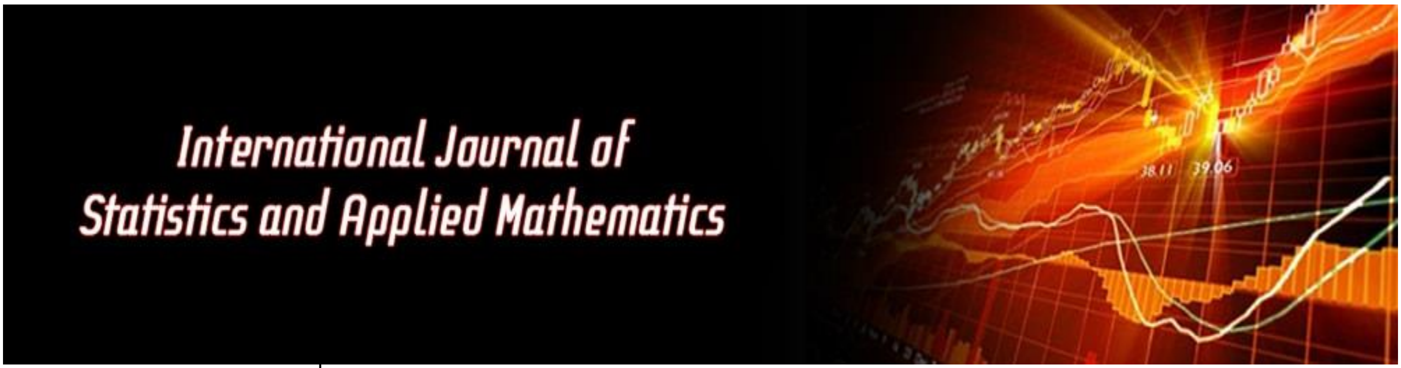

ISSN: 2456-1452
Maths 2023; 8(5): 224-229

https://www.mathsjournal.com
Received: 06-06-2023
Accepted: 08-07-2023

Preeti
Department of Mathematics and Statistics, CCS Haryana Agricultural University, Hisar, Haryana, India

Hemant Poonia
Department of Mathematics and Statistics, CCS Haryana Agricultural University, Hisar, Haryana, India

Ajay Sharma
Department of Mathematics and Statistics, CCS Haryana Agricultural University, Hisar, Haryana, India

Corresponding Author:
Preeti
Department of Mathematics and Statistics, CCS Haryana Agricultural University, Hisar, Haryana, India

Multi-objective fuzzy linear programming techniques: A critical review

Preeti, Hemant Poonia and Ajay Sharma

Abstract
This study delves into multi-objective optimization, which involves solving mathematical problems with multiple competing goals. It introduces key concepts and notations related to optimization. The paper examines several methods, such as Bellman and Zadeh's, Zimmerman's, and fuzzy versions of the Simplex method with different types of fuzzy numbers. It also explores the Fuzzy Decisive Set method. The study includes discussions on the algorithms for these techniques to find the best solutions.

Keywords: Fuzzy set theory, Fuzzy membership function, Fuzzy logic, optimal solution

Introduction
In our everyday descriptions of the real world, we often use imprecise language. For instance, when we ask someone's height, we might get a general answer like "tall" rather than a specific measurement. Similarly, when discussing the weather, we tend to use terms like "hot" or "cool" instead of giving exact temperatures. This imprecision is what we refer to as "fuzziness". Multi-objective optimization is a growing field within operations research that deals with problems where there are multiple objectives, each needing to be optimized simultaneously. The groundwork for formalizing this concept of fuzziness was laid by Lotfi A. Zadeh in the 1960s. Zadeh played a pivotal role in advancing and applying fuzzy logic to real-world problems. The idea of fuzzy linear programming (FLP) was introduced by Tanaka *et al.* (1974) [19], building on the fuzzy framework established by Bellman and Zadeh (1970) [4].
Zimmermann (1978) introduced the formulation of FLP problem and constructed a model of the problem also based on the fuzzy concepts of Bellman and Zadeh (1970) [4]. Optimization in fuzzy environment was further studied and was applied in various areas by many researchers 1 such as Tanaka *et al.* (1974) [19], Jana and Roy (2007) [11] etc. studied the multi objective intuitionistic fuzzy linear programming problem and applied it to transportation problem. There are many researchers who have given computational algorithms to solve the multi-objective fuzzy linear programming problems. Veeramani *et al.* (2011) [20] gave novel approach to solve the fuzzy multi-objective linear programming problem with linear membership function using the fuzzy decisive set method proposed by Gasimov and Yenilmez (2002) [9]. Shirin and Kamrunnahar (2014) [18] presented an optimization problem and optimal solutions has been computed by using three methods, such as Bellman-Zadeh’s method, Zimmerman’s method, and Fuzzy version of Simplex method, which are compared to each other. The main objective of this paper is to focus on the appropriate method in order to achieve the best optimal solution.
Rajarajeswari and Sudha (2014) [12] introduced a novel approach using Robust's ranking technique for solving fully fuzzy linear programming problems, providing an optimal solution. Beena (2016) [5] addressed multi-objective fuzzy linear programming with coefficients represented as triangular fuzzy numbers. The problem was converted into a crisp linear programming problem based on fuzzy decision concepts by Bellman and Zadeh (1970) [4], and solved using Pareto's optimality techniques. Abdulqader (2017) [3] proposed a technique to transform multiple optimization problems into a single fuzzy linear programming problem. The resulting compromise solution was obtained using a linear ranking function through the simplex method.

In agricultural planning, fuzzy optimization techniques play a pivotal role in achieving production goals. Authors like Sharma *et al.* (2007) [17], Sarkar and Samanta (2017) [15], Gupta and Umaid (2018) [10] have worked on crop planning models using fuzzy environments. Additionally, Angelov (1997) [2] extended fuzzy sets theory to intuitionistic fuzzy sets, leading to further research on optimization in this domain by authors like Dubey *et al.* (2012) [7] and Jana and Roy (2007) [11]. Bharati and Singh (2014) [6] delved into multi-objective linear programming problems in an intuitionistic fuzzy setting. This study aims to provide computational algorithms for solving multi-objective linear programming problems using various methods including those of Bellman and Zadeh, Zimmermann, the fuzzy version of the simplex method with triangular and trapezoidal fuzzy numbers, and the fuzzy decisive set method. The paper is organized into sections covering preliminaries of fuzzy optimization, computational algorithms, and concludes with results and references.

2 Preliminaries

Definitions used in fuzzy linear programming problem are given below.

2.1 Basic definitions

Fuzzy set theory: Fuzzy set theory deals with concepts on a spectrum, rather than in strict categories. It acknowledges that many real-world problems are complex and ambiguous, and provides a way to represent and work with uncertainties in a more nuanced manner.

Crisp set: Crisp sets operate on a strict membership basis, where elements are definitively either part of the set or not. This is determined by a characteristic function assigning a clear 1 or 0 values. For instance, a jelly bean is unquestionably classified as candy, while mashed potatoes are not. This clear distinction is a hallmark of crisp sets.

Fuzzy set: A fuzzy set extends the binary membership $\{0, 1\}$ of a conventional set to a spectrum in the interval $[0, 1]$. Fuzzy sets allow elements to be partially in a set. Mathematically, Let X is the universal set. $\tilde{A}$ is called a fuzzy set in X if $\tilde{A}$ is a set of ordered pairs $\tilde{A} = \{(x, \mu_{\tilde{A}}(x));\ x \in X\}$; where $\mu_{\tilde{A}}(x)$ is the membership function of $x \in \tilde{A}$.

Membership function: The evaluation function $\mu_{\tilde{A}}(x)$ is called the membership function or the grade of membership of x in $\tilde{A}$. Note that the membership function of $\tilde{A}$ is a characteristic (indicator) function for $\tilde{A}$ and it shows to what degree $x \in \tilde{A}$.

Types of membership function

Choosing an appropriate membership function is a crucial step in applying fuzzy logic. It falls upon the user to select a function that accurately represents the fuzzy concept in question. The widely used membership functions include:

1. Linear membership function
2. Triangular membership function
3. Trapezoidal membership function
4. Sigmoid membership function
5. $\pi -$ type membership function
6. Gaussian membership function

Fuzzy number

A fuzzy set $\tilde{A}$ on the set of real numbers R, is termed a fuzzy number if its membership function satisfies specific criteria

1. $\mu_{\tilde{A}} : R \rightarrow [0, 1]$ is continuous.
2. $\mu_{\tilde{A}}(x) = 0$ for all $x \in (-\infty, a) \cup (d, \infty)$.
3. $\mu_{\tilde{A}}(x)$ Strictly increasing on $[a, b]$ and strictly decreasing on $[c, d]$.
4. $\mu_{\tilde{A}}(x) = 1$ for all $x \in [b, c]$, where $a < b < c < d$.

Triangular fuzzy number

A fuzzy number is said to be a triangular fuzzy number if its membership function is defined by

$$\mu(x) = \begin{cases} 0 \ if \ x \le \alpha \\ \frac{x-\alpha}{\beta-\alpha} \ if \ \alpha \le x \le \beta \\ \frac{\beta-x}{\beta-\gamma} \ if \ \beta \le x \le \gamma \\ 0 \ if \ x > \beta \end{cases} \quad (1)$$

The above definition can be illustrated in figure 1

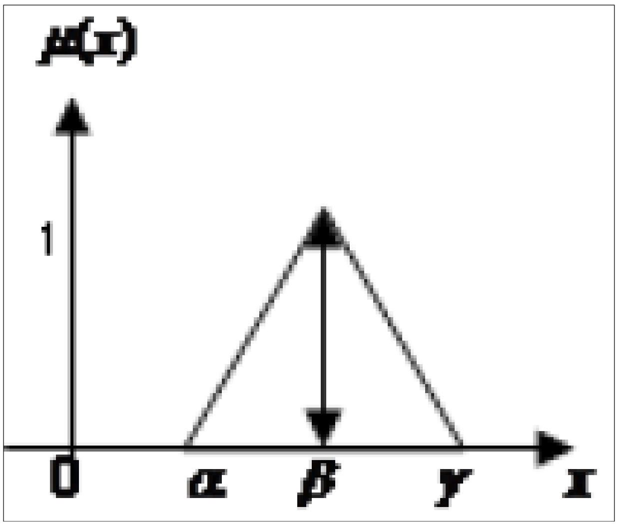

Fig 1: Triangular membership function

Trapezoidal fuzzy number: A fuzzy number is said to be a trapezoidal fuzzy number if its membership function is defined by

$$\mu(x) = \begin{cases} 0 \ if \ x < \alpha \\ \frac{x-\alpha}{\beta-\alpha} \ if \ \alpha \le x \le \beta \\ 1 \ if \ \beta \le x \le \gamma \\ \frac{\gamma-x}{\gamma-\delta} \ if \ \gamma \le x \le \delta \\ 0 \ if \ x \ge \delta \end{cases} \quad (2)$$

The above definition can be illustrated in figure 2.

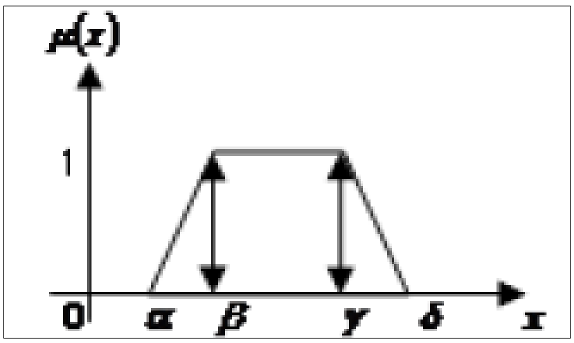

Fig 2: Trapezoidal membership functions

2.2 Fuzzy Linear Programming Problem

A fuzzy linear programming problem (FLPP) is defined as follows:

Maximize $\tilde{Z} = \tilde{C}\tilde{X}$
Subject to $\tilde{A}\tilde{X} \approx \tilde{b}$
$\tilde{X} \geq \tilde{0}$ (3)

Where $\tilde{A}$ is $m \times n$ matrix in fuzzy real field, i.e., $A^{\sim} \in F(P^{m \times n})$; $\tilde{X}, \tilde{C}$ are $1 \times n$ matrix in fuzzy real field, i.e., $X,^{\sim} C^{\sim} \in F(P^n)$; and $\tilde{b}$ is a $m \times 1$ matrix in fuzzy real field, i.e., $\tilde{b} \in F(P^m)$. The components of each matrix are fuzzy numbers.

2.3 Multi-objective Fuzzy Linear Programming Problem

In a fuzzy context, dealing with multiple objectives leads to the field of multi-criteria fuzzy programming. This area has seen substantial growth since the 1970s. Various methods have been devised to handle problems with multiple objectives, typically assuming real-valued objectives and well-defined actions. In summary, Multi-Objective Fuzzy Linear Programming (MOFLP) involves maximizing 'r' objectives and minimizing's-r' objectives, subject to constraints and decision variables.

$$Max\ \tilde{Z}_i(x) = \sum_{j=1}^{n} \tilde{C}_{ij}\tilde{X}_j; i = 1, \dots\dots, r$$

$$Min\ \tilde{Z}_i(x) = \sum_{j=1}^{n} \tilde{C}_{ij}\tilde{X}_j; i = r+1, \dots\dots, s \quad (4)$$

$Subject\ to\ \tilde{A}\tilde{X} \approx \tilde{b}$
$\tilde{X} \geq \tilde{0}$
Where $\tilde{A}, \tilde{b}, \tilde{C}, \tilde{X}$ *and* $\tilde{Z}$ is same as defined in equation (1).

3 Methodologies

This section outlines the methodology for multi-objective optimization techniques and discusses algorithms for different methods used in solving multi-objective fuzzy linear programming problems.

3.1 Bellman and Zadeh's approach

A fuzzy linear programming, in which only the right hand side constraints Bi are fuzzy numbers, is as follows:

$$Max \sum_{j=1}^{n} c_j\ x_j$$

$$subject\ to \sum_{j=1}^{n} a_{ij}\ x_j \leq B_i\ (i \epsilon N_m)\ \ x_j \geq 0\ (j \epsilon N_n)\ (5)$$

The steps for algorithm to obtain the fuzzy optimal solution by Bellman's Zadeh method are as follows:

1. Consider a fuzzy linear programming problem having right hand side as fuzzy numbers B_i having membership function defined as

$$B_i\ (x) = \begin{cases} 1, x \leq b_i \\ \frac{b_i + p_i - x}{p_i},\ b_i \leq x \leq b_i + p_i \\ 0, x \geq b_i + p_i \end{cases} \quad (6)$$

2. Calculate lower and upper bounds of the objective function z_l and z_u respectively.
3. Define the fuzzy set G of optimal values as.

$$G(z) = \begin{cases} 1, z_u \leq z \\ \frac{z - z_l}{z_u - z_l}, z_l \leq z \leq z_u \\ 0, z \leq z_l \end{cases} \quad (7)$$

4. Introduce a new variable λ which denotes the membership grade of the solution and also introduce a new constraint equation is $\lambda(z_u - z_l) - z \leq -z_l$.
5. Convert the fuzzy linear programming problem into the equivalent crisp linear programming problem as:

$Max\ \lambda$
$subject\ to\ \lambda(z_u - z_l) - z \leq -z_l$
$\lambda p_i + \sum_{j=1}^{n} a_{ij}\ x_j \leq b_i + p_i\ (i\ \epsilon N_m)$ (8)
$\lambda, x_j \geq 0\ (j \epsilon N_n)$

6. Solve it for the variable x_j and λ using LINGO 18.0 software.
7. Put the values of x_j into the objective function in order to get the optimal solution.

3.2 Zimmerman's Method

The formulation of Fuzzy linear programming in which only right hand side B_i and the objective function are fuzzy numbers, but the coefficients A_{ij} of the constant matrix are not fuzzy numbers, is as follows.

$$Max\ \sum_{j=1}^{n} A_{ij}\ x_j \leq B_i\ (i \epsilon N_m)\ x_j \geq 0\ (j \epsilon N_n) \quad (9)$$

In matrix form above fuzzy linear programming problem can be written as follows:
$Find\ x\ suchthat\ C^T\ x\ \gtrsim\ z$
$Ax\ \lesssim\ b\ x\ \geq\ 0$ (10)

Where $\gtrsim$ denotes the fuzzified version of $\geq$ and $\lesssim$ denotes the fuzzified version of $\leq$. This type of FLP is solved by using the algorithm of Zimmerman's Method and the algorithm is as follows:

1. We define the matrix A and b such that

$Bx\ \lesssim\ d, x\ \geq\ 0$
Where $\binom{-c^T}{A} = B$ and $\binom{-z}{b} = d$

2. Find the lower bound of tolerance intervalsd_i.
3. Spread the tolerance intervalsp_i.

Next define the membership function μ_i as follows.

$$\mu_i\ (x) = \begin{cases} 1, B_i\ x \leq d_i \\ 1 - \frac{B_i\ x - d_i}{p_i},\ d_i \leq B_i\ x \leq d_i + p_i \\ 0, B_i\ x > d_i + p_i \end{cases} \quad (11)$$

4. Introduce a new variable λ and calculate $\frac{\lambda p_i + B_i\ x}{p_i} \leq \frac{d_i + p_i}{p_i}, i = 1,2, \dots\dots\dots, m+1$.
5. Express the fuzzy linear programming as an equivalent crisp linear programming problem in the following manner.

$Max\ \lambda\ \lambda p_i + B_i\ x \leq d_i + p_i, i = 1,2, \dots\dots\dots, m+1$
$where\ x \geq 0\ x = (x_j), j \in N_n$ (12)

6. Solve it for the variables x and λ where the new variable λ denotes the membership grade of the solution and get the solution using LINGO 18.0 software.
7. Put the values of x_j into the objective function so that we can get the optimal solution.

3.3 Fuzzy version of simplex method having triangular fuzzy numbers

Fuzzy linear programming in which all the parameters and variables are considered as triangular fuzzy numbers are as follows.

$$Max\ z \approx \sum_{j=1}^{n} C_j X_j$$

$$s.t. \sum_{j=1}^{n} A_{ij} \preceq B_i \ (i \in N_m)$$

$X_j \succeq 0\ (j \in N_n)$ (13) Where A_{ij}, B_i, C_j are fuzzy numbers and X_j are variables whose states are fuzzy numbers ($i \in N_m, j \in N_n$); the operations of additions and multiplications are operations of fuzzy arithmetic; and $\preceq$, $\succeq$ denote the ordering of fuzzy numbers. The steps of the fuzzy version of simplex method are as follows:

1. Check whether the objective function of type given FLPP is to be minimized or maximized. If it is to be minimized then we convert it into a problem of maximizing it by using the result $Minimum\ \tilde{z} = -Maximum(-\tilde{z})$.
2. Check whether all $\widetilde{b_\iota}\ (i = 1,2, \ldots \ldots, m)$ are non-negative. If any one of $\widetilde{b_\iota}$ is negative then multiply the corresponding in equation of the constant by -1, so as to get all $\widetilde{b_\iota}\ (i = 1,2, \ldots \ldots, m)$ non-negative.
3. Convert all the inequalities of the constraints into equations by introducing slack and / or surplus fuzzy variables in the constraints. Put the cost of these variables equal to zero.
4. Obtain an initial basic feasible solution of the problem in the form $\tilde{X}_B \approx \tilde{B}^{-1}\tilde{b}$ and put in the first column of the Simplex table.
5. Compute the net evaluations $\tilde{z}_j - \tilde{c}_j\ (j = 1, 2, \ldots, n)$ by using the relation $\tilde{z}_j - \tilde{c}_j = \tilde{c}_B \tilde{y}_j - \tilde{c}_j$, where any column $\tilde{A}_j \approx \sum_{i=1}^{m} \tilde{y}_{ij}\ \widetilde{b_\iota} \approx \tilde{y}_{1j}\widetilde{b_1} + \tilde{y}_{2j}\widetilde{b_2} + \cdots + \tilde{y}_{rj}\widetilde{b_r} + \cdots + \tilde{y}_{mj}\widetilde{b_m} \approx \tilde{y}_j \tilde{B}$.
6. Further examine the sign of $\tilde{z}_j - \tilde{c}_j$.
   i. If all $(\tilde{z}_j - \tilde{c}_j) \succeq \tilde{0}$ then the initial basic feasible fuzzy solution $\tilde{x}_B$ is an optimum basic feasible fuzzy solution.
   ii. If at least one $(\tilde{z}_j - \tilde{c}_j) < \tilde{0}$, proceed on to the next step.
7. If there are more than one negative $(\tilde{z}_j - \tilde{c}_j)$, then choose the most negative of them. Let it be $(\tilde{z}_r - \tilde{c}_r)$ for some $j = r$.
   i. If all $\tilde{y}_{ir} < \tilde{0}\ (i = 1, 2, \ldots, m)$, then there is an unbounded solution to the given problem.
   ii. If at least one $\tilde{y}_{ir} < \tilde{0}\ (i = 1, 2, \ldots, m)$, then the corresponding vector $\tilde{y}_r$ enter the basis $\tilde{y}_B$.
8. Compute $\frac{\tilde{x}_{Bi}}{\tilde{y}_{ir}}, i = 1, 2, \ldots, m$ and choose minimum of them. Let minimum of these ratios be $\frac{\tilde{x}_{Br}}{\tilde{y}_{kr}}$. Then the vector $\tilde{y}_k$ will level the basis $\tilde{y}_B$. The common element $\tilde{y}_{kr}$, which in the k^{th} row and r^{th} column is known as leading triangular fuzzy number of the table.
9. Normalize the leading triangular fuzzy number by dividing its row by the number itself, and setting all other elements in its column to zero using the relation.

$$\cap\, \tilde{y} \approx \tilde{y}_{ij} - \left(\frac{\tilde{y}_{kj}}{\tilde{y}_{kr}}\right)\tilde{y}_{ir}, i = 1, 2, \ldots, m + 1; i \neq k \text{ and } \tilde{y}_{kj} \approx \frac{\tilde{y}_{kj}}{\tilde{y}_{kr}}, j = 0, 1, 2, \ldots, n.$$

10. Go to step 5 and repeat the computational procedure until either an optimum solution is obtained or these is an indication of an unbounded solution.

3.4 Fuzzy version of simplex method having trapezoidal fuzzy numbers

Follow these steps when solving a fuzzy linear programming problem with trapezoidal fuzzy numbers

1. Determine if the given FLPP objective function is for minimization or maximization. If it's for minimization, transform it into a maximization problem using the result: $Minimum\ \tilde{z} = -Maximum(-\tilde{z})$.
2. Check whether all $\widetilde{b_\iota}\ (i = 1,2, \ldots \ldots, m)$ are non-negative.. If any one of $\widetilde{b_\iota}$ is negative then multiply the corresponding in equation of the constant by -1, so as to get all $\widetilde{b_\iota}\ (i = 1,2, \ldots \ldots, m)$ non-negative.
3. Transform the inequalities into equations by introducing surplus or slack fuzzy variables in the constraints. Assign coefficients of (1, 1, 1, 1) to these variables and set their cost to zero.
4. The starting tableau for the Simplex method includes coefficients of decision variables from the original problem, as well as slack, surplus, and artificial variables introduced in the second step along with the constraints. The column C_b holds coefficients of the variables in the base. The first row consists of the objective function coefficients, while the last row contains the objective function value and reduced costs $C_j - Z_j$. The last row is calculated as follows: $Z_j = \sum C_{b_i} \times X_i$ for $i = 1, 2, \ldots, m$. Although this is the first tableau of the simplex method and all C_b are null, so the calculation can be simplified.
5. In maximization, if no negative values in last row, algorithm ends, Z_j is optimal. If all values in base's input variable column are non-positive, it's an unbounded problem with potential for improvement.
6. Select the input base variable using the highest value in Z_j row. Choose the basic variable from columns with equal coefficients (pivot column). Determine the output base variable by dividing independent terms by pivot column values, but only if both are strictly positive. Opt for the row with the smallest result. If any value is non-positive, skip the division. If all pivot column values are non-positive, it suggests an unbounded solution. The pivot row is found using the smallest positive quotient in the pivot column, indicating the slack variable leaving the base. The intersection of the pivot column and row indicates the pivot value.
7. The tableau's new coefficients are computed by either division (for the pivot row) or subtraction (for other rows), effectively normalizing the pivot.
8. In the last row if all the coefficients are ≤ 0; so the stop condition is fulfilled. The solution is optimal as $Z_j - C_j \leq 0$ for all j.

3.5 Fuzzy decisive set method

The fuzzy decisive set method, introduced by Sakawa and Yana (1989), combines the bisection method with the simplex method for obtaining a workable solution. In fuzzy decision-making, Bellman and Zadeh proposed maximizing a decision; later applied to mathematical programming by Tanaka *et al.* Zimmermann introduced a fuzzy approach to multi-objective linear programming. Shaocheng (1994) tackled fuzzy linear programming with fuzzy constraints, defuzzifying it by initially establishing an upper bound for the objective function. This crisp problem, solved using the fuzzy decisive set method, was also introduced by Sakawa and Yana (1989).

Max $Z_1(x) = C_1x$
Max $Z_2(x) = C_2x$
Max $Z_k(x) = C_kx$
subject to $Ax = \tilde{b}, x \geq 0, x\epsilon X$ (14)

Where x is an n-dimensional vector of decision variables. $Z_1(x), Z_2(x), \ldots\ldots, Z_k(x)$ are k-distinct linear objective function of the decision vector $x. C_1, C_2, \ldots\ldots., C_k$ are n-dimensional cost factor vectoes,A is an $m \times n$ constraint fuzzy matrix, $\tilde{b}$ is an m-dimensional constant fuzzy vector.
The membership function of the fuzzy matrix is $A^{\sim}$ is

$$\mu_{A^{\sim}}(x) = \begin{cases} 1, x \leq a_{ij} \\ \dfrac{a_{ij} + d_{ij} - x}{d_{ij}}, \ a_{ij} \leq x \leq a_{ij} + d_{ij} \\ 0, x \geq a_{ij} + d_{ij} \end{cases}$$

Where $x\epsilon R$ and $d_{ij} > 0$ $(tolerance\ level)$ for $i = 1,2, \ldots., m$ and $j = 1,2, \ldots\ldots.., n$
The membership function for the fuzzy resources $\tilde{b}$ is

$$\mu_{\tilde{b}}(x) = \begin{cases} 1, x \leq b_i \\ \dfrac{b_i + p_i - x}{p_i}, \ b_i \leq x \leq b_i + p_i \\ 0, x \geq b_i + p_i \end{cases}$$

Where $x\epsilon R$ and $p_i > 0$ $(tolerance\ level)$ for $i = 1,2, \ldots., m$

The solution methodology for the fuzzy decisive set method is as follows

1. Initially, we apply fuzzification to the objective function to subsequently defuzzify the problem. This involves determining the lower and upper bounds, denoted as Z_q^l and Z_q^u respectively. These bounds are obtained by solving standard linear programming problems. Let $Z_q^l = \min(Z_q^1, Z_q^2, Z_q^3, Z_q^4)$ The objective function takes the values between Z_q^l and Z_q^u while the technological coefficients take values between a_{ij} and a_{ij} $+d_{ij}$ and the right hand side numbers takes the values b_i and $b_i + p_i$.

Then, the fuzzy set of the i^{th} optimal value, $G_i$which subset forR^n is defined by

$$\mu_{G_j}(x) = \begin{cases} 0, \sum_{j=1}^{n} c_j x_j \leq Z_q^l \\ (\sum_{j=1}^{n} c_j x_j - Z_q^l)/(Z_q^u - Z_q^l), Z_q^l \leq \sum_{j=1}^{n} c_j x_j \leq Z_q^u \\ 1, \sum_{j=1}^{n} c_j x_j \geq Z_q^u \end{cases}$$

The fuzzy set of i (th) constraint, C_j which is the subset of R^n is defined by $\mu_{C_j}(x) =$

$$\begin{cases} 0, & b_i \leq \sum_{j=1}^{n} a_{ij} x_j \\ (b_i - \sum_{j=1}^{n} a_{ij} x_j)/(\sum_{j=1}^{n} d_{ij} x_j + p_i), & \sum_{j=1}^{n} a_{ij} x_j \leq b_i \leq \sum_{j=1}^{n} (a_{ij} + d_{ij}) x_j + p_i \\ 1, & \sum_{j=1}^{n} c_j x_j \geq Z_q^u \end{cases}$$

By using the definition of the fuzzy decision proposed by Bellman and Zadeh, we have

$$\mu_D(x) = min_j(\mu_{G_j}(x), min(\mu_{C_j}(x)))$$

In this case the optimal fuzzy decision is a solution of the problem

$$max_{x\geq 0}(\mu_D(x)) = max_{x\geq 0}(min_j(\mu_{G_j}(x), min(\mu_{C_j}(x))))$$

Consequently, the fuzzy linear programming problem is reduced to the following optimization problem

$$max\ \lambda$$
$$\lambda(Z_q^u - Z_q^l) - \sum_{j=1}^{n} c_j x_j + Z_q^u \leq 0$$
$$\sum_{j=1}^{n}(a_{ij} + d_{ij})x_j + \lambda p_i \text{-} b_i \leq 0 \quad (15)$$
$$x_j \geq 0, 0 \leq \lambda \leq 1$$

2. This method treats problem (15) by converting it into a linear programming problem for a fixedλ. The goal is to find the optimal λ^* that maximizes the feasible set.

Step1: Set $\lambda = 1$ and test whether a feasible set satisfying the constraints of the problem (15) exists or not using phase one of the simplex method. If a feasible set exists, set $\lambda = 1$ otherwise, setλ_L= 0and λ_R = 0 and go to the next step.
Step2: For the value of $\lambda = \frac{\lambda_L + \lambda_R}{2}$, update the value of λ_L and λ_R using the bisection method as follows:
$\lambda_L = \lambda$, if feasible set is nonempty for λ
$\lambda_R = \lambda$, if feasible set is empty for λ

Consequently, for each λ, test whether a feasible set of the problem (15) exists or not using phase one of the simplex methods and determine the maximum value of $\lambda*$ satisfying the constraints of the problem (15).

4. Results and Discussion

This review thoroughly examines 20 articles from the past two decades, evaluating their methodology and findings. It reveals effective techniques for tackling multi-objective fuzzy optimization problems, yielding satisfactory results. The paper provides both a concise history of fuzzy optimization and a detailed look at various methods for handling multi-objective scenarios in uncertain environments with fuzzy sets.

5. Conclusion

The paper reviews various techniques in multi-objective fuzzy optimization, particularly focusing on the recent emergence of fuzzy versions of simplex methods for MOFLP. These adaptations are expected to become prominent extensions of classical approaches due to the widespread applications of multi-objective optimization. However, their advantages over established methods in practical decision-making scenarios warrant further exploration. Researchers should not only consider theoretical generalizations but also investigate their real-world applications.

6. References

1. Atanassov KT. Intuitionistic fuzzy sets. Fuzzy Sets and Systems. 1986;20:87-96.
2. Angelov PP. Optimization in an intuitionistic fuzzy environment. Fuzzy Sets and Systems. 1997;86:299-306.
3. Abdulqader OH. A novel technique for solving multi objective fuzzy linear programming problems. ARO- The Scientific Journal of Koya University. 2017;1:1-8.
4. Bellman RE, Zadeh LA. Decision Making in a Fuzzy Environment. Management Sciences. 1970;17:141-164.

5. Beena TB. Solving multi-objective fuzzy linear optimization problem using fuzzy programming technique. IOSR Journal of Mathematics. 2016;05:18-21.
6. Bharati SK, Singh SR. Solving multi-objective linear programming problems using intuitionistic fuzzy environment optimization method: a comparative study. International Journal of Modeling and Optimization. 2014;4(1):10-16.
7. Dubey D, Chandra S, Mehra A. Fuzzy linear programming under interval uncertainty based on IFS representation. Fuzzy Sets and Systems. 2012;188:68-87.
8. Dubois D, Prade H. Operations on Fuzzy Numbers. International Journal of Systems Science. 1978;9(6):613-626.
9. Gasimov RN, Yenilmez K. Solving fuzzy linear programming with linear membership functions. Turk J Math. 2002;26:375-396.
10. Gupta S, Umaid IAM. Multiobjective optimization in agricultural production planning with fuzzy parameters. International Journal of Agricultural and Statistics Sciences. 2018;14(1):1-12.
11. Jana B, Roy TK. Multi objective intuitionistic fuzzy linear programming and its application in transportation model. NIFS. 2007;13:1-18.
12. Rajarajeswari P, Sudha AS. Solving a fully fuzzy linear programming problem by ranking. International Journal of Mathematics Trends and Technology. 2014;9(2):159-164.
13. Sakawa M, Yano H. Interactive decision making for multiobjective nonlinear programming problems with fuzzy parameters. Fuzzy Sets and Systems. 1989;29:315-326.
14. Sakawa M. Fuzzy Sets and Interactive Multiobjective Optimization. Plenum Press, New York; c1993. http://dx.doi.org/10.1007/978-1-4899-1633-4.
15. Sarkar B, Samanta S. Generalized fuzzy trees. International Journal of Computational Intelligence Systems. 2017;10(1):711-720.
16. Shaocheng T. Interval number and fuzzy number linear programming. Fuzzy Sets and Systems. 1994;66:301–306.
17. Sharma DK, Jana RK, Gaur A. Fuzzy goal programming for agricultural land allocation problems. Yugoslav Journal of Operations Research 2007;17(1): 31-42.
18. Shirin S, Kamrunnahar. Application of fuzzy optimization problem in fuzzy environment. Dhaka Univ. J Sci. 2014;62:119-125.
19. Tanaka H, Okuda T, Asai K. On Fuzzy-Mathematical Programming. Journal of Cybernetics. 1974;3:37-46.
20. Veeramani C, Duraisamy C, Nagoorgani A. Solving fuzzy multi-objective linear programming problem with linear membership functions. Australian Journal of Basic and Applied Sciences 2011;5:1163-1171.
21. Yager R. A procedure for ordering fuzzy subsets of the unit interval. Information Sciences1981;24:143-161.
22. Zadeh LA. Fuzzy sets. Information and Control. 1965;8:338-353.
23. Zimmerman, HJ. Fuzzy programming and linear programming with several objective functions. Int. J Fuzzy Sets Syst. 1978;1:45-56.