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A new generalization of quasi Amarendra distribution with some characterizations and its applications

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Abstract
In this manuscript, we have introduced a new model of quasi Amarendra distribution called as weighted quasi Amarendra distribution has been developed. The executed distribution has been addressed with different structural properties and its parameters are estimated by using the method of maximum likelihood estimation. Finally, a real data set has been analyzed and inspected to examine the applicability of a new distribution.

Keywords: Quasi Amarendra distribution, weighted distribution, order statistics, reliability measures, maximum likelihood estimation

1. Introduction

The idea of weighted distribution attained a greater flexibility and superiority in research areas related to reliability, ecology, biomedicine and other applied areas are of tremendous importance in mathematics, probability and statistics. Fisher (1934) [2] proposed the concept of weighted distribution to examine how the methods of ascertainment can influence the form of distribution of recorded observations and then Rao (1965) [10] significantly developed this concept in an integrative manner with respect to modeling statistical data were the standard distributions are not appropriate to record observations with equal probabilities. The weighted distributions will occur in a natural way in specifying probabilities of events as observed and recorded by making adjustments to probabilities of actual occurrence of events taking into account the method of ascertainment. Failure to make such adjustments can lead to wrong conclusions. The weighted distributions provide a unifying approach for correction of biases that exist in unequally weighted sample data. A remarkable contribution has been done by several authors to develop some weighted distributions along with their illustrations in various fields. Mohammad (2023) [4] introduced the new weighted Topp-Leone distribution along with simulation study and applications. Benchiha *et al.* (2021) [1] obtained weighted generalized quasi Lindley distribution with estimation and application for COVID-19 and engineering data. Oyedeji I. and Vitus C. (2020) [7] discussed on the length biased quasi-transmuted uniform distribution. Shukla, KK and Shanker (2019) [14] presented the weighted Ishita distribution and its application to survival data. Ramos and Louzada (2019) [8] proposed inverse weighted Lindley distribution with properties, estimation and application on a failure time data. Ganaie *et al.* (2023) [3] studied the weighted power Garima distribution with applications. Mudasir and Ahmad (2017) obtained the parameter estimation of weighted erlang distribution using R software. Mohiuddin *et al.* (2020) [5] discussed on the weighted devya distribution with properties and its applications.

Quasi Amarendra distribution is a recently developed two parametric distribution introduced by Rashid *et al.* (2019) [9] which is a special case of Amarendra distribution. Some of its structural properties including order statistics, moment about origin, mean, variance, coefficient of variation, dispersion, reliability measures, bonferroni and Lorenz curves, moment generating function and Renyi entropy have been discussed. Its parameters have also been estimated by applying the technique of maximum likelihood estimation. Shanker (2016) [13] discussed on Amarendra distribution and obtain its various statistical properties and estimate its parameter by the method of moments and method of maximum likelihood estimation.

2. Weighted Quasi Amarendra (WQA) Distribution

The probability density function of quasi Amarendra distribution is given by

$$f(x;\alpha,\theta)=\frac{\theta}{\left(\alpha^3+\alpha^2+2\alpha+6\right)}\left(\alpha^3+\alpha^2\theta x+\alpha(\theta x)^2+(\theta x)^3\right)e^{-\theta x};\ \theta,x>0;\ \alpha>-1 \quad (1)$$

And the cumulative distribution function of quasi Amarendra distribution is given by

$$F(x;\alpha,\theta)=1-\left(1+\left[\frac{x\theta(\alpha^2+2\alpha+6)+(x\theta)^2(\alpha+3)+(x\theta)^3}{\left(\alpha^3+\alpha^2+2\alpha+6\right)}\right]\right)e^{-\theta x};\theta,x>0;\ \alpha>-1 \quad (2)$$

Consider X be the non-negative random variable with probability density function $f(x)$.Let $w(x)$ be its non-negative weight function, then the probability density function of weighted random variable X_w is given by

$$f_w(x)=\frac{w(x)f(x)}{E(w(x))},\ x>0.$$

Where $w(x)$ be the non-negative weight function and $E(w(x))=\int w(x)f(x)dx<\infty.$

For various choices of weight function *w(x)* especially if $w(x) = x^c$, proposed distribution is termed as weighted distribution. In this paper, we have to introduce the weighted version of quasi Amarendra distribution called as weighted quasi Amarendra distribution. So, the weight function considered at $w(x) = x^c$, resulting distribution is called weighted distribution and its probability density function is given by

$$f_w(x)=\frac{x^c f(x)}{E(x^c)} \quad (3)$$

$$E(x^c)=\int_0^\infty x^c f(x)dx$$

$$E(x^c)=\frac{\alpha^3\Gamma(c+1)+\alpha^2\Gamma(c+2)+\alpha\theta^2\Gamma(c+3)+\theta^6\Gamma(c+4)}{\theta^c(\alpha^3+\alpha^2+2\alpha+6)} \quad (4)$$

Now substituting equations (1) and (4) in equation (3), we will get the required probability density function of weighted quasi Amarendra distribution.

$$f_w(x)=\frac{x^c\theta^{c+1}}{\left(\alpha^3\Gamma(c+1)+\alpha^2\Gamma(c+2)+\alpha\theta^2\Gamma(c+3)+\theta^6\Gamma(c+4)\right)}\left(\alpha^3+\alpha^2\theta x+\alpha(\theta x)^2+(\theta x)^3\right)e^{-\theta x} \quad (5)$$

And the cumulative distribution function of weighted quasi Amarendra distribution can be determined as

$$F_w(x)=\int_0^x f_w(x)dx$$

$$F_w(x)=\int_0^x\frac{x^c\theta^{c+1}}{\left(\alpha^3\Gamma(c+1)+\alpha^2\Gamma(c+2)+\alpha\theta^2\Gamma(c+3)+\theta^6\Gamma(c+4)\right)}\left(\alpha^3+\alpha^2\theta x+\alpha(\theta x)^2+(\theta x)^3\right)e^{-\theta x}dx$$

$$F_w(x)=\frac{1}{\left(\alpha^3\Gamma(c+1)+\alpha^2\Gamma(c+2)+\alpha\theta^2\Gamma(c+3)+\theta^6\Gamma(c+4)\right)}\int_0^x x^c\theta^{c+1}\left(\alpha^3+\alpha^2\theta x+\alpha(\theta x)^2+(\theta x)^3\right)e^{-\theta x}dx$$

$$F_w(x) = \frac{1}{\left(\alpha^3\Gamma(c+1)+\alpha^2\Gamma(c+2)+\alpha\theta^2\Gamma(c+3)+\theta^6\Gamma(c+4)\right)}\int_0^x x^c\theta^{c+1}\left(\alpha^3+\alpha^2\theta x+\alpha\theta^2x^2+\theta^3x^3\right)e^{-\theta x}dx$$

$$F_w(x) = \frac{1}{\left(\alpha^3\Gamma(c+1)+\alpha^2\Gamma(c+2)+\alpha\theta^2\Gamma(c+3)+\theta^6\Gamma(c+4)\right)}\begin{pmatrix}\alpha^3\theta^{c+1}\int_0^x x^c e^{-\theta x}dx+\alpha^2\theta^{c+2}\int_0^x x^{c+1}e^{-\theta x}dx \\ +\alpha\theta^{c+3}\int_0^x x^{c+2}e^{-\theta x}dx+\theta^{c+4}\int_0^x x^{c+3}e^{-\theta x}dx\end{pmatrix} \qquad (6)$$

$Put\ \theta x = t \Rightarrow \theta dx = dt \Rightarrow dx = \frac{dt}{\theta}$ Also $x = \frac{t}{\theta}$

When $x \to x, t \to \theta x$ and as when $x \to 0, t \to 0$

After the simplification of equation (6), we will obtain the cumulative distribution function of weighted quasi Amarendra distribution as

$$F_w(x) = \frac{1}{\left(\alpha^3\Gamma(c+1)+\alpha^2\Gamma(c+2)+\alpha\theta^2\Gamma(c+3)+\theta^6\Gamma(c+4)\right)}\left(\alpha^3\gamma(c+1,\theta x)+\alpha^2\gamma(c+2,\theta x)+\alpha\gamma(c+3,\theta x)+\gamma(c+4,\ \theta x)\right) \qquad (7)$$

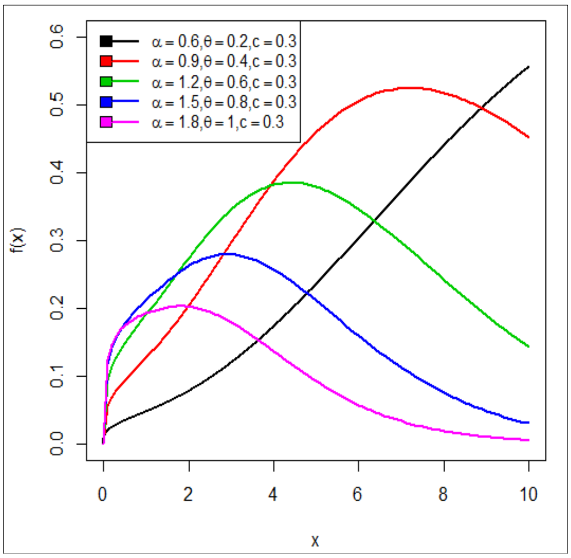

Fig 1: Pdf plot of weighted quasi Amarendra distribution

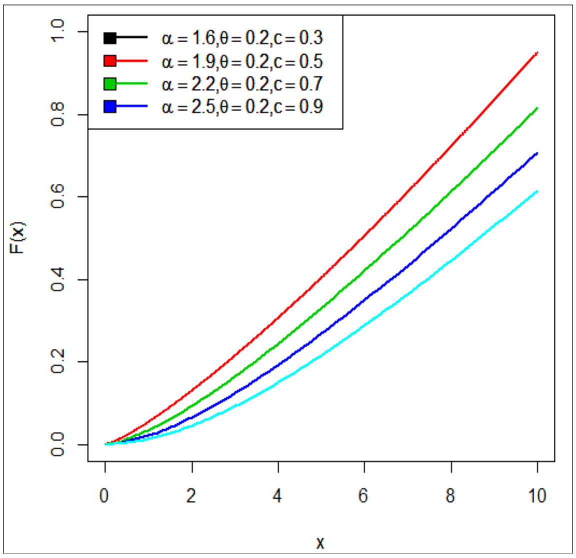

Fig 2: Cdf plot of weighted quasi Amarendra distribution

3. Reliability measures

In this section, we will discuss about the reliability function, hazard rate function, reverse hazard rate function and mills ratio of the proposed weighted quasi Amarendra distribution.

3.1 Reliability function

The reliability function or survival function of weighted quasi Amarendra distribution can be determined as

$$R(x) = 1 - F_w(x)$$

$$R(x) = 1 - \frac{1}{\left(\alpha^3\Gamma(c+1)+\alpha^2\Gamma(c+2)+\alpha\theta^2\Gamma(c+3)+\theta^6\Gamma(c+4)\right)}\left(\alpha^3\gamma(c+1,\theta x)+\alpha^2\gamma(c+2,\theta x)+\alpha\gamma(c+3,\theta x)+\gamma(c+4,\ \theta x)\right)$$

3.2 Hazard function

The hazard function is also known as hazard rate or failure rate and is given by

$$h(x) = \frac{f_w(x)}{1 - F_w(x)}$$

$$h(x) = \frac{x^c \theta^{c+1}\left(\alpha^3 + \alpha^2 \theta x + \alpha(\theta x)^2 + (\theta x)^3\right) e^{-\theta x}}{(\alpha^3 \Gamma(c+1) + \alpha^2 \Gamma(c+2) + \alpha\theta^2 \Gamma(c+3) + \theta^6 \Gamma(c+4)) - (\alpha^3 \gamma(c+1, \theta x) + \alpha^2 \gamma(c+2, \theta x) + \alpha \gamma(c+3, \theta x) + \gamma(c+4, \theta x))}$$

3.3 Reverse hazard function

The reverse hazard rate function is given by.

$$h_r(x) = \frac{f_w(x)}{F_w(x)}$$

$$h_r(x) = \frac{x^c \theta^{c+1}\left(\alpha^3 + \alpha^2 \theta x + \alpha(\theta x)^2 + (\theta x)^3\right) e^{-\theta x}}{(\alpha^3 \gamma(c+1, \theta x) + \alpha^2 \gamma(c+2, \theta x) + \alpha \gamma(c+3, \theta x) + \gamma(c+4, \theta x))}$$

3.4 Mills Ratio

$$M.R = \frac{1}{h_r(x)} = \frac{(\alpha^3 \gamma(c+1, \theta x) + \alpha^2 \gamma(c+2, \theta x) + \alpha \gamma(c+3, \theta x) + \gamma(c+4, \theta x))}{x^c \theta^{c+1}\left(\alpha^3 + \alpha^2 \theta x + \alpha(\theta x)^2 + (\theta x)^3\right) e^{-\theta x}}$$

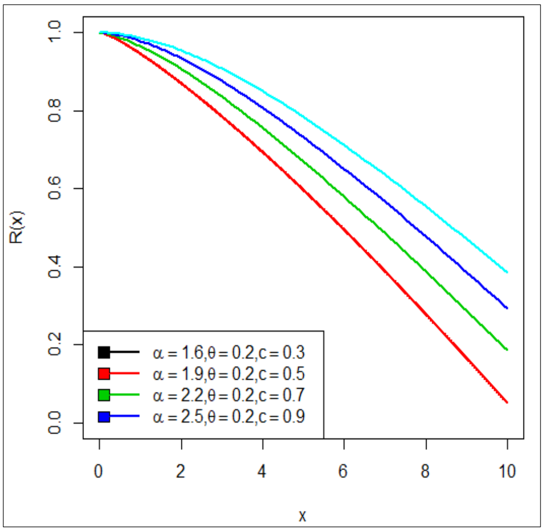

Fig 1: Reliability plot of weighted quasi Amarendra distribution

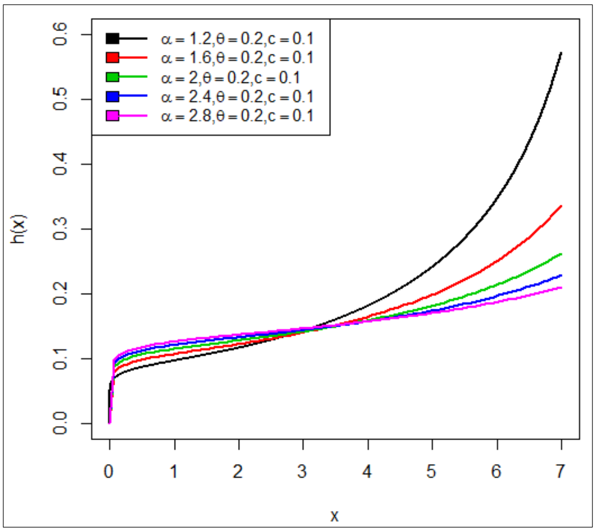

Fig 2: Hazad plot of weighted quasi Amarendra distribution

4. Order Statistics

Consider $X_{(1)}, X_{(2)}, \ldots, X_{(n)}$ be the order statistics of a random sample $X_1, X_2, \ldots, X_n$ from a continuous distribution with probability density function $f_x(x)$ and cumulative distribution function $F_X(x)$, then the probability density function of r^{th} order statistics $X_{(r)}$ is given by

$$f_{x(r)}(x) = \frac{n!}{(r-1)!(n-r)!} f_X(x)\left(F_X(x)\right)^{r-1}\left(1 - F_X(x)\right)^{n-r} \quad (8)$$

Now substituting equations (5) and (7) in equation (8), we will get the required probability density function of r^{th} order statistics of weighted quasi Amarendra distribution.

$$f_{x(r)}(x)=\frac{n!}{(r-1)!(n-r)!}\left(\frac{x^{c}\theta^{c+1}}{\left(\alpha^{3}\Gamma(c+1)+\alpha^{2}\Gamma(c+2)+\alpha\theta^{2}\Gamma(c+3)+\theta^{6}\Gamma(c+4)\right)}\left(\alpha^{3}+\alpha^{2}\theta x+\alpha(\theta x)^{2}+(\theta x)^{3}\right)e^{-\theta x}\right)$$

$$\times\left(\frac{1}{\left(\alpha^{3}\Gamma(c+1)+\alpha^{2}\Gamma(c+2)+\alpha\theta^{2}\Gamma(c+3)+\theta^{6}\Gamma(c+4)\right)}\left(\alpha^{3}\gamma(c+1,\,\theta x)+\alpha^{2}\gamma(c+2,\,\theta x)+\alpha\gamma(c+3,\,\theta x)+\gamma(c+4,\theta x)\right)\right)^{r-1}$$

$$\times\left(1-\frac{1}{\left(\alpha^{3}\Gamma(c+1)+\alpha^{2}\Gamma(c+2)+\alpha\theta^{2}\Gamma(c+3)+\theta^{6}\Gamma(c+4)\right)}\left(\alpha^{3}\gamma(c+1,\,\theta x)+\alpha^{2}\gamma(c+2,\,\theta x)+\alpha\gamma(c+3,\,\theta x)+\gamma(c+4,\theta x)\right)\right)^{n-r}$$

Therefore for $r = n$, we will obtain the probability density function of higher order statistic $X_{(n)}$ of weighted quasi Amarendra distribution as

$$f_{x(n)}(x)=\frac{n\,x^{c}\theta^{c+1}}{\left(\alpha^{3}\Gamma(c+1)+\alpha^{2}\Gamma(c+2)+\alpha\theta^{2}\Gamma(c+3)+\theta^{6}\Gamma(c+4)\right)}\left(\alpha^{3}+\alpha^{2}\theta x+\alpha(\theta x)^{2}+(\theta x)^{3}\right)e^{-\theta x}$$

$$\times\left(\frac{1}{\left(\alpha^{3}\Gamma(c+1)+\alpha^{2}\Gamma(c+2)+\alpha\theta^{2}\Gamma(c+3)+\theta^{6}\Gamma(c+4)\right)}\left(\alpha^{3}\gamma(c+1,\,\theta x)+\alpha^{2}\gamma(c+2,\,\theta x)+\alpha\gamma(c+3,\,\theta x)+\gamma(c+4,\theta x)\right)\right)^{n-1}$$

And for $r = 1$, we will obtain the probability density function of first order statistic $X_{(1)}$ of weighted quasi Amarendra distribution as

$$f_{x(1)}(x)=\frac{n\,x^{c}\theta^{c+1}}{\left(\alpha^{3}\Gamma(c+1)+\alpha^{2}\Gamma(c+2)+\alpha\theta^{2}\Gamma(c+3)+\theta^{6}\Gamma(c+4)\right)}\left(\alpha^{3}+\alpha^{2}\theta x+\alpha(\theta x)^{2}+(\theta x)^{3}\right)e^{-\theta x}$$

$$\times\left(1-\frac{1}{\left(\alpha^{3}\Gamma(c+1)+\alpha^{2}\Gamma(c+2)+\alpha\theta^{2}\Gamma(c+3)+\theta^{6}\Gamma(c+4)\right)}\left(\alpha^{3}\gamma(c+1,\,\theta x)+\alpha^{2}\gamma(c+2,\,\theta x)+\alpha\gamma(c+3,\,\theta x)+\gamma(c+4,\theta x)\right)\right)^{n-1}$$

5. Statistical Properties

In this section, we will discuss about the various statistical properties of weighted quasi Amarendra distribution that include moments, harmonic mean, moment generating function and characteristic function.

5.1 Moments

Consider X be the random variable following weighted quasi Amarendra distribution with parameters α, θ and c, then the r^{th} order moment $E(X^r)$ of executed distribution can be determined as

$$E(X^{r})=\mu_{r}'=\int_{0}^{\infty}x^{r}f_{w}(x)dx$$

$$E(X^{r})=\int_{0}^{\infty}x^{r}\frac{x^{c}\theta^{c+1}}{\left(\alpha^{3}\Gamma(c+1)+\alpha^{2}\Gamma(c+2)+\alpha\theta^{2}\Gamma(c+3)+\theta^{6}\Gamma(c+4)\right)}\left(\alpha^{3}+\alpha^{2}\theta x+\alpha(\theta x)^{2}+(\theta x)^{3}\right)e^{-\theta x}dx$$

$$E(X^r)=\int_0^\infty \frac{x^{c+r}\theta^{c+1}}{\left(\alpha^3\Gamma(c+1)+\alpha^2\Gamma(c+2)+\alpha\theta^2\Gamma(c+3)+\theta^6\Gamma(c+4)\right)}\left(\alpha^3+\alpha^2\theta x+\alpha(\theta x)^2+(\theta x)^3\right)e^{-\theta x}dx$$

$$E(X^r)=\frac{\theta^{c+1}}{\left(\alpha^3\Gamma(c+1)+\alpha^2\Gamma(c+2)+\alpha\theta^2\Gamma(c+3)+\theta^6\Gamma(c+4)\right)}\int_0^\infty x^{c+r}\left(\alpha^3+\alpha^2\theta x+\alpha(\theta x)^2+(\theta x)^3\right)e^{-\theta x}dx$$

$$E(X^r)=\frac{\theta^{c+1}}{\left(\alpha^3\Gamma(c+1)+\alpha^2\Gamma(c+2)+\alpha\theta^2\Gamma(c+3)+\theta^6\Gamma(c+4)\right)}\int_0^\infty x^{c+r}\left(\alpha^3+\alpha^2\theta x+\alpha\theta^2x^2+\theta^3x^3\right)e^{-\theta x}dx$$

$$E(X^r)=\frac{\theta^{c+1}}{\left(\alpha^3\Gamma(c+1)+\alpha^2\Gamma(c+2)+\alpha\theta^2\Gamma(c+3)+\theta^6\Gamma(c+4)\right)}\begin{pmatrix}\alpha^3\int_0^\infty x^{(c+r+1)-1}e^{-\theta x}dx+\alpha^2\theta\int_0^\infty x^{(c+r+2)-1}e^{-\theta x}dx\\ +\alpha\theta^2\int_0^\infty x^{(c+r+3)-1}e^{-\theta x}dx+\theta^3\int_0^\infty x^{(c+r+4)-1}e^{-\theta x}dx\end{pmatrix} \quad (9)$$

After the simplification of equation (9), we obtain

$$E(X^r)=\mu_r'=\frac{\alpha^3\Gamma(c+r+1)+\alpha^2\Gamma(c+r+2)+\alpha\theta^2\Gamma(c+r+3)+\theta^6\Gamma(c+r+4)}{\theta^r(\alpha^3\Gamma(c+1)+\alpha^2\Gamma(c+2)+\alpha\theta^2\Gamma(c+3)+\theta^6\Gamma(c+4))} \quad (10)$$

Now putting r = 1, 2, 3 and 4 in equation (10), we will obtain the first four moments of weighted quasi Amarendra distribution as

$$E(X)=\mu_1'=\frac{\alpha^3\Gamma(c+2)+\alpha^2\Gamma(c+3)+\alpha\theta^2\Gamma(c+4)+\theta^6\Gamma(c+5)}{\theta(\alpha^3\Gamma(c+1)+\alpha^2\Gamma(c+2)+\alpha\theta^2\Gamma(c+3)+\theta^6\Gamma(c+4))}$$

$$E(X^2)=\mu_2'=\frac{\alpha^3\Gamma(c+3)+\alpha^2\Gamma(c+4)+\alpha\theta^2\Gamma(c+5)+\theta^6\Gamma(c+6)}{\theta^2(\alpha^3\Gamma(c+1)+\alpha^2\Gamma(c+2)+\alpha\theta^2\Gamma(c+3)+\theta^6\Gamma(c+4))}$$

$$E(X^3)=\mu_3'=\frac{\alpha^3\Gamma(c+4)+\alpha^2\Gamma(c+5)+\alpha\theta^2\Gamma(c+6)+\theta^6\Gamma(c+7)}{\theta^3(\alpha^3\Gamma(c+1)+\alpha^2\Gamma(c+2)+\alpha\theta^2\Gamma(c+3)+\theta^6\Gamma(c+4))}$$

$$E(X^4)=\mu_4'=\frac{\alpha^3\Gamma(c+5)+\alpha^2\Gamma(c+6)+\alpha\theta^2\Gamma(c+7)+\theta^6\Gamma(c+8)}{\theta^4(\alpha^3\Gamma(c+1)+\alpha^2\Gamma(c+2)+\alpha\theta^2\Gamma(c+3)+\theta^6\Gamma(c+4))}$$

$$\text{Var.}=\frac{\alpha^3\Gamma(c+3)+\alpha^2\Gamma(c+4)+\alpha\theta^2\Gamma(c+5)+\theta^6\Gamma(c+6)}{\theta^2(\alpha^3\Gamma(c+1)+\alpha^2\Gamma(c+2)+\alpha\theta^2\Gamma(c+3)+\theta^6\Gamma(c+4))}-\left(\frac{\alpha^3\Gamma(c+2)+\alpha^2\Gamma(c+3)+\alpha\theta^2\Gamma(c+4)+\theta^6\Gamma(c+5)}{\theta(\alpha^3\Gamma(c+1)+\alpha^2\Gamma(c+2)+\alpha\theta^2\Gamma(c+3)+\theta^6\Gamma(c+4))}\right)^2$$

$$S.D=\sqrt{\left(\frac{\alpha^3\Gamma(c+3)+\alpha^2\Gamma(c+4)+\alpha\theta^2\Gamma(c+5)+\theta^6\Gamma(c+6)}{\theta^2(\alpha^3\Gamma(c+1)+\alpha^2\Gamma(c+2)+\alpha\theta^2\Gamma(c+3)+\theta^6\Gamma(c+4))}-\left(\frac{\alpha^3\Gamma(c+2)+\alpha^2\Gamma(c+3)+\alpha\theta^2\Gamma(c+4)+\theta^6\Gamma(c+5)}{\theta(\alpha^3\Gamma(c+1)+\alpha^2\Gamma(c+2)+\alpha\theta^2\Gamma(c+3)+\theta^6\Gamma(c+4))}\right)^2\right)}$$

5.2 Harmonic mean

The harmonic mean for proposed weighted quasi Amarendra distribution can be determined as

$$H.M=E\left(\frac{1}{x}\right)=\int_0^\infty \frac{1}{x} f_w(x)dx$$

$$H.M=\int_0^\infty \frac{1}{x}\frac{x^c\theta^{c+1}}{\left(\alpha^3\Gamma(c+1)+\alpha^2\Gamma(c+2)+\alpha\theta^2\Gamma(c+3)+\theta^6\Gamma(c+4)\right)}\left(\alpha^3+\alpha^2\theta x+\alpha(\theta x)^2+(\theta x)^3\right)e^{-\theta x}dx$$

$$H.M=\int_0^\infty \frac{x^{c-1}\theta^{c+1}}{\left(\alpha^3\Gamma(c+1)+\alpha^2\Gamma(c+2)+\alpha\theta^2\Gamma(c+3)+\theta^6\Gamma(c+4)\right)}\left(\alpha^3+\alpha^2\theta x+\alpha(\theta x)^2+(\theta x)^3\right)e^{-\theta x}dx$$

$$H.M=\frac{\theta^{c+1}}{\left(\alpha^3\Gamma(c+1)+\alpha^2\Gamma(c+2)+\alpha\theta^2\Gamma(c+3)+\theta^6\Gamma(c+4)\right)}\int_0^\infty x^{c-1}\left(\alpha^3+\alpha^2\theta x+\alpha(\theta x)^2+(\theta x)^3\right)e^{-\theta x}dx$$

$$H.M=\frac{\theta^{c+1}}{\left(\alpha^3\Gamma(c+1)+\alpha^2\Gamma(c+2)+\alpha\theta^2\Gamma(c+3)+\theta^6\Gamma(c+4)\right)}\begin{pmatrix}\alpha^3\int_0^\infty x^{(c+1)-2}e^{-\theta x}dx+\alpha^2\theta\int_0^\infty x^{(c+1)-1}e^{-\theta x}dx\\ +\alpha\theta^2\int_0^\infty x^{(c+2)-1}e^{-\theta x}dx+\theta^3\int_0^\infty x^{(c+3)-1}e^{-\theta x}dx\end{pmatrix} \quad (11)$$

After simplification, we obtain from equation (11)

$$H.M=\frac{(\alpha^3\Gamma(c+1)+\alpha^2\theta\Gamma(c+1)+\alpha\theta^3\Gamma(c+2)+\theta^7\Gamma(c+3))}{(\alpha^3\Gamma(c+1)+\alpha^2\Gamma(c+2)+\alpha\theta^2\Gamma(c+3)+\theta^6\Gamma(c+4))}$$

5.3 Moment generating function and characteristic function

Consider X be the random variable following weighted quasi Amarendra distribution with parameters α, θ and c, then the moment generating function of introduced distribution can be determined as

$$M_X(t)=E\left(e^{tx}\right)=\int_0^\infty e^{tx} f_w(x)dx$$

$$=\int_0^\infty\left(1+tx+\frac{(tx)^2}{2!}+....\right)f_w(x)dx$$

$$=\int_0^\infty \sum_{j=0}^\infty \frac{t^j}{j!}x^j f_w(x)dx$$

$$=\sum_{j=0}^\infty \frac{t^j}{j!}\mu_j'$$

$$=\sum_{j=0}^\infty \frac{t^j}{j!}\left(\frac{\alpha^3\Gamma(c+j+1)+\alpha^2\Gamma(c+j+2)+\alpha\theta^2\Gamma(c+j+3)+\theta^6\Gamma(c+j+4)}{\theta^j(\alpha^3\Gamma(c+1)+\alpha^2\Gamma(c+2)+\alpha\theta^2\Gamma(c+3)+\theta^6\Gamma(c+4))}\right)$$

$$M_X(t)=\frac{1}{(\alpha^3\Gamma(c+1)+\alpha^2\Gamma(c+2)+\alpha\theta^2\Gamma(c+3)+\theta^6\Gamma(c+4))}\sum_{j=0}^\infty \frac{t^j}{j!\theta^j}\begin{pmatrix}\alpha^3\Gamma(c+j+1)+\alpha^2\Gamma(c+j+2)\\ +\alpha\theta^2\Gamma(c+j+3)+\theta^6\Gamma(c+j+4)\end{pmatrix}$$

Similarly, the characteristic function of introduced new distribution can be obtained.

$$\varphi_x(t) = M_X(it)$$

$$M_X(it) = \frac{1}{(\alpha^3\Gamma(c+1)+\alpha^2\Gamma(c+2)+\alpha\theta^2\Gamma(c+3)+\theta^6\Gamma(c+4))}\sum_{j=0}^{\infty}\frac{it^j}{j!\theta^j}\begin{pmatrix}\alpha^3\Gamma(c+j+1)+\alpha^2\Gamma(c+j+2)\\ +\alpha\theta^2\Gamma(c+j+3)+\theta^6\Gamma(c+j+4)\end{pmatrix}$$

6. Bonferroni and Lorenz Curves

The bonferroni and Lorenz curves are also known as classical curves are mostly being used to measure the inequality in income or how wealth is distributed. The bonferroni and Lorenz curves are given by.

$$B(p) = \frac{1}{p\mu_1'}\int_0^q x\, f_w(x)dx$$

and $L(p) = pB(p) = \frac{1}{\mu_1'}\int_0^q x\, f_w(x)dx$

Where $\mu_1' = \frac{\alpha^3\Gamma(c+2)+\alpha^2\Gamma(c+3)+\alpha\theta^2\Gamma(c+4)+\theta^6\Gamma(c+5)}{\theta(\alpha^3\Gamma(c+1)+\alpha^2\Gamma(c+2)+\alpha\theta^2\Gamma(c+3)+\theta^6\Gamma(c+4))}$ and $q = F^{-1}(p)$

$$B(p) = \frac{\theta(\alpha^3\Gamma(c+1)+\alpha^2\Gamma(c+2)+\alpha\theta^2\Gamma(c+3)+\theta^6\Gamma(c+4))}{p(\alpha^3\Gamma(c+2)+\alpha^2\Gamma(c+3)+\alpha\theta^2\Gamma(c+4)+\theta^6\Gamma(c+5))}\int_0^q x\frac{x^c\theta^{c+1}\left(\alpha^3+\alpha^2\theta x+\alpha(\theta x)^2+(\theta x)^3\right)e^{-\theta x}}{\left(\alpha^3\Gamma(c+1)+\alpha^2\Gamma(c+2)+\alpha\theta^2\Gamma(c+3)+\theta^6\Gamma(c+4)\right)}dx$$

$$B(p) = \frac{\theta(\alpha^3\Gamma(c+1)+\alpha^2\Gamma(c+2)+\alpha\theta^2\Gamma(c+3)+\theta^6\Gamma(c+4))}{p(\alpha^3\Gamma(c+2)+\alpha^2\Gamma(c+3)+\alpha\theta^2\Gamma(c+4)+\theta^6\Gamma(c+5))}\int_0^q \frac{x^{c+1}\theta^{c+1}\left(\alpha^3+\alpha^2\theta x+\alpha(\theta x)^2+(\theta x)^3\right)e^{-\theta x}}{\left(\alpha^3\Gamma(c+1)+\alpha^2\Gamma(c+2)+\alpha\theta^2\Gamma(c+3)+\theta^6\Gamma(c+4)\right)}dx$$

$$B(p) = \frac{\theta^{c+2}}{p(\alpha^3\Gamma(c+2)+\alpha^2\Gamma(c+3)+\alpha\theta^2\Gamma(c+4)+\theta^6\Gamma(c+5))}\int_0^q x^{c+1}\left(\alpha^3+\alpha^2\theta x+\alpha(\theta x)^2+(\theta x)^3\right)e^{-\theta x}dx$$

$$B(p) = \frac{\theta^{c+2}}{p(\alpha^3\Gamma(c+2)+\alpha^2\Gamma(c+3)+\alpha\theta^2\Gamma(c+4)+\theta^6\Gamma(c+5))}\int_0^q x^{c+1}\left(\alpha^3+\alpha^2\theta x+\alpha\theta^2x^2+\theta^3x^3\right)e^{-\theta x}dx$$

$$B(p) = \frac{\theta^{c+2}}{p(\alpha^3\Gamma(c+2)+\alpha^2\Gamma(c+3)+\alpha\theta^2\Gamma(c+4)+\theta^6\Gamma(c+5))}\begin{pmatrix}\alpha^3\int_0^q x^{(c+2)-1}e^{-\theta x}dx+\alpha^2\theta\int_0^q x^{(c+3)-1}e^{-\theta x}dx\\ +\alpha\theta^2\int_0^q x^{(c+4)-1}e^{-\theta x}dx+\theta^3\int_0^q x^{(c+5)-1}e^{-\theta x}dx\end{pmatrix}$$

After simplification, we obtain

$$B(p) = \frac{\theta^{c+2}}{p(\alpha^3\Gamma(c+2)+\alpha^2\Gamma(c+3)+\alpha\theta^2\Gamma(c+4)+\theta^6\Gamma(c+5))}\left(\alpha^3\gamma(c+2,\theta q)+\alpha^2\theta\gamma(c+3,\theta q)+\alpha\theta^2\gamma(c+4,\theta\theta)+\theta^3\gamma(c+5,\theta q)\right)$$

$$L(p) = \frac{\theta^{c+2}}{(\alpha^3\Gamma(c+2)+\alpha^2\Gamma(c+3)+\alpha\theta^2\Gamma(c+4)+\theta^6\Gamma(c+5))}\left(\alpha^3\gamma(c+2,\theta q)+\alpha^2\theta\gamma(c+3,\theta q)+\alpha\theta^2\gamma(c+4,\theta q)+\theta^3\gamma(c+5,\theta q)\right)$$

7. Maximum Likelihood Estimation and Fisher's Information Matrix

In this section, we will discuss the technique of maximum likelihood estimation to estimate the parameters of weighted quasi Amarendra distribution. Consider $X_1, X_2, \ldots, X_n$ be the random sample of size n drawn from the weighted quasi Amarendra distribution, then the likelihood function can be defined as.

$$L(x)=\prod_{i=1}^{n} f_w(x)$$

$$L(x)=\prod_{i=1}^{n}\left(\frac{x_i^{\,c}\theta^{c+1}}{\left(\alpha^3\Gamma(c+1)+\alpha^2\Gamma(c+2)+\alpha\theta^2\Gamma(c+3)+\theta^6\Gamma(c+4)\right)}\left(\alpha^3+\alpha^2\theta x_i+\alpha(\theta x_i)^2+(\theta x_i)^3\right)e^{-\theta x_i}\right)$$

$$L(x)=\frac{\theta^{n(c+1)}}{\left(\alpha^3\Gamma(c+1)+\alpha^2\Gamma(c+2)+\alpha\theta^2\Gamma(c+3)+\theta^6\Gamma(c+4)\right)^n}\prod_{i=1}^{n}\left(x_i^{\,c}\left(\alpha^3+\alpha^2\theta x_i+\alpha(\theta x_i)^2+(\theta x_i)^3\right)e^{-\theta x_i}\right)$$

$$L(x)=\frac{\theta^{n(c+1)}}{\left(\alpha^3\Gamma(c+1)+\alpha^2\Gamma(c+2)+\alpha\theta^2\Gamma(c+3)+\theta^6\Gamma(c+4)\right)^n}\prod_{i=1}^{n}\left(x_i^{\,c}\left(\alpha^3+\alpha^2\theta x_i+\alpha\theta^2 x_i^{\,2}+\theta^3 x_i^{\,3}\right)e^{-\theta x_i}\right)$$

The log likelihood function is given by

$$\log L=n(c+1)\log\theta-n\log\left(\alpha^3\Gamma(c+1)+\alpha^2\Gamma(c+2)+\alpha\theta^2\Gamma(c+3)+\theta^6\Gamma(c+4)\right)+c\sum_{i=1}^{n}\log x_i$$

$$+\sum_{i=1}^{n}\log\left(\alpha^3+\alpha^2\theta x_i+\alpha\theta^2 x_i^{\,2}+\theta^3 x_i^{\,3}\right)-\theta\sum_{i=1}^{n}x_i \qquad (12)$$

Now by differentiating the log likelihood equation (12) with respect to parameters α, θ and c and must satisfy the following normal equations

$$\frac{\partial\log L}{\partial\alpha}=-n\left(\frac{3\alpha^2\Gamma(c+1)+2\alpha\Gamma(c+2)+\theta^2\Gamma(c+3)}{\alpha^3\Gamma(c+1)+\alpha^2\Gamma(c+2)+\alpha\theta^2\Gamma(c+3)+\theta^6\Gamma(c+4)}\right)+\sum_{i=1}^{n}\left(\frac{3\alpha^2+2\alpha\theta x_i+\theta^2 x_i^{\,2}}{(\alpha^3+\alpha^2\theta x_i+\alpha\theta^2 x_i^{\,2}+\theta^3 x_i^{\,3})}\right)=0$$

$$\frac{\partial\log L}{\partial\theta}=\frac{n(c+1)}{\theta}-n\left(\frac{2\alpha\theta\Gamma(c+3)+6\theta^5\Gamma(c+4)}{\alpha^3\Gamma(c+1)+\alpha^2\Gamma(c+2)+\alpha\theta^2\Gamma(c+3)+\theta^6\Gamma(c+4)}\right)+\sum_{i=1}^{n}\left(\frac{\alpha^2 x_i+2\alpha\theta x_i^{\,2}+3\theta^2 x_i^{\,3}}{(\alpha^3+\alpha^2\theta x_i+\alpha\theta^2 x_i^{\,2}+\theta^3 x_i^{\,3})}\right)$$

$$-\sum_{i=1}^{n}x_i=0$$

$$\frac{\partial\log L}{\partial c}=n\log\theta-n\psi\left(\alpha^3\Gamma(c+1)+\alpha^2\Gamma(c+2)+\alpha\theta^2\Gamma(c+3)+\theta^6\Gamma(c+4)\right)+\sum_{i=1}^{n}\log x_i=0$$

Where ψ (.) is the digamma function.

Because of complicated form of above likelihood equations, algebraically it is very difficult to solve the system of nonlinear equations. Therefore, we use the technique like Newton-Raphson for estimating the parameters of introduced distribution.

In order to use the asymptotic normality result for obtaining the obtain confidence interval. We have that if $\hat{\lambda}=(\hat{\alpha},\hat{\theta},\hat{c})$ denotes the MLE of $\lambda=(\alpha,\theta,c)$. we can determine the results as

$$\sqrt{n}(\hat{\lambda}-\lambda)\rightarrow N_3(0,I^{-1}(\lambda))$$

Where $I^{-1}(\lambda)$ is Fisher's informatio n matrix.i.e

$$I(\lambda)=-\frac{1}{n}\begin{pmatrix} E\left(\frac{\partial^2 \log L}{\partial\alpha^2}\right) & E\left(\frac{\partial^2 \log L}{\partial\alpha\,\partial\theta}\right) & E\left(\frac{\partial^2 \log L}{\partial\alpha\,\partial c}\right) \\ E\left(\frac{\partial^2 \log L}{\partial\theta\,\partial\alpha}\right) & E\left(\frac{\partial^2 \log L}{\partial\theta^2}\right) & E\left(\frac{\partial^2 \log L}{\partial\theta\,\partial c}\right) \\ E\left(\frac{\partial^2 \log L}{\partial c\,\partial\alpha}\right) & E\left(\frac{\partial^2 \log L}{\partial c\,\partial\theta}\right) & E\left(\frac{\partial^2 \log L}{\partial c^2}\right) \end{pmatrix}$$

$$E\left(\frac{\partial^2 \log L}{\partial\alpha^2}\right)=-n\left(\frac{(\alpha^3\Gamma(c+1)+\alpha^2\Gamma(c+2)+\alpha\theta^2\Gamma(c+3)+\theta^6\Gamma(c+4))(6\alpha\,\Gamma(c+1)+2\Gamma(c+2)) - (3\alpha^2\Gamma(c+1)+2\alpha\,\Gamma(c+2)+\theta^2\Gamma(c+3))^2}{(\alpha^3\Gamma(c+1)+\alpha^2\Gamma(c+2)+\alpha\theta^2\Gamma(c+3)+\theta^6\Gamma(c+4))^2}\right)$$

$$+\sum_{i=1}^{n}\left(\frac{(\alpha^3+\alpha^2\theta x_i+\alpha\theta^2 x_i^{\,2}+\theta^3 x_i^{\,3})(6\alpha+2\theta x_i)-(3\alpha^2+2\alpha\theta x_i+\theta^2 x_i^{\,2})^2}{(\alpha^3+\alpha^2\theta x_i+\alpha\theta^2 x_i^{\,2}+\theta^3 x_i^{\,3})^2}\right)$$

$$E\left(\frac{\partial^2 \log L}{\partial\theta^2}\right)=-\frac{n(c+1)}{\theta^2}-n\left(\frac{(\alpha^3\Gamma(c+1)+\alpha^2\Gamma(c+2)+\alpha\theta^2\Gamma(c+3)+\theta^6\Gamma(c+4))(2\alpha\,\Gamma(c+3)+30\,\theta^4\Gamma(c+4)) - (2\alpha\theta\Gamma(c+3)+6\theta^5\Gamma(c+4))^2}{(\alpha^3\Gamma(c+1)+\alpha^2\Gamma(c+2)+\alpha\theta^2\Gamma(c+3)+\theta^6\Gamma(c+4))^2}\right)$$

$$+\sum_{i=1}^{n}\left(\frac{(\alpha^3+\alpha^2\theta x_i+\alpha\theta^2 x_i^{\,2}+\theta^3 x_i^{\,3})(2\alpha x_i^{\,2}+6\theta x_i^{\,3})-(\alpha^2 x_i+2\alpha\theta x_i^{\,2}+3\theta^2 x_i^{\,3})^2}{(\alpha^3+\alpha^2\theta x_i+\alpha\theta^2 x_i^{\,2}+\theta^3 x_i^{\,3})^2}\right)$$

$$E\left(\frac{\partial^2 \log L}{\partial c^2}\right)=-n\psi'\left(\alpha^3\Gamma(c+1)+\alpha^2\Gamma(c+2)+\alpha\theta^2\Gamma(c+3)+\theta^6\Gamma(c+4)\right)$$

$$E\left(\frac{\partial^2 \log L}{\partial\theta\,\partial\alpha}\right)=-n\left(\frac{(\alpha^3\Gamma(c+1)+\alpha^2\Gamma(c+2)+\alpha\theta^2\Gamma(c+3)+\theta^6\Gamma(c+4))(2\theta\,\Gamma(c+3))-(2\alpha\theta\Gamma(c+3)+6\theta^5\Gamma(c+4)) (3\alpha^2\Gamma(c+1)+2\alpha\Gamma(c+2)+\theta^2\Gamma(c+3))}{(\alpha^3\Gamma(c+1)+\alpha^2\Gamma(c+2)+\alpha\theta^2\Gamma(c+3)+\theta^6\Gamma(c+4))^2}\right)$$

$$+\sum_{i=1}^{n}\left(\frac{(\alpha^3+\alpha^2\theta x_i+\alpha\theta^2 x_i^{\,2}+\theta^3 x_i^{\,3})(2\alpha x_i+2\theta x_i^{\,2})-(\alpha^2 x_i+2\alpha\theta x_i^{\,2}+3\theta^2 x_i^{\,3})(3\alpha^2+2\alpha\theta x_i+\theta^2 x_i^{\,2})}{(\alpha^3+\alpha^2\theta x_i+\alpha\theta^2 x_i^{\,2}+\theta^3 x_i^{\,3})^2}\right)$$

$$E\left(\frac{\partial^2 \log L}{\partial\theta\,\partial c}\right) = \frac{n}{\theta} - n\psi\left(\frac{2\alpha\theta\Gamma(c+3) + 6\theta^5\Gamma(c+4)}{\alpha^3\Gamma(c+1) + \alpha^2\Gamma(c+2) + \alpha\theta^2\Gamma(c+3) + \theta^6\Gamma(c+4)}\right)$$

$$E\left(\frac{\partial^2 \log L}{\partial\alpha\,\partial c}\right) = -n\psi\left(\frac{3\alpha^2\Gamma(c+1) + 2\alpha\Gamma(c+2) + \theta^2\Gamma(c+3)}{\alpha^3\Gamma(c+1) + \alpha^2\Gamma(c+2) + \alpha\theta^2\Gamma(c+3) + \theta^6\Gamma(c+4)}\right)$$

Where $\psi(.)'$ is the first order derivative of digamma function.

Since λ being unknown, we estimate $I^{-1}(\lambda)$ by $I^{-1}(\hat{\lambda})$ and this can be used to obtain asymptotic confidence intervals for α, θ and c.

8. Application

In this section, we have analyzed and inspected a real lifetime data set for fitting weighted quasi Amarendra distribution in order to show that weighted quasi Amarendra distribution provides a better fit over quasi Amarendra and Amarendra distributions. The following real data set is given below as

The following real data set given below in table 1 represents the strength of 1.5cm glass fibres measured at the National physical laboratory England reported by Smith and Naylor (1987) [12] and the data set is given below as.

Table 1: Data represents the strength of 1.5cm glass fibres reported by Smith and Naylor (1987) [12]

0.55	2	1.82	1.76	1.7	1.7	0.93	0.74	2.01	1.84.	1.77	1.78
1.25	1.04	0.77	2.24	1.84	1.89	1.36	1.27	1.11	0.81	0.84	1.49
1.39	1.28	1.13	1.24	1.52	1.49	1.42	1.29	1.3	1.61	1.59	1.54
1.5	1.51	1.64	1.61	1.6	1.55		1.68	1.66	1.62	1.61	1.61
1.73	1.68	1.66	1.62	1.63	1.81	1.76	1.69	1.66	1.67	1.58	1.53
1.5	1.48	1.48									

To compute the model comparison criterions along with the estimation of unknown parameters, the technique of R software is applied. In order to compare the performance of weighted quasi Amarendra distribution over quasi Amarendra and Amarendra distributions, we use the criterion values *AIC* (Akaike Information Criterion), *BIC* (Bayesian Information Criterion), *AICC* (Akaike Information Criterion Corrected) and *-2logL*. The distribution is better which shows lesser the corresponding criterion values of *AIC*, *BIC*, *AICC* and *-2logL*. For determining the criterion values like *AIC*, *BIC*, *AICC* and *-2logL* following formulas are used.

$$AIC = 2k - 2\log L,\ \ BIC = k\log n - 2\log L \quad \text{and} \quad AICC = AIC + \frac{2k(k+1)}{n-k-1}$$

Where k is the number of parameters in the statistical model, n is the sample size and $-2\log L$ is the maximized value of the log-likelihood function under the considered model.

Table 2: Shows MLE, S.E, Criterion values (AIC, BIC, AICC), -2logL and Performance of Fitted Distributions

Distributions	MLE	S.E	-2logL	AIC	BIC	AICC
Weighted Quasi Amarendra	$\hat{\alpha} = 3.455999$ $\hat{\theta} = 1.157467$ $\hat{c} = 1.644094$	$\hat{\alpha} = 1.500600$ $\hat{\theta} = 2.072457$ $\hat{c} = 3.078178$	47.90317	53.90317	60.33257	54.3099
Quasi Amarendra	$\hat{\alpha} = 0.0010000$ $\hat{\theta} = 2.6544101$	$\hat{\alpha} = 0.3553131$ $\hat{\theta} = 0.1787381$	93.67924	97.67924	101.9655	97.8792
Amarendra	$\hat{\theta} = 1.7200935$	$\hat{\theta} = 0.1070047$	151.0372	153.0372	155.1804	153.1027

From results given above in table 2, it has been clearly realized that the weighted quasi Amarendra distribution has the lesser *AIC*, *BIC*, *AICC* and *-2logL* values as compared to the quasi Amarendra and Amarendra distributions. Hence, it can be revealed that the weighted quasi Amarendra distribution provides a better fit over quasi Amarendra and Amarendra distributions.

9. Conclusion

In this paper, a new class of quasi Amarendra distribution called as weighted quasi Amarendra distribution has been thoroughly presented and studied. The proposed new distribution is introduced by using the weighted technique to its baseline distribution. Its different structural properties including shape of behaviour of pdf and cdf, raw moments, mean and variance, harmonic mean, order statistics, bonferroni and lorenz curves, survival function, hazard rate function, reverse hazard function and moment generating function have been discussed. For estimating the parameters of introduced distribution, the technique of maximum likelihood estimation has been used. Finally, a real lifetime data set has been examined and analyzed to demonstrate the significance of a new distribution and hence it is also concluded that the proposed weighted quasi Amarendra distribution provides a quite satisfactory result over quasi Amarendra and Amarendra distributions.

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