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## MHD convective of TiO<sub>2</sub> nanofluid over a cylindrical permeable plate: Heat and mass transfer coefficients

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### Abstract

In this work, magnetohydrodynamics convective of TiO<sub>2</sub> nanofluid over a cylindrical permeable plate: heat and mass transfer coefficients was analyzed. The Frobenius Method was used to solve the governing equations modeled in partial differential equations (PDE) which was non-dimensionalized, and converted to ordinary differential equations (ODE) using the regular perturbation approach with boundary conditions, each solution of these equations separately, yielding analytic solutions. The analytic approximate solutions for temperature, concentration, and velocity were perturbed to examine the Nusselt parameter, the Sherwood number, and the Skin friction which were numerically computed in tabular form. The Nusselt parameter and the skin friction declined as the radiation parameter was raised from 0.5 to 2.5 and an increment of the chemical reaction term from 2.35 to 5.35 diminished the skin friction and Sherwood number. The skin friction increased from  $2.88093 \times 10^{-1}$  to  $1.55554 \times 10^2$  as the magnetic field was increasing from 1 to 5; the porosity parameter when increased from 0.3 to 1.5, lowered the skin friction, also the Nusselt parameter and the Sherwood number declined as the nanoparticle volume fraction was raised from 0.1 to 0.5.

**Keywords:** Heat transfer, sherwood number, nanofluid, convection, cylindrical porous plate

### Introduction

Heat transfer is the process of moving heat from one location to another. Energy transfer is a term used to describe how heat produced by convection can be effective in a variety of physical circumstances. Heat is transmitted through convection. Heat transfer between a flowing fluid and a surface when their temperatures are different is known as convection. Ethylene glycol, water, and mineral oil are common convective heat transfer fluids used in a variety of industries, including transportation, power generation, air conditioning, chemical manufacture, and microelectronics (Yimin & Li, 2000) <sup>[1]</sup>. Free convection, mixed convection, and forced convection are the three types of convection that occur. When a fluid is mechanically pumped through a thermal system, (caused by external pressure gradient or object motion) it is called forced convection while free convection is induced by buoyancy. A combination of the occurrence of forced and free convection to heat transfer is mixed convection and readily realized in the flow of the fluid in a channel (Sebdani *et al.*, 2012; Sheikhzabeh, 2012; Al-Salem *et al.*, 2012; Nadeem & Saleem, 2004) <sup>[2, 3, 4, 5]</sup>. Thermal radiation is a basic property of matter that depends on its temperature and is one of the three basic forms of heat transfer (Abramzon and Sazhin, 2006) <sup>[7]</sup>. Makinde (2005) <sup>[6]</sup> investigated mass transfer through a permeable vertical plate in conjunction with thermal radiation and the free convection boundary layer. Bakier (2001) <sup>[8]</sup> considered a research work on thermal radiation effect on mixed convection from vertical surfaces in saturated porous media. Chamkha *et al.* (2003) <sup>[9]</sup> examined “the impact of thermal radiation on MHD forced convection flow next to a non-isothermal wedge”. The focus of the investigation was on the forced convective boundary-layer flow of an incompressible Newtonian fluid over a non-isothermal wedge that is in a steady-state, absorbing or heat-generating, and electrically conducting. This flow was caused by hydromagnetic forces.

Similarity analysis of thermal radiation and magnetic field effects on forced convection flow, magneto-hydrodynamic forced convection heat transfer from radiate, was examined by Damseh *et al.* (2006) [10]. Narahari and Nayan (2011) [11] studied an analytical view of free convection flow near an impulsively started infinite vertical plate with constant mass diffusion, and Newtonian heating in the presence of thermal radiation was performed. Xaman *et al.* (2008) [12] conducted research on the topic of "laminar and turbulent natural convection combined with surface thermal radiation in a square cavity with a glass wall". The nanofluid suspensions' significantly improved mass and heat transfer properties which make them ideal media (Muhammad *et al.*, 2018) [13]. Ibrahim and Makinde (2011) [30] examined the "radiation effect on chemically reacting magnetohydrodynamics (MHD) boundary layer flow of heat and mass transfer through a porous vertical flat plate". Mathematically, a model was presented for a two-dimensional, electrically conducting, viscous, incompressible, and laminar free convection boundary layer flow. Using thermal radiation, chemical reaction, and viscous dissipation, effects in mind, Haile and Shankar (2014) [15] examined research on heat and mass transfer through a permeable media of MHD flow of Nanofluids. In a variable porosity regime, Prasad *et al.* (2012) [16] examined the effects of thermal radiation on magnetohydrodynamic free convection heat and mass transfer from a sphere. Mabood *et al.* (2016) [17] carried out an "investigation on MHD stagnation flow of water-based nanofluids in which the mass and heat transfer" includes the effects of radiation volume fraction of nanoparticles, viscous dissipation, and chemical reaction. Zueco and Ahmed (2010) [18] investigated the combined mass and heat transfer by mixed convection MHD flow along with a permeable plate with chemical reaction in presence of heat source. Studies such as Xuan and Roetzel (2000) [19], Wang *et al.* (2006) [20], Roy *et al.* (2004) [21] examined the convective heat transfer properties of nanofluids, and found that as thermal Grashof number and nanoparticle volume fraction increased, so did the average Nusselt number. Compared to the heat conductivity ratio of nanofluids, heat transfer performance is a more accurate indicator. In order to improve heat transmission and the performance of energy devices, researchers are paying more attention to the study of heat and mass transfer coefficients (Bergman *et al.*, 2011) [22]. The radiation and magnetic field in a mixture of tin-oxide nanoparticles in water to generate tin-oxide nanofluid are thoroughly examined in this study in relation to the effects of heat and mass transfer coefficients. The results shall be compared with the works of Ngiangia and Orukari (2021) [23], Wang *et al.* (2006) [20] and karthikeyan *et al.* (2013) [24].

## Materials and Methods

### Materials

We considered an oscillatory, unsteady, radiating, incompressible tin-oxide nanofluid flow where  $v$  represents the velocity of the nanofluid in the presence of thermal and concentration buoyancy effects (Lawson *et al.*, 2023) [25].

$$\frac{\partial \rho_{nf}}{\partial t^t} + \frac{1}{r^c} \frac{\partial}{\partial r^c} (\rho_{nf} r^c v_r) = 0 \quad (1)$$

$$\rho_{nf} \left( \frac{\partial v_r}{\partial t^t} + v_r \frac{\partial v_r}{\partial r^c} \right) = \mu_{nf} \left[ \frac{\partial}{\partial r^c} \left( \frac{1}{r^c} \frac{\partial}{\partial r^c} (r^c v_r) \right) \right] + g \rho_{nf} \beta'_{nf} (C^c - C_\infty) + g \rho_{nf} \beta_{nf} (T - T_\infty) - \frac{\mu_{nf}}{k^t} v_r - \sigma \beta_0^2 v_r = 0 \quad (2)$$

$$(\rho C_p)_{nf} \left( \frac{\partial T}{\partial t^t} + v_r \frac{\partial T}{\partial r^c} \right) = K_{nf} \left[ \frac{1}{r^c} \frac{\partial}{\partial r^c} r^c \left( \frac{\partial T}{\partial r^c} \right) \right] - \frac{\partial q}{\partial r} \quad (3)$$

$$(\rho C_p)_{nf} \left( \frac{\partial C^c}{\partial t^t} + v_r \frac{\partial C^c}{\partial r^c} \right) = D_{nf} \left[ \frac{1}{r^c} \frac{\partial}{\partial r^c} r^c \left( \frac{\partial C^c}{\partial r^c} \right) \right] - K_r^2 C^c \quad (4)$$

With the boundary conditions,

$$T(0,t) = 0, \quad T(1,t) = 1, \quad v_r(0,t) = 0, \quad v_r(1,t) = 1, \quad C^c(0,t) = 0, \quad C^c(1,t) = 1$$

(Lawson *et al.*, 2023; ) [25].

where  $T(r,t)$  is the nanofluid temperature,  $C^c(r,t)$  is the nanofluid concentration,  $v_r(r,t)$  denotes the nanofluid velocity,  $r^c$  is the nanoparticles radius,  $\mu_{nf}$  is the dynamic viscosity coefficient of the nanofluid,  $\rho_{nf}$  is the density of nanofluids,  $t$  is the time,  $\sigma$  is the conductivity of the base fluid,  $g$  is the gravity due acceleration,  $k_r^2$  is the chemical reaction term,  $B_0^2$  is the magnetic field,  $k^t$  is the porosity parameter or permeability,  $\rho_{nf} \beta_{nf}$  is the temperature of thermal expansion coefficient of the nanofluids,  $(\rho C_p)_{nf}$  is the heat capacitance of the nanofluids,  $D_{nf}$  is the Chemical molecular diffusivity of the nanofluid,

$\rho_{nf} \beta_{nf}$  is the Concentration nanofluids of the thermal expansion coefficient,  $K_{nf}$  is the thermal conductivity of the nanofluid,  $q$  is the radiative heat flux.

The models for dynamic viscosity and thermal conductivity developed by Hamilton and Crosser (1962) [27] are applied in this study since they may be applied to nanoparticles with spherical or non-spherical shapes.

$$\mu_{nf} = \mu_f (1 + a\phi + b\phi^2) \tag{5}$$

$$\frac{k_{nf}}{k_f} = \frac{k_s + (n-1)k_f + (n-1)(k_s - k_f)\phi}{k_s + (n-1)k_f - (k_s - k_f)\phi} \tag{6}$$

According to Asma *et al.* (2015) [29], and Loganathan *et al.* (2013) [28]

$$\left. \begin{aligned} \rho_{nf} &= (1 - \phi)\rho_f + \phi\rho_s \\ (\rho\beta)_{nf} &= (1 - \phi)(\rho\beta)_f + \phi(\rho\beta)_s \tag{7} \\ (\rho C_p)_{nf} &= (1 - \phi)(\rho C_p)_f + \phi(\rho C_p)_s \end{aligned} \right\}$$

where  $\rho_f$  is the densities of the base fluids and  $\rho_s$  is the densities of the solids particles,  $\phi$  is the nanoparticles volume fraction,  $\beta_f$  is the thermal expansion due to temperature of base fluids and  $\beta_s$  is the thermal expansion due to temperature of solids nanoparticles,  $K_f$  and  $K_s$  are the thermal conductivity of the fluid and solids nanoparticles,  $\mu_f$  is the dynamic viscosity coefficient of the fluid,  $(c_p)_f$  and  $(c_p)_s$  are the specific heat capacities of base nanoparticles and solid fluids at constant pressure,  $a$  and  $b$  are constants and depend on the particle shapes as given in Table 1.

The empirical shape factor,  $n = 3/\Psi$ , which appears in Equation (6), is the sphericity, which is defined as the ratio of the sphere's surface area to the surface area of the actual particle with equal volumes. Table 2 lists the values of  $\Psi$  for the various particle shapes.

In addition to the previously mentioned, Table 3 provides some physical characteristics of the nanoparticles and base fluid. Makinde and Mhone (2005) [6] assumed that the plate temperatures  $T_0, T_w$  were highly sufficient to cause radiative heat transfer. According to Cogley *et al.* (1968) [31] state that the radiative heat flux for an optically thin medium with a relatively low density is given by

$$\frac{\partial q}{\partial y} = 4\delta^2(T - T_0) \tag{8}$$

where  $\delta$  is the coefficient of the radiation absorption.

**Table 1:** Constant  $a$  and  $b$  empirical shape factors, (Timofeeva *et al.*, 2009).

Model	$a$	$b$
Platelet	37.1	612.6
Blade	14.6	123.3
Cylinder	13.5	904.4
Brick	1.9	471.4

**Table 2:** Sphericity  $\Psi$  for the different nanoparticles shapes, (Aaiza *et al.*, 2015 and Timofeeva *et al.*, 2009) [33, 32].

Model	$\Psi$
Platelet	0.52
Blade	0.36
Cylinder	0.62
Brick	0.81

Using Buckingham's  $\pi$ - theorem we have the following dimensionless variables

$$r = \frac{r^c}{d} \quad v = \frac{v_r}{v_0} \quad t = \frac{t'v_0}{d} \quad \frac{1}{k} = \frac{d^2 \rho_f}{k' \rho_{nf}} \quad \theta = \frac{T - T_\infty}{T_w - T_\infty} \quad C = \frac{C^c - C_\infty}{C_w - C_\infty} \quad k_\infty = \frac{k_r T_\infty}{v_0^2} \quad N = \frac{4\sigma^2 d^2}{k_f} \quad M = \frac{\sigma B_0^2 \mu_{nf} \rho_f}{\mu_f \rho_{nf} v_r}$$

$$Re^{-1} = \frac{\mu_{nf} \rho_f}{\mu_f \rho_{nf}} \quad Sc^{-1} = \frac{D}{\nu} \quad Pr = \frac{\nu \rho C_p}{(\rho C_p)_{nf} k_f} \quad Gr_{\theta} = \frac{g \beta_{nf} (T - T_{\infty}) \mu_{nf}}{U_0 \nu_r^2 \beta_f} \quad Gr_C = \frac{g \beta'_{nf} (C^c - C_{\infty}) \mu_{nf}}{U_0 \nu_r^2 \beta'_f}$$

Applying dimensionless variables, we have

$$\frac{\partial}{\partial t} \left( \frac{\rho_{nf}}{\rho_f} \right) + \frac{1}{r} \frac{\partial}{\partial r} \left( \frac{\rho_{nf} r \nu}{\rho_f} \right) = 0 \tag{9}$$

$$\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial r} = L_1 Re^{-1} \left( \frac{\partial^2 v}{\partial r^2} + \frac{1}{r} \frac{\partial v}{\partial r} - \frac{v}{r^2} \right) + Gr_{\theta} \theta L_2 + Gr_C C L_3 - \left( \frac{L_6}{k} + M \right) v = 0 \tag{10}$$

$$\frac{\partial \theta}{\partial t} + v \frac{\partial \theta}{\partial r} = L_5 Pr^{-1} \left( \frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} \right) - N \theta L_4 \tag{11}$$

$$\frac{\partial C}{\partial t} + v \frac{\partial C}{\partial r} = L_4 Sc^{-1} \left( \frac{\partial^2 C}{\partial r^2} + \frac{1}{r} \frac{\partial C}{\partial r} \right) - L_4 k_{\infty} C \tag{12}$$

Subject to the boundary conditions

$$v(0, t) = 0, v(1, t) = e^{\omega t} \quad t > 0 \tag{13a}$$

$$\theta(0, t) = 0, \theta(1, t) = e^{\omega t} \quad t > 0 \tag{13b}$$

$$C(0, t) = 0, C(1, t) = e^{\omega t} \quad t > 0 \tag{13c}$$

where  $d$  is the diameter,  $r$  is the dimensionless coordinate,  $N$  is the dimensionless radiation parameter,  $k$  is the dimensionless porosity parameter,  $k_{\infty}$  is the dimensionless chemical reaction term,  $M$  is the Hartmann number and also called magnetic parameter,  $Sc$  is the Schmidt number,  $Re$  is called the Reynold number,  $Pr$  is the Prandtl number,  $Gr_C$  is the modified Grashof number,  $Gr_{\tau}$  is the thermal Grashof number.

Furthermore, we define

$$L_1 = \frac{1 + a\phi + b\phi^2}{1 - \phi - \phi \frac{\rho_s}{\rho_f}} \quad L_3 = 1 - \phi - \phi \frac{\beta'_s}{\beta'_f} \quad L_5 = \frac{k_s + (n-1)k_f + (n-1)(k_s - k_f)\phi}{k_s + (n-1)k_f - (k_s - k_f)\phi} \quad L_4 L_2 = 1 - \phi - \phi \frac{\beta_s}{\beta_f} \quad L_4 =$$

$$\left[ 1 - \phi - \phi \left( \frac{\rho C_p}{\rho C_p} \right)_f \right]^{-1} \quad L_6 = 1 + a\phi + b\phi^2 \quad J_1 = Re^{-1} L_1 \quad J_3 = Gr_C L_3 \quad w_1 = Pr^{-1} L_5 \quad E_1 = Sc^{-1} L_4 \quad J_2 = Gr_{\theta} L_2$$

$$J_4 = \frac{L_6}{k} + M \quad w_2 = L_4 N \quad E_2 = L_4 k_{\infty}$$

**Table 3:** Thermophysical properties of tin-oxide nanoparticles and water (Asma *et al.*, 2015 and Loganathan *et al.*, 2013) <sup>[29, 28]</sup>.

Model	H <sub>2</sub> O	TiO <sub>2</sub>
$\rho(kgm^{-3})$	997.1	4250
$C\rho(kg^{-1}k^{-1})$	4179	686.2
$k(Wm^{-1}k^{-1})$	0.613	8.9528
$\beta \times 10^{-5}(k^{-1})$	21	0.9

**Method of Solution:** For engineering and scientific applications, it is supposed that the nanofluid under consideration is incompressible, equation (9) can be solved to produce.

$$\frac{1}{r} \frac{\partial}{\partial t} \left( \frac{r \rho_{nf} v}{\rho_f} \right) = 0v = \frac{Q}{r} \quad (14)$$

where  $Q$  is an integration constant. Using the transformation technique used by Lawson *et al.* (2023) <sup>[25]</sup>

$$v(r, t) = v_0(r) + v_1(r)e^{-\omega t} \quad (15a)$$

$$\theta(r, t) = \theta_0(r) + \theta_1(r)e^{-\omega t} \quad (15b)$$

$$C(r, t) = C_0(r) + C_1(r)e^{-\omega t} \quad (15c)$$

is selected

where  $\omega$  is the free stream frequency. Equation (14) is inserted into equations (10) – (12) and the regular perturbation technique is applied by replacing equations (15a)–(15c) in the simplified equations. This results in:

$$J_1 \frac{d^2 v_0}{dr^2} + \left( \frac{J_1 - Q}{r} \right) \frac{dv_0}{dr} - \left( \frac{J_1}{r^2} + J_4 \right) v_0 + J_3 C_0 + J_2 \theta_0 = 0 \quad (16a)$$

$$J_1 \frac{d^2 v_1}{dr^2} + \left( \frac{J_1 - Q}{r} \right) \frac{dv_1}{dr} - \left( \frac{J_1}{r^2} + J_4 - \omega \right) v_1 + J_3 C_1 + J_2 \theta_1 = 0 \quad (16b)$$

$$w_1 \frac{d^2 \theta_0}{dr^2} + \left( \frac{w_1 - Q}{r} \right) \frac{d\theta_0}{dr} - w_2 \theta_0 = 0 \quad (17a)$$

$$w_1 \frac{d^2 \theta_1}{dr^2} + \left( \frac{w_1 - Q}{r} \right) \frac{d\theta_1}{dr} + (\omega - w_2) \theta_1 = 0 \quad (17b)$$

$$E_1 \frac{d^2 C_0}{dr^2} + \left( \frac{E_1 - Q}{r} \right) \frac{dC_0}{dr} - E_2 C_0 = 0 \quad (18a)$$

$$E_1 \frac{d^2 C_1}{dr^2} + \left( \frac{E_1 - Q}{r} \right) \frac{dC_1}{dr} + (\omega - E_2) C_1 = 0 \quad (18b)$$

with the associated boundary conditions

$$v_0(0, t) = 0, \quad v_0(1, t) = e^{\omega t}, \quad v_1(0, t) = 0, \quad v_1(1, t) = e^{\omega t}, \quad \theta_0(0, t) = 0, \quad \theta_0(1, t) = e^{\omega t},$$

$$\theta_1(0, t) = 0, \quad \theta_1(1, t) = e^{\omega t}, \quad C_0(0, t) = 0, \quad C_0(1, t) = e^{\omega t}, \quad C_1(0, t) = 0, \quad C_1(1, t) = e^{\omega t}$$

Equation (16a) – (18b) is tested and shown to be finite in the presence of a singular point (Raisinghania, 2011, Dass and Rama, 2000, Gupta, 2005) <sup>[35-37]</sup>. Using the Frobenius method, one can find a solution of this type.

$$\theta_0(r) = \sum_{k=0}^{\infty} a_k r^{(m+k)} \quad (19)$$

is assumed. Substituting equations (19) into equations (17a) after simplification, the expression procure an indicial equation of the form

$$m[w_1(m-1) + w_1 - Q]a_0 = 0 \quad (a_0 \neq 0) \quad (20)$$

the solution is attaining as

$$m = 0, \text{ or } m = \frac{Q}{w_1} \quad (21)$$

the recurrence relation is

$$a_{k+2} = \frac{w_2}{(m+k+2)[w_1(m+k+1)+w_1-Q]} a_k \tag{22}$$

Imposition of the boundary conditions is given by

$$\theta_0(r) = \frac{e^{w_1 r}}{\left(1 + \frac{w_2}{2Q + 4w_1} + \dots\right)} \left( r^{\frac{Q}{w_1}} + \frac{w_2 r^{2+\frac{Q}{w_1}}}{2Q + 4w_1} + \dots \right) \tag{23}$$

Imposition of the boundary conditions equation (17b) is given as

$$\theta_1(r) = \frac{e^{w_1 r}}{\left(1 + \frac{(\omega - w_2)}{2Q + 4w_1} + \dots\right)} \left( r^{\frac{Q}{w_1}} + \frac{(\omega - w_2) r^{2+\frac{Q}{w_1}}}{2Q + 4w_1} + \dots \right) \tag{24}$$

Equation (18a) using the same approach after imposing the boundary conditions yield to

$$C_0(r) = \frac{e^{w_1 r}}{\left(1 + \frac{E_2}{2Q + 4E_1} + \dots\right)} \left( r^{\frac{Q}{E_1}} + \frac{w_2 r^{2+\frac{Q}{E_1}}}{2Q + 4E_1} + \dots \right) \tag{25}$$

Using the same approach gives

$$C_1 = \frac{e^{w_1 r}}{\left(1 + \frac{(\omega - E_2)}{2Q + 4E_1} + \dots\right)} \left( r^{\frac{Q}{E_1}} + \frac{(\omega - E_2) r^{2+\frac{Q}{E_1}}}{2Q + 4E_1} + \dots \right) \tag{26}$$

Rewriting equation (16a) yields

$$J_1 \frac{d^2 V_0}{dr^2} + r^{-1}(J_1 - Q) \frac{dV_0}{dr} - r^{-2} J_1 V_0 + J_4 V_0 = -J_2 r^a (A_{11} + A_{12} r^2) - J_3 r^b (B_{11} + B_{12} r^2) \tag{27}$$

Where

$$a = \frac{Q}{w_1}, \quad A_{11} = \frac{e^{\omega r} (2Q + 4w_1)}{2Q + 4w_1 + w_2}, \quad A_{12} = \frac{w_2 e^{\omega r}}{2Q + 4w_1 + w_2}, \quad b = \frac{Q}{E_1}, \quad B_{11} = \frac{e^{\omega r} (2Q + 4E_1)}{2Q + 4E_1 + E_2}, \quad B_{12} = \frac{E_2 e^{\omega r}}{2Q + 4E_1 + E_2}$$

The complimentary part in equation (27) yields

$$v_c = A_5 r^{m_1} \left( 1 + \frac{J_4 r^2}{(m_1 + 2)(J_1 m_1 + 2J_1 - Q) - J_1} + \dots \right) + B_5 r^{m_2} \left( 1 + \frac{J_4 r^2}{(m_2 + 2)(J_1 m_2 + 2J_1 - Q) - J_1} + \dots \right) \tag{28}$$

$$v_p = a_{11} r^a + a_{12} r^{2+a} + a_{13} r^{1+a} + a_{14} r^a + b_{11} r^b + b_{12} r^{2+b} + b_{13} r^{1+b} + b_{14} r^b \tag{29}$$

Equation (27) can be solved for the particular integral to produce

$$v_p = \frac{J_2 A_{12}}{J_4} r^{2+a} + \frac{J_3 B_{12}}{J_4} r^{2+b} \tag{30}$$

imposing the boundary conditions, the full solution, which consists of the particular integral and the complimentary function, takes the following shape:

$$V_0 = A_5 r^{M_1} \left( 1 + \frac{J_4 r^2}{(M_1 + 2)(J_1 M_1 + 2J_1 - Q) - J_1} + \dots \right) + B_5 r^{M_1} \left( 1 + \frac{J_4 r^2}{(M_2 + 2)(J_1 M_2 + 2J_1 - Q) - J_1} + \dots \right) + \frac{J_2 A_{12}}{J_4} r^{2+a} + \frac{J_3 B_{12}}{J_4} r^{2+b} \tag{31}$$

Adopting the same approach and procedure:

$$V_1 = A_6 r^{M_1} \left( 1 + \frac{(\omega - J_4) r^2}{(M_1 + 2)(J_1 M_1 + 2J_1 - Q) - J_1} + \dots \right) + B_6 r^{M_2} \left( 1 + \frac{(\omega - J_4) r^2}{(M_2 + 2)(J_1 M_2 + 2J_1 - Q) - J_1} + \dots \right) + \frac{J_2 R_{12}}{(\omega - J_4)} r^{2+a} + \frac{J_3 S_{12}}{(\omega - J_4)} r^{2+b} \tag{32}$$

After the imposition of the boundary conditions, simplify equations (32) and (31) by using the Taylor series expansion about the point  $r = 1$  and ignoring powers of  $r > 1$ . Additionally, substitute equations (23), (24), (25), (26), (31) and (32) into equations (15a), (15b) and (15c)

$$\theta(r, t) = \frac{e^{\omega t} (2Q + 4w_1)}{2Q + 4w_1 + w_2} r^{\frac{Q}{w_1}} + \frac{w_2}{2Q + 4w_1 + w_2} r^{2 + \frac{Q}{w_1}} + \frac{(2Q + 4w_1)}{2Q + 4w_1 + w_2} r^{\frac{Q}{w_1}} + \frac{(\omega - w_2)}{2Q + 4w_1 + w_2} r^{2 + \frac{Q}{w_1}} \tag{33}$$

$$C(r, t) = \frac{e^{\omega t} (2Q + 4E_1)}{2Q + 4E_1 + E_2} r^{\frac{C}{E_1}} + \frac{E_2}{2Q + 4E_1 + E_2} r^{2 + \frac{Q}{E_1}} + \frac{(2Q + 4E_1)}{2Q + 4E_1 + E_2} r^{\frac{Q}{E_1}} + \frac{(\omega - w_2)}{2Q + 4E_1 + E_2} r^{2 + \frac{Q}{E_1}} \tag{34}$$

$$v(r, t) = A_5 r^{M_1} \left( 1 + \frac{J_4 r^2}{(M_1 + 2)(J_1 M_1 + 2J_1 - Q) - J_1} + \dots \right) + B_5 r^{M_2} \left( 1 + \frac{J_4 r^2}{(M_2 + 2)(J_1 M_2 + 2J_1 - Q) - J_1} + \dots \right) + \frac{J_2 A_{12}}{J_4} r^{2+a} + \frac{J_3 B_{12}}{J_4} r^{2+b} + e^{-\omega t} \left( A_6 r^{M_1} \left( 1 + \frac{(\omega - J_4) r^2}{(M_1 + 2)(J_1 M_1 + 2J_1 - Q) - J_1} + \dots \right) + B_6 r^{M_2} \left( 1 + \frac{(\omega - J_4) r^2}{(M_2 + 2)(J_1 M_2 + 2J_1 - Q) - J_1} + \dots \right) + \frac{J_2 R_{12}}{(\omega - J_4)} r^{2+a} + \frac{J_3 S_{12}}{(\omega - J_4)} r^{2+b} \right) \tag{35}$$

Where

$$M_1 = \frac{Q + \sqrt{Q^2 + 4J_1^2}}{2J_1} \quad M_2 = \frac{Q - \sqrt{Q^2 + 4J_1^2}}{2J_1}$$

**Heat Transfer Coefficient or Nusselt Number (Nu)**

It has been demonstrated in the literature that the heat transfer coefficient, also known as the Nusselt number, provides a more accurate indication of the temperature effect on nanofluids than the effective thermal conductivity of the nanofluid in question, particularly in the case of coolants and other applications where the nanofluid is utilized in industry. By using the Taylor series expansion of equation (33) around the point  $r=1$  and ignoring powers of  $r > 1$ , one can determine the Nusselt number.

$$Nu = - \left( \frac{d\theta(r, t)}{dr} \right)_{r=0} = - \frac{e^{\omega t} (2Q + 4w_1)}{2(2Q + 4w_1 + w_2)} - \frac{3w_2}{4(2Q + 4w_1 + w_2)} - \frac{(2Q + 4w_1)}{2(2Q + 4w_1 + w_2)} - \frac{3(\omega - w_2)}{4(2Q + 4w_1 + w_2)} \tag{36}$$

**Mass Transfer Coefficient or Sherwood Number (Sh)**

The Sherwood number has been found by applying the Taylor series expansion of equation (34), focusing on the point  $r=1$  and disregarding powers of  $r > 1$ .



$$Sh = -\left(\frac{dC(r,t)}{dr}\right)_{r=0} = -\frac{e^{\omega t}(2Q+4E_1)}{2(2Q+4E_1+E_2)} - \frac{3E_2}{4(2Q+4E_1+E_2)} - \frac{(2Q+4E_1)}{2(2Q+4E_1+E_2)} - \frac{3(\omega-E_2)}{4(2Q+4E_1+E_2)} \tag{37}$$

**Skin Friction**

With respect to powers of  $r > 1$ , the skin friction is determined by applying the Taylor series expansion of equation (35) about the point  $r = 1$ .

$$\begin{aligned} \tau = \left(\frac{dv(r,t)}{dr}\right)_{r=0} &= A_5 \left( M_1 + \frac{J_4(2+M_1)}{(M_1+2)(J_1M_1+2J_1-Q)-J_1} \right) + B_5 \left( M_2 + \frac{J_4M_2}{(M_2+2)(J_1M_2+2J_1-Q)-J_1} \right) \\ &+ \frac{3}{4} \left( \frac{J_2A_{12}}{J_4} \right) + \frac{3}{4} \left( \frac{J_3B_{12}}{J_4} \right) + e^{-\omega t} A_6 r^{M_1} \left( M_1 + \frac{(\omega-J_4)(2+M_1)}{(M_1+2)(J_1M_1+2J_1-Q)-J_1} + \dots \right) \\ &+ e^{-\omega t} B_6 r^{M_2} \left( 1 + \frac{(\omega-J_4)M_2}{(M_2+2)(J_1M_2+2J_1-Q)-J_1} + \dots \right) + \frac{3}{4} \left( \frac{J_2R_{12}}{(\omega-J_4)} \right) + \frac{3}{4} \left( \frac{J_3S_{12}}{(\omega-J_4)} \right) \end{aligned} \tag{38}$$

**Results and Discussion**

**Results**

Lawson *et al.* (2023) [25] used the following parameter values to obtain physical insight and numerical validation into the problem  $Pr = 0.71, 1.71, 2.71, 3.71, 4.71, M = 1, 2, 3, 4, 5; Sc = 0.6, 0.8, 1.0, 1.2, 1.4; k_\infty = 1.350, 2.350, 3.350, 4.350, 5.350; Gr_C = 1, 3, 5, 7, 9; \phi = 0.10, 0.20, 0.30, 0.40, 0.50; K = 0.30, 0.60, 0.90, 1.20, 1.50, Gr_\theta = 0.78, 1.56, 2.34, 3.12, 3.90 \omega = 0.2, 0.4, 0.6, 0.8, 1.0; N = 0.5, 1.0, 1.5, 2.0, 2.5; Re = 1000, 1500, 2000, 2500, 3000; t = 0.5; Q = 1$

**Table 4:** Showing values of Nusselt number (Nu) and Skin friction ( $\tau$ ) for varying some material parameters.

Pr	N	$\phi$	$\omega$	Nu	$\tau$
0.71	0.5	0.1	0.2	$-7.85633 \times 10^1$	$2.88092 \times 10^1$
0.81				$-6.40782 \times 10^1$	$2.88092 \times 10^1$
0.91				$-5.38142 \times 10^1$	$2.88092 \times 10^1$
1.01				$-4.62390 \times 10^1$	$2.88092 \times 10^1$
1.11				$-4.04619 \times 10^1$	$2.88091 \times 10^1$
	1.0			$-8.24083 \times 10^1$	$2.88091 \times 10^1$
	1.5			$-8.62574 \times 10^1$	$2.88090 \times 10^1$
	2.0			$-9.01102 \times 10^1$	$2.88089 \times 10^1$
	2.5			$-9.39660 \times 10^1$	$2.88087 \times 10^1$
		0.2		$-1.843050 \times 10^2$	$-2.11832 \times 10^1$
		0.3		$-5.03890 \times 10^2$	$-4.59087 \times 10^1$
		0.4		$-1.7451172 \times 10^3$	$1.00299 \times 10^3$
		0.5		$-1.1836032 \times 10^4$	$1.46363 \times 10^3$
			0.4	$-8.83390 \times 10^2$	$1.40259 \times 10^2$
			0.6	$-9.89533 \times 10^2$	$1.3866 \times 10^2$
			0.8	$-1.10494 \times 10^2$	$1.3783 \times 10^2$
			1.0	$-1.23060 \times 10^2$	$1.3555 \times 10^2$

**Table 5:** Showing values of Sherwood number (Sh) and Skin friction ( $\tau$ ) for varying some material parameters.

Sc	$K_\infty$	$\phi$	$\omega$	Sh	$\tau$
0.6	0.5	0.1	0.2	$-6.57538 \times 10^1$	$2.88092 \times 10^1$
0.8				$-4.50688 \times 10^1$	$2.88092 \times 10^1$
1.0				$-3.38993 \times 10^1$	$2.88091 \times 10^1$
1.2				$-2.73399 \times 10^1$	$2.88091 \times 10^1$
1.4				$-2.30892 \times 10^1$	$2.88091 \times 10^1$
	2.35			$-7.12871 \times 10^1$	$3.96028 \times 10^{-5}$
	3.35			$-7.68296 \times 10^1$	$8.56845 \times 10^{-6}$
	4.35			$-8.23827 \times 10^1$	$2.36854 \times 10^{-6}$
	5.35			$-8.79441 \times 10^1$	$6.81276 \times 10^{-7}$
		0.2		$-9.75756 \times 10^1$	$-2.11832 \times 10^1$
		0.3		$-1.61929 \times 10^2$	$-4.59087 \times 10^1$
		0.4		$-3.66034 \times 10^2$	$1.00299 \times 10^3$
		0.5		$-1.51977 \times 10^3$	$1.46363 \times 10^3$
			0.4	$-7.45118 \times 10^1$	$1.40259 \times 10^2$
			0.6	$-8.39806 \times 10^1$	$1.3866 \times 10^2$
			0.8	$-9.42384 \times 10^1$	$1.3783 \times 10^2$
			1.0	$-1.053678 \times 10^2$	$1.3555 \times 10^2$



**Table 6:** Showing values of Skin friction ( $\tau$ ) for varying some material parameters.

Re	M	K	$Gr_{\theta}$	$Gr_c$	$\tau$
1000	1	0.3	0.78	1	$2.88093 \times 10^1$
1500					$1.87716 \times 10^2$
2000					$1.63118 \times 10^3$
2500					$1.75871 \times 10^2$
3000					$2.92230 \times 10^2$
	2				$1.44148 \times 10^2$
	3				$1.47950 \times 10^2$
	4				$1.51752 \times 10^2$
	5				$1.55554 \times 10^2$
		0.6			$6.81420 \times 10^1$
		0.9			$4.40743 \times 10^1$
		1.2			$3.20594 \times 10^1$
		1.5			$2.48353 \times 10^1$
			1.56		$2.88093 \times 10^1$
			2.34		$2.88093 \times 10^1$
			3.12		$2.88093 \times 10^1$
			3.90		$2.88093 \times 10^1$
				3	$2.88093 \times 10^1$
				5	$2.88094 \times 10^1$
				7	$2.88094 \times 10^1$
				9	$2.88095 \times 10^1$

### Discussions

Table 4 shows the Nusselt number and skin friction of different values for varying some material parameters. It was discovered that as the Prandtl number gained from 0.71 to 1.11, the Nusselt number also increased from -7.85633 to -4.404619 and the skin friction diminished from 2.88092 to 2.88091. As the radiation term increased from 0.5 to 2.5, the Nusselt number decreased from -7.85633 to -9.39660. This observation was consistent with Wang *et al.* (2006)<sup>[20]</sup> and Karthikeyan *et al.* (2013)<sup>[24]</sup>. An increase in the free stream frequency from 0.2 to 1.0 corresponded to an increase in the Nusselt number from  $-7.85633 \times 10^1$  to  $-1.23060 \times 10^1$  and decreased the skin friction. The increased volume fraction from 0.1 to 0.5 showed a decrease in the Nusselt number (Ngiangia *et al.*, 2021)<sup>[23]</sup> and decreased in the skin friction and later increased it.

Table 5 shows the Sherwood number and skin friction of different values for varying some material parameters. It was observed that the Sherwood number was enhanced while the skin friction diminished as there was an increment in the Schmidt number from 0.6 to 1.4. A raise in the chemical reaction parameter from 0.5 to 5.35 reduced the Sherwood number  $-6.57538 \times 10^1$  to  $-8.79441 \times 10^1$  (Babu *et al.*, 2011)<sup>[39]</sup> and diminished skin friction  $2.88092 \times 10^1$  to  $6.81276 \times 10^{-7}$ . As the volume fraction was increased, the Sherwood reduced and the skin friction also reduced and later increased. As the skin friction reduced, the Sherwood number increased due to an increase in the free stream frequency.

Table 6 shows skin friction of different values for varying Reynolds, magnetic field, Grashof, and modified Grashof parameters. Skin friction was enhanced as the Reynolds number was increased, which showed that the tin oxide nanofluid flow was of the laminar type and this result was in agreement with the work of Polidori *et al.* (2007)<sup>[40]</sup>, Mansour *et al.* (2007)<sup>[41]</sup>, and consistent with the work of Wang *et al.* (2006)<sup>[20]</sup>. An increment in the magnetic field parameter from 2 to 5 enhanced skin friction from  $1.44148 \times 10^2$  to  $1.51752 \times 10^2$ . The skin friction diminished as the porosity parameter was increased from 0.3 to 1.5. As the Grashof parameter and the modified Grashof parameter increased from 1 to 9 and 0.78 to 3.90 respectively, the skin friction got enhanced. This aligned with the research conducted by Ngiangia and Orukari (2021)<sup>[23]</sup>.

### Conclusion

The following noteworthy findings were noted in this study: as there was an increment in the nanofluid volume fraction, the temperature profile, velocity profile, and concentration profile of the tin oxide nanofluid all improved. It has been found that the nanoparticle volume fraction, the Nusselt number, and the skin friction decreased as the radiation number increased are among the most important parameters in the variety of nanofluids. The Sherwood number and skin friction decreased as the chemical reaction term increased. The skin friction increased with an increase in the magnetic field; the skin friction decreased with an enhancement in the porosity parameter, the skin friction dropped with an increase in the nanoparticle volume fraction; and the skin friction reduced with a raise in the free stream frequency.

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