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Optimize the Taguchi method, the signal-to-noise ratio, and the sensitivity

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Abstract
Statistical quality control methods are a collection of strategies and scientific techniques to study many different complex production problems, which is an essential aspect of ensuring. The product is of a good enough quality to meet expectations. Quality control is a broad field with many subfields to research. One of the key parameters used in quality control is the signal-to-noise ratio (S/N). Which measures a product's quality by comparing the desired signal with the noise or unwanted variation output. The Taguchi method is an important statistical technique in quality control. Taguchi made contributions to the advancement and creation of sophisticated techniques in quality control models. The S/N ratio is an integral part of the Taguchi. That method is widely used in various manufacturing fields. The sensitivity analysis method is another important aspect of quality control that involves analyzing the impact of variations in the input factors on the output quality. Using the S/N ratio and sensitivity analysis, the Taguchi method can effectively improve product quality, reduce production costs, and minimize output variation. This makes the Taguchi method a good tool for quality control in various industries.

Keywords: Response optimization, orthogonal array, S/N ratio taguchi method

Introduction

Statistical quality control methods are a collection of strategies and scientific techniques to study many different problems and have a large area of study. Statistical quality control came from Dr. Walter Shewhart in 1924 [6].

A crucial component of making sure a good or service fulfills the required standard of quality is quality control. It entails keeping an eye on and testing the good or service to find and fix any flaws or problems. Quality control is essential in ensuring customer satisfaction, reducing costs, and improving overall efficiency.

Quality control using the S/N ratio is an integral part of the Taguchi method, a systematic approach to G.Taguchi developed the new method to optimize the design of manufacturing processes product.

One of the key parameters used in quality control is the signal-to-noise (S/N) ratio, to evaluate the quality of a product or service by comparing the desired signal with the noise or unwanted variation in the output. The Taguchi method is a statistical technique that uses the S/N ratio to optimize product design and manufacturing processes.

Modern industry aims to reduce costs while putting more of an emphasis on the steps an organization takes to ensure it is producing a high-quality good or service. Measurable characteristics and specifications must be created in order to compare the actual qualities of the good or service with them in order to apply quality control effectively.

Since 1960, we have been using Taguchi methods to improve the quality starting with the specification and product planning [3, 4].

The signal-to-noise ratio (SNR) and the noise-to-signal ratio (NSR), are two main types of S/N ratio.

SNR measures of ratio of the power of the signal to the power of noise. Both SNR and NSR are significant metrics in many other domains and are used in quality control to assess a product's quality.

There are three different kinds of noise factors: between-product noise, inner noise, and external noise. The validity of several of Taguchi's techniques is debatable theoretically. One of them is his utilization of exterior arrays, which might be a component of the standard experimental layout. However, issues with experimental design are not our main concern. His advice on using signal-to-noise (S/N) ratios to create robust designs is more pertinent.
Good mean settings must be used in a robust design for the product to be resilient to variation. In other words, we are interested in both means and variances. However, according to Taguchi, data can be transformed using S/N ratios in such a way that both mean and variance can be optimized simultaneously [3, 5, 11].
The signal-to-noise (SN) ratio is then analyzed to identify the preferable parameter values, with the best factor levels that maximize the right SN ratio depending on the intended performance response, there 12
The signal-to-noise ratio serves as a gauge of a measuring system's quality. The quality increases with an increasing SN ratio.
To evaluate measuring methods as well as the functionality of goods and processes, Taguchi expanded the SN ratio idea as employed in the communication sector [5].
Different types of standard SN ratios kinds of noise factors
Smaller is better: In this case,the magnitude that we want to minimize in this instance is the squared response. Alternatively, the following transformation can be minimized to achieve this:

$$\frac{S}{N} = -10\mathrm{Log}_{10}\left[\frac{1}{n}\sum_{i=1}^{n} x_i^2\right] \tag{1}$$

Nominal is best: Here, Taguchi looks at the statistic

$$\frac{S}{N} = 10\mathrm{Log}_{10}\,\frac{\text{mean squared response}}{\text{variance}} = 10\mathrm{Log}_{10}\left[\frac{\overline{x^2}}{s^2}\right] \tag{2}$$

Larger is better: In this case we may select l/x, as the response that we should minimize, leading to

$$\frac{S}{N} = -10\mathrm{Log}_{10}\left[\frac{1}{n}\sum_{i=1}^{n}\frac{1}{x_i^2}\right] \tag{3}$$

To get the desired mean value, the main strategy is to choose settings that have a high signal-to-noise ratio, or a relatively strong signal, and then use other factors to adjust the mean.
The United States has been using a new quality engineering method known as the Taguchi method since the 1980s. This method is distinct from the quality engineering detailed method.
Taguchi's method is a robust design based on the following three procedures:

1. Orthogonal array.
2. SN ratio,
3. Loss function.

These three procedures are not robust design purses because they are used to evaluate technical means or products [6, 7].
Prior to designing robustness, we must comprehend what robustness means. When the true values of the signal factor levels are unknown, the variation of samples is used.
After calculating the signal factor variation (SM) and subtracting it from the total variation (ST), the residual is regarded as the error term. As in the next two examples, the overall variation varies based on what is anticipated from the linear relationship.

Proportional equation ST =SM+ Se

Linear equation: ST =Sm+ SM + Se

These two cases a basic concept: Calculation of the effect, S_M, of the signal factor, M, and the general mean, S_m, with different degrees of freedom.
cannot calculate the total variation when the true value is known.
The total effect (S_T) and (S_M), is obtained as follows:

$$S_T = y_{11}^2 + y_{12}^2 + y_{13}^2 + \cdots + y_{kr0}^2 \ (f = k_{r0}) \tag{4}$$

$$S_M = \frac{y_1^2 + + y_2^2 + + y_3^2 + \cdots + y_k^2}{r_0} \ (f = \ k\ ) \tag{5}$$

Where k is the degrees of freedom of the signal variation (S_M).
The error variation as equation:

$$S_e = S_T - S_M$$

Where the degrees of Freedom of the error is:

$$f_e = k_{ro} - k$$

The error variance is becomes

$$V_e = \frac{1}{k_{ro} - k} S \quad (6)$$

The variance of signal factor is obtained as:

$$V_M = \frac{1}{k} S_M \quad (7)$$

The S/N ratio is determined by:

$$\eta = 10\, log \frac{(\frac{1}{r_0})(V_M - V_e)}{V_e} \quad (8)$$

Because all production processes can produce different outputs for different products, the dynamic SN ratio is an important application to improve the robustness of the process product function to develop a group of products.

No Signal Factor (Non-dynamic SN Ratio) [2, 3, 4, 8]

The non-dynamic S/N ratio is a type of signal-to-noise ratio that considers the mean and standard deviation of a response variable. Usually, there are upper and lower limits to specifications. The type I case is one in which all of the data are positive. Data of type II is mix up of both positive and negative values.

The first stage in two-step optimization for a nominal-the-best application is to minimize variability around the average by maximizing the SN ratio. The average must be adjusted to the target in the second step.

When the nominal-optimal S/N ratio is explained as follows

$$\text{SN} = \frac{\text{desired output}}{\text{undesired output}}$$

$$\text{SN} = \frac{\text{deffect of the average}}{\text{variability around the average}}$$

In applications where nominal-the-best is used, the data is positive. This is particularly true if the output response is energy-related, in which case there shouldn't be any negative data because zero input should result in zero output from the system. Here are the calculations for a nominal-the-best SN ratio.as follows.

$$y_1, y_2, \ldots, y_n \ (f = n)$$

The Average variation, SN is

$$S_n = \frac{(y_1 + y_2 + y_3 + \cdots + y_n)^2}{n} \ (f = n) \quad (9)$$

The error variation, Se is

$$S_e = S_T - S_m \ (f = n - 1)$$

The error variance, V_e, is the error variation divided by its degrees of freedom:

$$V_e = \frac{S_e}{n-1} \quad (10)$$

The S/N ratio, denoted by Ƞ is given by:

$$Ƞ = 10\, log \frac{\left(\frac{1}{n}\right)(S_m - V_e)}{V_e} \quad (11)$$

The Sensitivity is given by:

$$S = 10 \log \frac{1}{n} (S_m - V_e) \quad (12)$$

When negative values are included in the experiment results, the positive and negative values cancel each other out, rendering the Sm calculation useless. As a result, it is impossible to gather data on average or sensitivity. In this instance, the S/N ratio computation reveals only variability. Compared to type I, it is less informative. From a conceptual standpo, the SN ratio gives the following information:

$$SN = \frac{1}{Variability\ around\ the\ average}$$

The S/N ratio is calculated as follows where there are n pieces of data: y_1, y_2,., y_n.
The error variation is:

$$S_e = y_1^2 + y_2^2 + y_3^2 + \cdots + y_n^2 - \frac{(y_1+y_2+y_3 \ldots\ldots + y_n)^2}{n} \ (f = m - 1) \tag{13}$$

The error variance is:

$$V_e = \frac{S_e}{n-1} \tag{14}$$

And the S/N ratio is given by:

$$\eta = 10 \log \frac{1}{V_e} = -10 \log V_e \tag{15}$$

Two Types of S/N ratio

1-Smaller-the-Better,

When minimizing the output value is the goal and there are no negative data, the smaller-the-better type is employed. The target value in this scenario is zero.
A smaller-the-better SN ratio is calculated as:

$$\eta = -10\ log \frac{1}{n} \left(y_1^2 + y_2^2 + \cdots . + y_n^2\right). \tag{16}$$

2. Larger-the-Better

When maximizing output is the goal, the larger-the-better approach is adopted as long as there are no negative data points. The target in this scenario is infinity.
It is advised to combine this kind of SN ratio with the smaller-is-better kind when creating an SN ratio for a non-dynamic operating window [9, 10, 12].
The larger-the-better SN ratio is given by:

$$\eta = -10\ log \frac{1}{n} \left(\frac{1}{y_1^2} + \frac{1}{y_2^2} + \cdots . + \frac{1}{y_n^2}\right) \tag{17}$$

The Gain and Optimum of S/N Ratios [1, 5]

The Gain is the optimum of S/N ratio calculate as this equation: The equation that follows to define the gain as well:

Gain = (Average S/N Ratio) Level 1 – (Average S/N Ratio) Level 2

Variability Re duction=1-$(0.5)^{Gain/6}$ …17

In order to predict the optimum condition, the expected mean at the optimal settings (Y) is calculated by using the following model.

$$Y_{Actual}\ S/N = T - (A1-T) + (B1-T) + (C1-T) + \tag{18}$$

k = Number of variables
R = The value of S/N ratio of level
T = Total of SN

Role of orthogonal arrays

The primary function of orthogonal as in robust engineering is to enable engineers to assess a product design in terms of cost and robustness against noise. An inspection tool called the OA is used to stop "poor designs" from moving "downstream." These arrays are typically represented as Ln (Bx) arrays, with factors at multiple levels; the most frequent factors are two and three t many levels, although two and three factors are the most commonly encountered.
Where:
L = denotes Latin square design
n = Number of experiments
B = Number of levels
x = Number of columns

By reducing the number of experiments, the robust design's use of orthogonal arrays aims to estimate the effects of various factors and necessary interactions. and to determine variables that are useful while using the fewest possible variable combinations. The variables are allocated to the array's various array [1, 5].

Numerical Application

This paper used the current data of Lime Saturation Factors (L.S.F) of cement. The L.S.F value depends on five chemical elements such as (SiO_2, Al_2O_3, Fe_2O_3,CaO, and SO_3), where the value of L.S.F is an equation:

shown L.S. F= (CaO_2-0.8*SO_3)/(2.5*SiO_3+2.5*Al_2O_3+0.5*Fe_2O_3).

The data are collected from chemical laborers with sample sizes (7*16) as shown in table (1).

To calculate the S/N ratio for the data using the Taguchi Orthogonal Array Design of 16 runs with 5 variables with two levels using L16(25). The corresponding S/N ratios, mean and the sensitivity of the SN ratio is calculated by equation (12) as shown in the table (1).

Table 1: LSF. Data, SN, Average and Sensitive

X1	X2	X3	X4	X5	X6	X7	SN	Av	Sensitive
0.952	0.951	0.946	0.941	0.978	0.939	0.948	37.301	0.951	1.99
0.949	0.958	0.942	0.941	0.937	0.945	0.954	42.153	0.947	1.95
0.937	0.961	0.939	0.948	0.945	0.937	0.949	40.73	0.945	1.941
0.932	0.952	0.948	0.941	0.931	0.940	0.944	41.803	0.941	1.903
0.940	0.948	0.952	0.938	0.936	0.957	0.943	41.785	0.945	1.938
0.967	0.952	0.943	0.935	0.933	0.948	0.946	38.364	0.946	1.948
0.955	0.943	0.946	0.941	0.928	0.958	0.961	38.274	0.948	1.962
0.975	0.946	0.942	0.936	0.930	0.964	0.939	35.335	0.948	1.962
0.970	0.942	0.939	0.941	0.933	0.957	0.944	37.439	0.947	1.954
0.970	0.939	0.947	0.941	0.927	0.952	0.948	37.23	0.946	1.950
0.955	0.947	0.948	0.937	0.925	0.952	0.946	39.366	0.944	1.932
0.968	0.948	0.945	0.934	0.928	0.953	0.937	36.946	0.945	1.936
0.969	0.945	0.936	0.933	0.936	0.955	0.932	36.652	0.944	1.927
0.971	0.936	0.947	0.931	0.925	0.955	0.940	35.618	0.944	1.927
0.968	0.947	0.952	0.933	0.929	0.953	0.955	36.91	0.948	1.967
0.974	0.952	0.957	0.941	0.932	0.958	0.951	37.033	0.952	2.006

Table (2) shows the response of S/N ratios, response of average and response of mean of data with two levels and calculating the delta and rank for two levels. And the first rank of SN is X1, for average is X5 and for mean is X2.

Table 2: Response Table for SN, Average and Mean

Response Table for SN						Response Table for Average					Response Table for Means				
Level	A	B	C	D	E	A	B	C	D	E	A	B	C	D	E
1	39.47	39.12	38.32	38.56	38.25	1.95	1.945	1.949	1.952	1.952	0.946	0.946	0.9460	0.9464	0.947
2	37.15	37.5	38.3	38.06	38.36	1.95	1.955	1.951	1.948	1.947	0.946	0.947	0.9463	0.9460	0.946
Delta	2.32	1.62	0.02	0.5	0.11	0.00	0.010	0.003	0.003	0.005	0	0.001	0.0003	0.0004	0.001
Rank	1	2	5	3	4	5	4	3	2	1	5	1	4	3	2

Discussion

1. According to Fig.(1)'s SN ratio, from level 1 the value (X1, X2, and, X4) has a higher SN ratio, which means that it has a relatively strong impact on variability, these variables are significant and useful.
   from Level 2- the value of (X5)has a higher SN ratio. It has a relatively weak impact on variability. Variables (X3) are harming discrimination, and are not significant.
2. In Fig (2)'s mean SN ratio, for the Level, the higher value of SN ratio is (X4, X5) indicating a relatively strong influence on variability. These are important and practical variables. the variable (X2, and X3) of Level 2 has a higher SN ratio. It barely affects variability at all. Variables (X1) are not significant but hurt discrimination.
3. Sensitive Fig (3), shows that the higher SN ratio of level 1 is (X4, X5), which it is means that it has a relatively strong impact on variability. These variables are important significant and useful.

The SN ratio is higher in Level 2 (X2, and X3). It has a relatively weak impact on variability. Variables (X1) are harming discrimination, and are not significant. From response table (2) of the S/N ratio used to determin the optimum S/N ratio, as Y_{Opt} S/N, denotes the minimum value from Level 1,2 for the (5) variables.

Where T is the total value.

The original equation of Y_{Opt}, Y_{Actual}, and variability reduction, as shown in table(3)

where the Y_{Actual} SN denotes the maximum value. The gain between current and optimal

Gain = (Average S/N Ratio) Level 1 – (Average S/N Ratio) Level 2

where the variability reduction=1-$(0.5)^{Gain/6}$

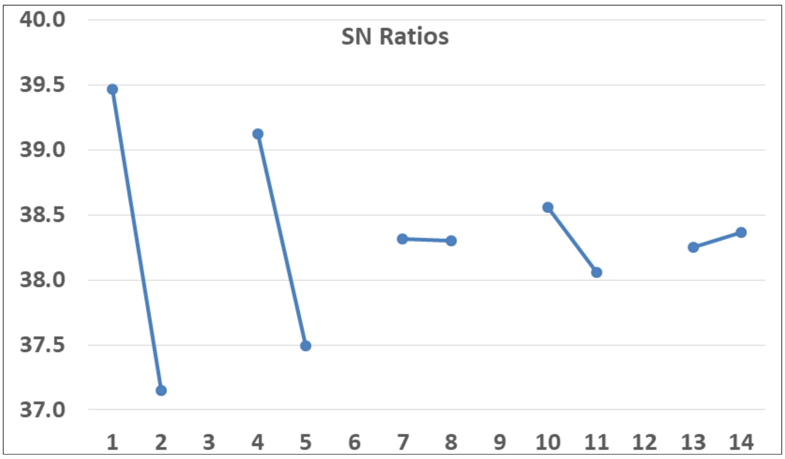

Fig 1: Main Effects for SN ratio

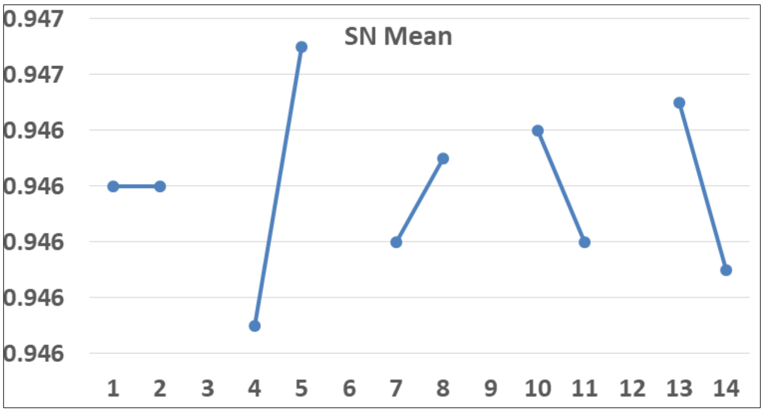

Fig 2: Main Effects for Mean

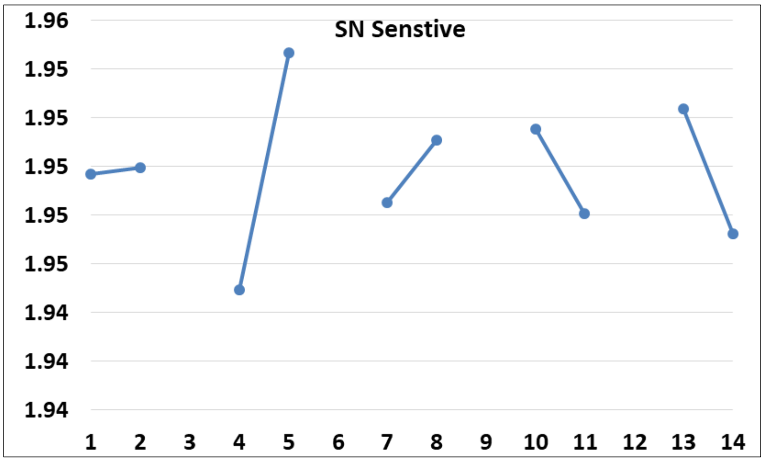

Fig 3: Main Effects for Sensitive

From Table (3) it is seen that the gain of the optimum mean for (5) variables is (0.009) and the gain between the current and optimal condition is translated into a 0.101% reduction in energy transformation variability.
Y_{Actual} SN denotes the maximum value of the original 5 variables in Level1 and Level 2
Total of SN = 38.309
The optimized Y_{Opt} S/N and Y_{Actual} SN and gain are shown in table (3).

Table 3: Denoted the original equation of optimum, actual, gain and Variability Reduction of responsible level

Average	$Y_{Optim.}$ Min	Y_{Actual}	The Gain	~~%~~-Variability Reduction
38.31	36.034	36.025	0.009	0.101

Conclusion

The results of this paper consider Taguchi's quality for calculating the SN ratio to the implication of a finite target ratio, to identify the useful set of origin variables simply. To show the higher level of SN ratio for (X1, X2, X3, X4, X5), which means that it has a relatively strong impact on variability.

To separate between which variables are significant and useful, and not useful. Determine the Y_{Opt}, Y_{Actual}, variability reduction, and the gain between current and optimal. The value of optimized Y_{Opt} S/N and Y_{Actual} SN for the original 5 variables in Level 1 and Level 2 are (36.034,236.029), and the value of gain is (0.009) with Variability Reduction is 0.101%.

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