressions of the copulas

Copula	Expression	Parameter (θ)	Relation with Kendall's Tau
Gaussian	$\emptyset_G[\emptyset^{-1}(u_1), \emptyset^{-1}(u_2)]$	$-1 \le \theta \le 1$	$\tau = \frac{6}{\pi} arcsin\left(\frac{\theta}{2}\right)$
Clayton	$max\left([u_1^{-\theta} + u_2^{-\theta} - 1]^{-\frac{1}{\theta}}, 0\right)$	$\theta \in -1, \infty)/\{0\}$	$\tau = \frac{\theta}{\theta + 2}$
Gumbel	$exp\left(-[(-\ln u_1)^\theta + (-\ln u_2)^\theta]^{\frac{1}{\theta}}\right)$	$\theta \in 1, \infty)$	$\tau = \frac{\theta - 1}{\theta}$
Frank	$-\frac{1}{\theta} ln\left(1 + \frac{(e^{-\theta u_1} - 1)(e^{-\theta u_2} - 1)}{e^{-\theta} - 1}\right)$	$\theta \in (-\infty, \infty)/\{0\}$	$\tau = 1 - \frac{4}{\theta}[1 - D_1(\theta)]$ $D_k(x)$ is the Debye function $D_k(x) = \frac{k}{x^k}\int_0^x \frac{t^k}{e^t - 1} dt$

2.5 Univariate and Bivariate Return Periods

The return periods of drought events are useful to relate the magnitude of drought to the frequency of occurrence by using probability distributions. The univariate return periods (Kim *et al.*, 2003) [6] of drought duration and severity are denoted by T_D and T_S and given as follows:

$$T_D = \frac{N}{n[1 - F_D(d)]} \quad (4)$$

$$T_S = \frac{N}{n[1 - F_S(s)]} \quad (5)$$

where $F_D(d)$ and $F_S(s)$ are the best fitted marginal probability distributions of drought duration and severity, respectively. N is the length of study period in years and n is the total number of drought events.

The bivariate return periods are given in terms of AND- and OR-case (Salvadori and De Michele, 2004) [9]. The AND case bivariate return period is given as:

$$T_{DS}^{AND} = \frac{N}{n[P(D \ge d \text{ and } S \ge s)]} = \frac{N}{n[1 - F_D(d) - F_S(s) + C(F_D(d), F_S(s))]} \quad (6)$$

$$T_{DS}^{OR} = \frac{N}{n[P(D \ge d \text{ or } S \ge s)]} = \frac{N}{n[1 - C(F_D(d), F_S(s))]} \quad (7)$$

3. Results and Discussion

The number of months under moderate, severe, and extreme droughts for different scales of SPEI are shown in Figure 1. The number of drought months decreased with the aggregation of time scale for moderate and extreme categories. The SPEI-9 was an exception for the moderate drought category wherein the drought months were higher than the preceding time scale. The number of drought months increased with aggregation of time scale for the extreme drought category. The percentage of moderate, severe, and extreme droughts for different SPEI is shown in Figure 2. The moderate drought was highest for SPEI-1. The severe drought was highest for SPEI-12. The extreme drought was highest for SPEI-3. The drought characteristics identified by the Run theory are presented in Table 3. As expected, the duration and severity of drought increased with the aggregation of time scales. Thus, highest duration of drought was 13 months (from July-2002) for SPEI-12. The highest severity was 23.03 for SPEI-12 during July-2002.

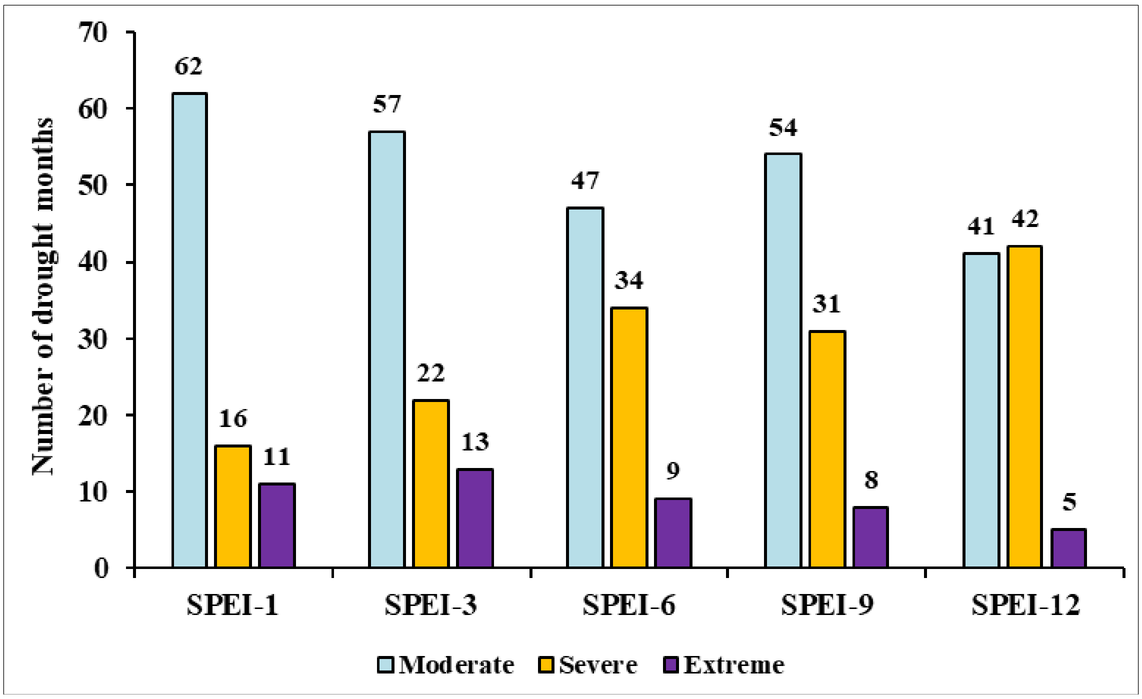

Fig 1: Number of months under moderate, severe, and extreme droughts for different scales of SPEI

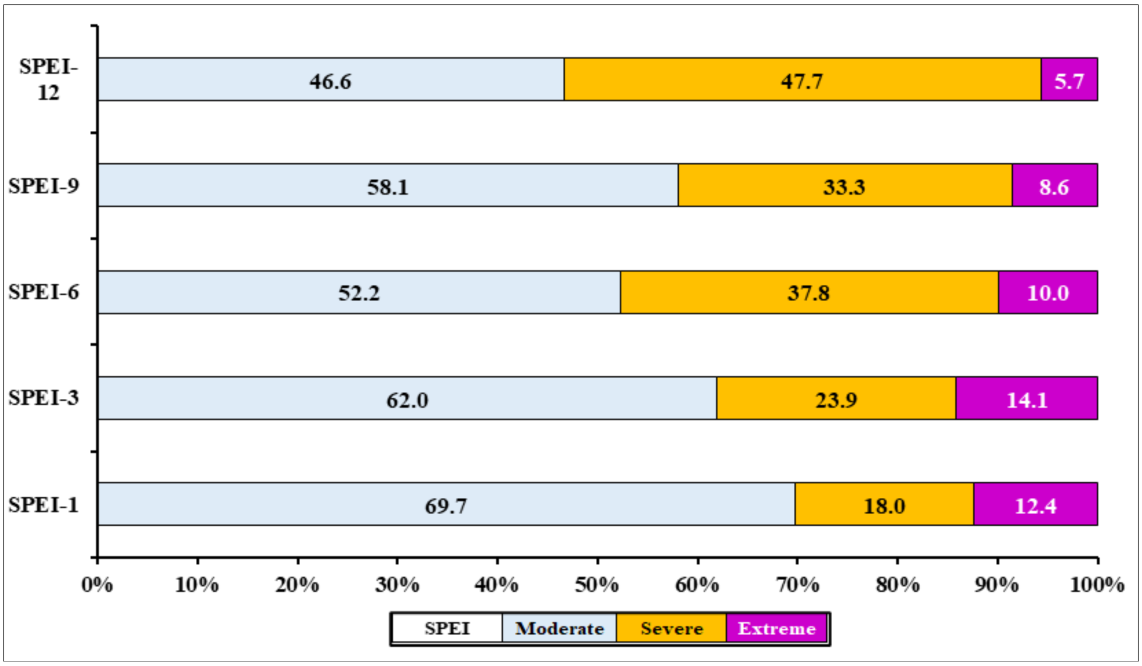

Fig 2: Percentage of moderate, severe, and extreme droughts for different SPEI

Table 3: Drought characteristics identified by Run theory

Drought Characteristic	Statistic	SPEI-1	SPEI-3	SPEI-6	SPEI-9	SPEI-12
Duration (months)	Min	1	1	1	1	1
	Max (Starting year)	4 (Jul-1987)	7 (Aug-1987 and Mar-2004)	11 (Aug-1987)	12 (Aug-1972, May-1986, Aug-1987, and May-2002)	13 (Jul-2002)
	Mean	1.31	2.09	3.21	5.17	7.33
Severity	Min	1.02	1.06	1.03	1.05	1.01
	Max (Starting year)	6.93 (Jul-1987)	13.94 (Aug-1987)	18.84 (Aug-1987)	20.42 (Aug-1987)	23.03 (Jul-2002)
	Mean	1.93	3.14	4.87	7.78	11.27

The fitted marginal distributions for drought duration and severity are given in Table 4 (a and b). The least values of AIC, BIC, and K-S statistic values were used to identify the best fitted marginal distribution. Based on the AIC, BIC, and K-S statistic values for the drought duration, the Log-Logistic distribution was fitted best for SPEI-1. The Log-Normal distribution was fitted best for SPEI-3 and SPEI-6. The exponential distribution was fitted best was SPEI-9 and SPEI-12 values. For the drought severity, the log-logistic and exponential distributions fitted best for SPEI-1 and SPEI-12 values, respectively. The log-normal distribution fitted best for SPEI-3, SPEI-6, and SPEI-9 values.

Table 4(a): Fitted marginal distributions for drought duration

SPEI	Best Fit Measure	Distribution						
		Normal	Weibull	Log-Normal	Gamma	Exponential	Logistic	Log-Logistic
SPEI-1	AIC	142.495	125.468	88.208	104.384	174.601	124.515	77.242
	BIC	150.826	133.798	96.539	112.715	178.767	132.846	85.573
	K-S Statistic	0.471	0.440	0.480	0.481	0.534	0.419	0.416
SPEI-3	AIC	164.841	144.376	132.682	139.622	154.909	160.100	136.065
	BIC	172.301	151.836	140.142	147.082	158.639	167.560	143.525
	K-S Statistic	0.286	0.289	0.271	0.321	0.380	0.326	0.305
SPEI-6	AIC	143.011	123.960	118.633	122.985	123.386	142.218	121.355
	BIC	149.568	130.517	125.190	129.541	126.664	148.774	127.912
	K-S Statistic	0.286	0.260	0.238	0.280	0.267	0.251	0.268
SPEI-9	AIC	111.830	99.120	97.222	99.103	97.120	113.993	99.513
	BIC	117.503	104.793	102.894	104.776	99.956	119.666	105.185
	K-S Statistic	0.349	0.291	0.250	0.298	0.238	0.319	0.290
SPEI-12	AIC	76.428	74.875	77.270	75.292	73.818	78.002	78.559
	BIC	81.290	79.736	82.131	80.153	76.249	82.863	83.420
	K-S Statistic	0.281	0.327	0.329	0.329	0.263	0.328	0.304

Note: Best fitted values are shown in bold figures

Table 4(b): Fitted marginal distributions for drought severity

SPEI	Best Fit Measure	Distributions							
		Normal	GEV	Weibull	Log-Normal	Gamma	Exponential	Logistic	Log- Logistic
SPEI-1	AIC	226.069	170.753	198.657	171.401	185.754	227.082	214.637	118.717
	BIC	234.399	179.084	206.988	179.732	194.085	231.247	222.968	131.213
	K-S Statistic	0.270	0.194	0.237	0.244	0.261	0.413	0.255	0.079
SPEI-3	AIC	220.814	175.086	188.015	160.278	184.363	190.705	210.788	177.448
	BIC	228.274	182.547	195.475	171.469	191.823	194.435	218.248	184.908
	K-S Statistic	0.229	0.184	0.186	0.121	0.193	0.273	0.231	0.163
SPEI-6	AIC	175.166	143.251	148.631	121.860	148.377	146.653	172.439	145.855
	BIC	181.722	149.807	155.188	131.694	154.933	149.931	178.995	152.411
	K-S Statistic	0.232	0.205	0.200	0.156	0.220	0.192	0.251	0.187
SPEI-9	AIC	129.152	111.679	113.493	101.792	113.616	111.857	131.084	113.935
	BIC	134.825	117.351	119.165	110.301	119.288	114.693	136.757	119.608
	K-S Statistic	0.327	0.348	0.235	0.199	0.247	0.276	0.295	0.211
SPEI-12	AIC	89.391	89.127	86.043	88.000	86.122	84.129	90.916	89.385
	BIC	94.253	96.419	90.905	92.862	90.984	86.560	95.778	94.246
	K-S Statistic	0.253	0.244	0.245	0.278	0.251	0.238	0.243	0.255

Note: Best fitted values are shown in bold figures

The fitted copulas alongwith their parameter values are shown in Table 5. Based on the minimum RMSE, AIC, and BIC values, the Clayton copula was fitted best for duration and severity combination in SPEI-1, SPEI-3, SPEI-6, and SPEI-9 whereas Gaussian copula was best fitted for SPEI-12. The univariate and bivariate return periods are shown in Table 6. The duration and severity increased with increase in return period across all SPEI. The AND-case bivariate return period was more than the univariate return period whereas the OR-case return period was less than the univariate return period. For example, for the case of 10-year return period for SPEI-3, the duration was 3.6 months and the severity was 5.7. The return period for $P(D \geq 3.6 \text{ and } S \geq 5.7)$ is 24.1 years which is more than 10 years. Similarly, the return period for $P(D \geq 3.6 \text{ or } S \geq 5.7)$ is 9.4 years which is less than 10 years.

Table 5: Fitted copulas for bivariate distribution of drought duration and severity

SPEI	Copula	θ	RMSE	AIC	BIC
SPEI-1	Gaussian	0.78	0.199	155.49	157.71
	Clayton	2.60	0.192	148.75	150.97
	Frank	7.06	0.195	153.35	155.56
	Gumbel	2.30	0.201	158.56	160.78
SPEI-3	Gaussian	0.96	0.127	100.15	101.93
	Clayton	9.11	0.125	98.20	99.98
	Frank	20.43	0.127	100.92	102.71
	Gumbel	5.55	0.129	101.63	103.42
SPEI-6	Gaussian	0.98	0.113	61.82	63.15
	Clayton	13.92	0.111	60.82	62.16
	Frank	30.10	0.113	62.23	63.57
	Gumbel	7.96	0.115	62.55	63.89
SPEI-9	Gaussian	0.96	0.152	41.39	42.28
	Clayton	9.53	0.144	40.26	41.15
	Frank	21.28	0.151	41.86	42.75
	Gumbel	5.77	0.156	42.08	42.97
SPEI-12	Gaussian	0.99	0.125	26.39	26.88
	Clayton	21.10	0.128	26.42	26.91
	Frank	44.50	0.130	26.62	27.11
	Gumbel	11.55	0.129	26.54	27.03

Note: Best fitted values are shown in bold figures

Table 6: Univariate and bivariate return periods of drought duration and severity

SPEI	Return Period (Years)	Duration (Months)	Severity	T_{DS}^{AND} (Years)	T_{DS}^{OR} (Years)
SPEI-1	5	1.5	2.5	5.7	3.4
	10	1.7	3.1	15.8	9.3
	25	2.0	4.0	27.6	15.6
	50	2.3	4.9	58.4	31.0
	75	2.5	5.5	88.7	55.9
	100	2.6	5.9	108.2	67.7
SPEI-3	5	2.7	4.0	6.3	3.6
	10	3.6	5.7	24.1	9.4
	25	4.9	8.0	30.0	13.9
	50	5.9	10.1	62.3	35.5
	75	6.5	11.4	89.7	55.8

	100	7.0	12.4	124.1	74.2
SPEI-6	5	3.0	4.3	11.2	2.1
	10	4.9	7.4	18.7	6.4
	25	7.8	12.8	40.1	14.8
	50	10.5	18.0	61.8	34.2
	75	12.2	21.5	93.2	57.6
	100	13.5	24.3	128.4	70.6
SPEI-9	5	3.0	3.3	11.3	2.9
	10	6.6	8.3	28.1	9.3
	25	11.4	18.2	46.4	20.4
	50	14.9	29.0	70.5	38.9
	75	17.0	37.0	97.2	63.1
	100	18.5	43.4	125.4	76.6
SPEI-12	5	1.3	2.1	14.4	2.1
	10	6.4	9.9	30.2	9.2
	25	13.1	20.2	42.9	19.8
	50	18.2	28.0	71.5	38.9
	75	21.2	32.6	90.0	57.2
	100	23.3	35.8	136.0	77.2

4. Summary and Conclusion

Drought is a complex phenomenon which cannot be understood by analysing a single drought characteristic. Thus, bivariate or higher dimensional analysis of drought characteristics is essential to gain an in-depth knowledge of the underlying drought processes. In this study, the meterological drought was assessed for Jaisalmer district of Rajasthan during the period of 50 years i.e., 1971-2020. The SPEI was used to identify the drought events. Various marginal probability distributions were fitted to the drought characteristics. A relatively newer statistical technique known as the Copula theory was used to establish the bivariate distributions of drought duration and severity across multi-scalar SPEI values. The analysis of return periods exhibited that the AND-case return period was more than the OR-case return period. The univariate return period was in between the bivariate return periods. Thus, the bivariate approach is beneficial in assessing the risk of drought which could be underestimated or overestimated with the univariate technique. The results of this study can provide effective information for the study area to assess drought risk and contribute to the decision-making process to reduce the associated risks. This can result in optimizing the allocation of water resources and reduce the impact of drought on the Jaisalmer district in the future.

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