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Multi-objective transportation problem using fuzzy programming techniques based on exponential membership functions

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Abstract
In this paper a Fuzzy Multi Objective Transportation problem [FMOTP]is first reduced to crisp Multi Objective transportation problem using ranking function. The crisp Multi Objective transportation problem is then solved by Zimmerman Technique using exponential membership functions. The results are compared with those obtained using trapezoidal and hyperbolic membership functions in Zimmerman Technique.

Keywords: Multi objective linear programming problem, fuzzy multi-objective linear programming problem, exponential membership function. transportation problem

1. Introduction

Most of the real-world problems are inherently characterized by multiple, conflicting and incommensurate aspect of evaluation. These areas of evolution are generally operationalized by objective functions to be optimized in the framework of multiple objective linear programming models. Furthermore, when addressing real world problems, frequently the parameters are imprecise numerical quantities. Fuzzy quantities are very adequate for modeling these situations. Bellmann and Zadeh [3] introduced the concept of fuzzy quantities and also the concept of fuzzy decision making. The most common approach to solve fuzzy linear programming problem is to change them into corresponding deterministic linear program. Some methods based on comparison of fuzzy numbers have been suggested by H.R. Maleki [12], A. Ebrahimnejad, S.H. Nasser [14], F. Roubens [9]. L. Campos [7], A. Munoz. Zimmermann [4, 5] have introduced fuzzy programming approach to solve crisp multi objective linear programming problem. H.M. Nehi *et al.* [13]. used ranking function suggested by Delgodo *et al.* [11] to solve fuzzy MOLPP. Leberling [6] used a special type non-linear (hyperbolic) membership function for the vector maximum linear programming problem. Dhingra and Moskowitz [6] defined other type of non-linear (exponential, quadratic and logarithmic) membership functions and applied them to an optimal design problem. Verma, Biswal and Biswas [8] used the fuzzy programming technique with some non-linear (hyperbolic and exponential) membership functions to solve a multi objective transportation problems. R.B. Dash and P.D.P Dash [13] introduced a method in which a fuzzy MOLLP is first reduced to crisp MOLLP using ranking function suggested by F. Roubens [9]. Then he solved crisp MOLPP using Zimmerman technique based on trapezoidal membership function. In this paper, following R.B. Dash [15] we reduce Fuzzy MOLPP to crisp MOLPP using Rouben's Ranking function. Then we solve the crisp problem applying exponential membership function. Finally, we obtain the membership functions of Fuzzy MOTP. These results are compared with those obtained using trapezoidal and Hyperbolic membership functions in Zimmerman's Technique.

2. Multi-Objective transportation problem

Mathematical Model

In a classical transportation problem, a homogeneous product is to be transported from each of m sources to n destinations.

The sources are production facilities, warehouses, or supply point, characterized by available capacities a_i (i=1,2,…,m). The destinations are consumption facilities, warehouses, or demand points, characterized by required levels of demand b_j (j = 1,2, …, n). A penalty C_{ij} is associated with transportation of a unit of the product from sources i to destination j. The penalty could represent transportation cost, delivery time, quantity of goods delivered, under used capacity, etc. A variable X_{ij} represents the unknown quantity to be transported from origin O_i to destination D_j. In the real would, however, transportation problems are not all-single objective type, we may have more than one objective in a transportation problem.

A Multi-objective transportation problem may be stated mathematically as:

$$Z_p = \text{Minimize} \begin{cases} \sum_{i=1}^{m}\sum_{j=1}^{n} c^1_{ij}x_{ij} \\ \sum_{i=1}^{m}\sum_{j=1}^{n} c^2_{ij}x_{ij} \\ \vdots \\ \sum_{i=1}^{m}\sum_{j=1}^{n} c^P_{ij}x_{ij} \end{cases} \qquad (1)$$

subject to

$$\sum_{j=1}^{n} x_{ij} = a_i, \quad i = 1,2,\dots,m \quad (2)$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \quad j = 1,2,\dots,n \quad (3)$$

$$x_{ij} \geq 0 \text{ for all } i, j \quad (4)$$

Where the subscript on Z_p and superscript on C^p_{ij} denote the p^{th} penalty criterion; $a_i >$ o for all i, $b_j >$ o, for all j, $C^p_{ij} \geq$ o for all i, j, and

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

(Balanced condition)

The balanced condition is treated as a necessary and sufficient condition for the existence of a feasible solution to the balanced linear transportation problem. A standard transportation problem has exactly (m + n) constraints and (m n) variables.

The LINDO package handles the transportation problem in an explicit equation form and thus solves the problem as a standard linear programming problem.

An Exponential membership function is defined by

$$\mu^E Z_p(x) = \begin{cases} 1, & \text{if } Z_p \le L_p \\ \dfrac{e^{-s\psi_p(x)} - e^{-s}}{1-e^{-s}}, & \text{if } L_p \le Z_p \le U_p \\ 0, & \text{if } Z_p \le U_p \end{cases} \qquad (5)$$

where $\psi_p(x) = (Z_p(x) - L_p) / U_p - L_p$, p = 1,2,…,P

and s is a non –zero parameter prescribed by the decision maker. A fuzzy number A = (a, b, c) is said to be a triangular fuzzy number if its membership function is given by

$$\begin{cases} \dfrac{x-a}{b-a} \\ 1 \\ \dfrac{x-b}{b-c} \\ 0 \end{cases}$$

Assume that R: F → *R*. R is linear ordered function that maps each fuzzy number into the real number, in which F denotes the whole fuzzy numbers. Accordingly for any two fuzzy numbers a and b we have.

$\tilde{a} \geq_R \tilde{b}$ iff $R(\tilde{a}) \geq R(\tilde{b})$

$\tilde{a} >_R \tilde{b}$ iff $R(\tilde{a}) > R(\tilde{b})$

$\tilde{a} \equiv_R \tilde{b}$ iff $R(\tilde{a}) = R(\tilde{b})$

We restrict our attention to linear ranking function, that is a ranking function *R* such that.

$R(k\tilde{a} + \tilde{b}) = k R(\tilde{a}) + R(\tilde{b})$

For any a and b in F and any k ∈ R.

A. Rouben's ranking function

The ranking function suggested by F. Rouben is defined by

$R(\tilde{a}) = \frac{1}{2}\int_1^0 ((\inf \tilde{a}_\alpha + \sup \tilde{a}_\alpha)\, dx$

This reduces to

$R(a) = \frac{1}{2}(a^L + a^U + \frac{1}{2}(\beta - \alpha))$

for a trapezoidal number

$(a,^L - a, a^L, a^V, a^V + \beta)$

B. Solving Fuzzy multi objective Transportation problem Problem (FMOTP)

A fuzzy multi objective Transportation problem is defined as followed. An initial fuzzy model of the problem (1-4) can be stated as: -

Find (x_{ij}, i =1, 2, …,m; j = 1, 2, …,n;) so as to satisfy

$$Z_p \lesssim L_p, \quad p = 1,2,\dots,P \qquad (6)$$

subject to

$$\sum_{j=1}^{n} x_{ij} = a_i, \quad i = 1,2,\dots,m \qquad (7)$$

$$\sum_{i=1}^{m} x_{ij} = b_j \quad , j = 1,2,\ldots,n \qquad (8)$$

$$x_{ij} \geq 0 \text{ for all } i, j \qquad (9)$$

$\lesssim$ fuzzification symbol indicates nearly less than equal to $\tilde{a}ij$ and $\tilde{c}$ pj are in the above relation are in trapezoidal form as

$\tilde{a}i = (a^1ij, a^2ij, a^3ij, a^4ij)$

$\tilde{c}$ $pj = (c^1ij, c^2ij, c^3ij, c^4ij)$

C. Definition (6-9)

x ∈ X is said to be feasible solution to the FMOTP problem (6-9) if it satisfies constraints of (7-9).

D. Definition

x ∈ X is said to be an optimal solution to this FMOTP problem (6-9) if there does not exist another x ϵ X such that $\tilde{z}_i(x) \geq \tilde{z}_i(x^*)$ for all i =1, 2…q. Now the FMOTP can be transformed to a classic form of a MOTP by applying ranking function *R* as follows.

Max $R(\tilde{z}_p) = \sum_j R(\tilde{c}_{pij})\, x_{ij}$ p = 1, 2…q

s.t. $R(a_{ij})\, x_{ij} \leq R(b_j)$ i,j = 1, 2…m

$x_{ij} \geq 0$ for all i, j

So we have

Max $z_p' = \sum_j cpj\ \ x_{ij}$ p=1, 2…q

s.t. $\sum a_{ij}\ \ x_{ij} \leq b_i$ i,j = 1, 2…m

$x_{ij} \geq 0$

Where a_{ij} ′, b_i ′, c_j′ are real numbers corresponding to the fuzzy numbers $\tilde{a}_{ij}$, $\tilde{b}_i$, $\tilde{c}_i$ respectively which are obtained by applying the ranking function *R*.

3. Fuzzy programming technique

To solve Multi-objective transportation problem may be stated mathematically as:

$$Z_p = \text{Minimize} \begin{cases} \sum_{i=1}^{m}\sum_{j=1}^{n} c^1_{ij}x_{ij} \\ \sum_{i=1}^{m}\sum_{j=1}^{n} c^2_{ij}x_{ij} \\ \vdots \\ \sum_{i=1}^{m}\sum_{j=1}^{n} c^P_{ij}x_{ij} \end{cases} \qquad (10)$$

$$\text{subject to } \sum_{j=1}^{n} x_{ij} = a_i \quad , i = 1,2,\ldots,m \qquad (11)$$

$$\sum_{i=1}^{m} x_{ij} = b_j \quad , j = 1,2,\ldots,n \qquad (12)$$

$$x_{ij} \geq 0 \text{ for all } i, j$$

We use fuzzy programming technique suggested by Zimmermann. The method is presented briefly in the following steps.

Step-1

Solve the multi objective linear programming problem by considering one objective at a time and ignoring all others. Repeat the process ‘q’ times for ‘q’ different objective functions. Let X_1, X_2,….,Xq be the ideal situations for respective functions.

Step-2

Using all the above q ideal solutions in the step-1 construct a pay-off matrix of size q by q. Then from pay-off matrix find the lower bound(Lp) and upper bound(Up) for the objective function. zp ′

as: Lp ≤ zp ′ ≤ Up, p=1, 2,…q

Step-3

If we use the exponential membership function as defined (3.1) then an equivalent crisp model for the fuzzy model can be formulated as follows.

An Exponential membership function is defined by

$$\mu^E Z_p(X) = \begin{cases} 1, & \text{if } Z_p \leq L_p \\ \dfrac{e^{-s\psi_p(x)} - e^{-s}}{1-e^{-s}}, & \text{if } L_p \leq Z_p \leq U_p \\ 0, & \text{if } Z_p \leq U_p \end{cases} \qquad (13)$$

where $\psi_p(x) = (Z_p(x) - L_p) / U_p - L_p$, p = 1,2,…,P

and s is a non –zero parameter prescribed by the decision maker. If we use the exponential membership function as defined (13) then an equivalent crisp model for the fuzzy model can be formulated as follows:

Maximize λ (14) subject to

$$\lambda \leq \frac{e^{-s\psi_p(x)} - e^{-s}}{1-e^{-s}}, \quad p = 1,2,\text{-----}P \qquad (15)$$

$$\text{subject to } \sum_{j=1}^{n} x_{ij} = a_i \quad , i = 1,2,\ldots,m \qquad (16)$$

$$\sum_{i=1}^{m} x_{ij} = b_j \quad , j = 1,2,\ldots,n \qquad (17)$$

$$x_{ij} \geq 0 \text{ for all } i, j \qquad (18)$$

The above problem (14-18) can be further simplified as:

Maximize X_3 (19)

Subject to

$s\{1-\psi_p(x)\} \geq X_3$ p = 1,2,-----P (20)

subject to $\sum_{j=1}^{n} x_{ij} = a_i$, i = 1,2,…,m (21)

$\sum_{i=1}^{m} x_{ij} = b_j$, j = 1,2,…,n (22)

$x_{ij} \geq 0$ for all i, j $X_3 \geq 0$

where $X_3 = \log\{1+\lambda(e^s-1)\}$

Step-4: Solve crisp model to find the optimal compromise solutions. Evaluate the values of objective functions at the compromise solutions.

4. Numerical example

Min: z 1 x = $16x_{11} + 19x_{12} + 12x_{13} + 22_{21} + 13x_{22} + 19x_{23} + 14x_{31} + 28x_{32} + 8x_{33}$

Min : z 2 x = $9x_{11} + 14x_{12} + 12x_{13} + 16x_{21} + 10x_{22} + 14x_{23} + 8x_{31} + 20x_{32} + 6x_{33}$

s.t

$$\sum_{j=1}^{3} x_{1j} = 14, \quad \sum_{j=1}^{3} x_{2j} = 16, \quad \sum_{j=1}^{3} x_{3j} = 12$$

$$\sum_{i=1}^{3} x_{i1} = 10, \quad \sum_{i=1}^{3} x_{i2} = 15, \quad \sum_{i=1}^{3} x_{i3} = 17,$$

$$x_{ij} \geq 0, \; i = 1,2,3. \quad j = 1,2,3.$$

Where
16 = (15.9, 16, 16. 1, 16.2)
19 = (18.8, 18.9, 19, 19.6)
12 = (11.6, 11.7, 12, 12.5)
22 = (21.8, 21.9, 22, 22.4)
13 = (12.2, 12.3, 13.3, 13.8)
19 = (18.8,18.9, 19, 19.5)
14 = (13.9,1 3.9, 14.2, 14.8)
28 = (27.8, 27.9, 28, 28.3)
8 = (7.4, 7.6, 8, 8.6)
9 = (8.8, 8.9 9.2,9.3,)
1 0 = (9.2, 9.4, 10.2, 10.4)
20 = (10.3, 10.6, 11.2, 11.5)
1 5 = (14.4, 14.5, 15.1, 15.6)
6 = (5.9, 6, 6.5)
17 = (16.9,17, 17.5, 17.6)
1 = (0.94, 1, 1.1)

Using ranking function suggested by Rouben [7] the problem reduces to

Min : $z_1(x) = R(\widetilde{16})x_{11} + R(\widetilde{19})x_{12} + R(\widetilde{12})x_{13} + R(\widetilde{22})_{21} + R(\widetilde{13})x_{22} + R(\widetilde{19})x_{23} + R(\widetilde{14})x_{31} + R(\widetilde{28})x_{32} + (\widetilde{8})x_{33}$ (23)

Min : $z_2(x) = R(\widetilde{9})x_{11} + R(\widetilde{14})x_{12} + R((\widetilde{12})x_{13} + R(\widetilde{16})x_{21} + R(\widetilde{10})x_{22} + R(\widetilde{14})x_{23} + R(\widetilde{8})x_{31} + R(\widetilde{20})x_{32} + R(\widetilde{6})x_{33}$ (24)

Subject to

$R(\tilde{1})x_{11} + R(\tilde{1})x_{12} + R(\tilde{1})x_{13} = R(\widetilde{14})$

$R(\tilde{1})x_{21} + R(\tilde{1})x_{22} + R(\tilde{1})x_{23} = R(\widetilde{16})$

$R(\tilde{1})x_{31} + R(\tilde{1})x_{32} + R(\tilde{1})x_{33} = R(\widetilde{12})$ (25)

$R(\tilde{1})x_{11} + R(\tilde{1})x_{21} + R(\tilde{1})x_{31} = R(\widetilde{10})$

$R(\tilde{1})x_{12} + R(\tilde{1})x_{22} + R(\tilde{1})x_{32} = R(\widetilde{15})$

$R(\tilde{1})x_{13} + R(\tilde{1})x_{23} + R(\tilde{1})x_{33} = R(\widetilde{17})$

$x_{ij} \geq 0$ for all i,= 1,2,3 j=1,2,3.

↔ Min : z 1 x = $16.05x_{11} + 19.05x_{12} + 11.9x_{13} + 21.8_{21} + 13.4x_{22} + 19.2x_{23} + 13.4x_{31} + 27.5x_{32} + 8.8x_{33}$ (26)

Min : z 2 x = $9.05x_{11} + 13.4x_{12} + 11.2x_{13} + 15.06x_{21} + 9.9x_{22} + 14.05x_{23} + 8.8x_{31} + 19.09x_{32} + 6.01x_{33}$ (27)

Subject to
$1.01x11 + 1.02x_{12} + 1.01x_{13} = 14.9$

$1.01x_{21} + 1.01x_{22} + 1.02x_{23} = 15.9$

$1.02x_{31} + 1.01x_{32} + 1.01x_{33} = 12.05$

$1.02x_{11} + 1.01x_{21} + 1.01x_{31} = 10.05$ (28)

$1.01x_{12} + 1.02x_{22} + 1.01x_{32} = 14.8$

$1.01x_{13} + 1.02x_{23} + 1.01x_{33} = 16.9$

$x_{ij} \geq 0$ for all i,= 1,2,3 j=1,2,3

Step 1 and step 2

Optimal solution which minimizes Z_1 subject to constraints (28) is as follows:

$X_{11} = 9, X_{13} = 5, X_{21} = 1, X_{22} = 15, X_{33} = 12,$

with $Z_R(X_1) = 517, Z_R(X_2) = 518$

Optimal solution which minimizes Z_2 subject to constraints (28) is as follows:

$X_{11} = 10, X_{13} = 4, X_{21} = 15, X_{23} = 1, X_{33} = 12,$

with $Z_C(X_1) = 374, Z_C(X_2) = 379$

Step 3:

$U_1 = 518, L_1 = 517, U_2 = 379\ L_2 = 374$

Find { x_{ij}, i = 1,2,3; j = 1,2,3.} so as to satisfy

$$Z_R \leq 517, \; Z_C \leq 374$$

and constraints (28). If we use exponential membership function with the parameter s=1, an equation crisp model can be formulated as Min X3 (29)

s.t
$s[z1(x)] + x4(U1 - L1) \geq s(U1)$

$s[z2(x)] + x4(U2 - L2) \geq s(U2)$ (30)

subject to $\sum_{j=1}^{n} x_{ij} = a_i$, i = 1,2,…,m (31)

$\sum_{i=1}^{m} x_{ij} = b_j$, j = 1,2,…,n (32)

$x_{ij} \geq 0$ for all i, j $X_3 \geq 0$

where $X_3 = \log\{1+ \lambda(e^s -1)\}$

Using exponential function the problem reduces to
Min X3

$16.05x_{11} + 19.05x_{12} + 11.9x_{13} + 21.8_{21} + 13.4x_{22} + 19.2x_{23} + 13.4x_{31} + 27.5x_{32} + 8.8x_{33} + X_4 \geq 518$

$9.05x_{11} + 13.4x_{12} + 11.2x_{13} + 15.06x_{21} + 9.9x_{22} + 14.05x_{23} + 8.8x_{31} + 19.09x_{32} + 6.01x_{33} + 5X_4 \geq 379$

Subject to
$1.01x11 + 1.02x_{12} + 1.01x_{13} = 14.9$

$1.01x_{21} + 1.01x_{22} + 1.02x_{23} = 15.9$

$1.02x_{31} + 1.01\, x_{32} + 1.01\, x_{33} = 12.05$

$1.02\, x_{11} + 1.01x_{21} + 1.01x_{31} = 10.05$

$1.01x_{12} + 1.02x_{22} + 1.01x_{32} = 14.8$

$1.01x_{13} + 1.02\, x_{23} + 1.01\, x_{33} = 16.9$

$x_{ij} \geq 0$ for all i,= 1,2,3 j=1,2,3 and $X_3 \geq 0$

The optimal solution is presented as follows

$$\begin{cases} X_{11} = 9.5, \quad X_{13} = 4.5, \quad X_{21} = 0.5 \\ X_{22} = 15, \quad X_{23} = 0.5, \quad X_{33} = 12 \\ rest\ all\ \ Xij's\ \ are\ \ zeros \end{cases}$$

$$Z_1 = 517.5 \quad Z_2 = 376.5 \quad \lambda = 0.50$$

5. Conclusions

We have proposed a simple method to find the efficient solution of multi-objective fuzzy transportation problem involving trapezoidal fuzzy numbers. and those using exponential membership function in the Zimmerman's algorithm. The proposed method is easy to apply and also reduces the computational work. Thus this is an alternative solution to the Fuzzy MOTLPP.

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