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Estimation of location (θ) and scale (λ) of Two-Parameter Half Logistic -Rayleigh Distribution (HLRD) using least square regression methods

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Abstract
In this paper, we propose the estimation of Location (θ) and Scale (λ) parameters using the Least Square Regression Method. We also computed Average Estimate (AE), Variance (VAR), Standard Deviation (STD), Mean Absolute Deviation (MAD), Mean Square Error (MSE), Simulated Error (SE) and Relative Absolute Bias (RAB) for both the parameters under complete sample based on 1000 simulations to assess the performance of the estimators.

Keywords: Two parameter HLRD, least square regression method, montecarlo simulation

1. Introduction

Generally in many of the situations, we face some type of situations of non monotonic failure rates to supervise the reliability analysis of the data. In order to model such data, proposed by Aarset *et al* (1987) [1] and Venkataraman *et al* (1988) [12] proposed Least squares estimators and Weighted Least squares estimators of a Beta distribution present an extension of the Weibull family that not only contains unimodel distribution with bath tub failure rates but also allows for a broader class of monotone hazard rates and is computationally convenient for censored data. They named their extended version as "Exponentiated Weibull Family". On similar lines Gupta and Kundu (2001b) [8] proposed a new model called generalized exponential distribution. A generalized (type – II) version of logistic distribution was considered and some interesting properties of the distribution were derived by Balakrishnan and Hassain (2007) [5]. Ramakrishna (2008) [7] studied the Type I generalized half logistic distribution scale (σ) and shape (θ) parameters estimation using the least square method in two step estimation methods. Torabi and Bagheri (2010) [13] considered different parameter estimation methods in extended generalized half logistic distribution for censored as well as complete sample. Rama Mohan and Anjaneyulu (2011) [10] studied how the least square method be good for estimating the parameters in two parameter Weibull distribution from an optimally constructed grouped sample.

In Section - 2, we discuss the procedure for estimating the Location (θ) and Scale (λ) parameters of the HLRD using the least squares regression method. We therefore employ these approximations as the least squares method.

In Section - 3 we present the observations and the conclusions are based on the simulation results with the numerical example.

Let $x_1, x_2,..., x_n$ be a random sample of size n from HLRD (θ,λ), its Probability Density Density Function (PDF), cumulative distribution function(CDF) and Hazard Function (HF) are given by

$$f(x;\theta,\lambda)=\frac{4\lambda(x-\theta)e^{-\lambda(x-\theta)^2}}{\left[1+e^{-\lambda(x-\theta)^2}\right]^2};x>\theta,\lambda>0 \qquad \text{... (1.1)}$$

$$F(x;\theta,\lambda)=\frac{1-e^{-\lambda(x-\theta)^2}}{1+e^{-\lambda(x-\theta)^2}};x>\theta,\lambda>0 \quad ... (1.2)$$

$$h(x) = H(x;\theta,\lambda) = \frac{2\lambda(x-\theta)}{\left[1+e^{-\lambda(x-\theta)^2}\right]} \quad ... (1.3)$$

2. Estimation of Location (θ) and Scale (λ) parameters of two parameter HLRD

A. HLRD using Least Square Regression Method

Let $x_{(1)}< x_{(2)}< x_{(3)}\ldots\ldots\ldots.< x_{(N)}$ be an ordered sample of size 'N' from Half Logistic Rayleigh Distribution with the parameters Location (θ) and Scale (λ). Then the cdf is given in equation (1.2), can be written as

$$1+F(x)=\frac{2}{1+e^{-\lambda(x-\theta)^2}} \quad ... (2.1)$$

Here, $F(X) = \frac{i}{N+1}$

and

$$1-F(x)=\frac{2e^{-\lambda(x-\theta)^2}}{1+e^{-\lambda(x-\theta)^2}} \quad ... (2.2)$$

$$\frac{1+F(x)}{1-F(x)}=e^{\lambda(x-\theta)^2} \quad ... (2.3)$$

Taking Logarithm on both side of equation (2.3), we get

$$x = \theta+\frac{1}{\sqrt{\lambda}}[\log Y]^{1/2} \quad ... (2.4)$$

Where $Y = \left(\frac{1+F(X)}{1-F(X)}\right)$

From the least square parameter estimation method (also known as regression analysis), let us consider

$A = \theta$, $\hat{\theta} = \hat{A}$, $B = \frac{1}{\sqrt{\lambda}}$, $\hat{\lambda} = \frac{1}{\sqrt{B}}$. ...(2.5)

And

$$U=\log\left(\frac{1+F(x)}{1-F(x)}\right)^{1/2}, V=x \quad ...(2.6)$$

$$\tilde{A}=\frac{\left(\sum_{i=1}^{n}v_i^2\right)\left(\sum_{i=1}^{n}u_i\right)-\left(\sum_{i=1}^{n}v_i\right)\left(\sum_{i=1}^{n}v_iu_i\right)}{\left(n\sum_{i=1}^{n}v_i^2\right)-\left(\sum_{i=1}^{n}v_i\right)^2} \quad ... (2.7)$$

$$\tilde{B}=\frac{n\left(\sum_{i=1}^{n}v_iu_i\right)-\left(\sum_{i=1}^{n}v_i\right)\left(\sum_{i=1}^{n}u_i\right)}{\left(n\sum_{i=1}^{n}v_i^2\right)-\left(\sum_{i=1}^{n}v_i\right)^2} \quad ... (2.8)$$

B. Simulation study

In order to obtain the Least Square Regression method estimators of Location (θ)and Scale (λ) is used to obtain estimators and to study their predictive properties by Average Estimate (AE), Variance (VAR), Mean Square Error (MSE), Relative Absolute Bias (RAB) and Relative Error (RE). If $\hat{\xi}_{lm}$ is Median Ranks Method estimate of $\hat{\xi}_m$, m=1, 2 where $\xi_m$is a general notation that can be replaced by $\xi_1 = \lambda, \xi_2 = \theta$ based on sample l, (l=1,2,…,r), then the Average Estimate (AE), Variance (VAR), Mean Absolute Deviation (MAD), Mean Square Error (MSE) and Relative Absolute Bias (RAB) and Relative Error (RE) are given respectively by

Average Estimate $(\hat{\psi}_m) = \frac{\sum_{i=1}^{r}\hat{\psi}_{lm}}{r}$

Variance$(\hat{\psi}_m) = \frac{\sum_{i=1}^{r}(\hat{\psi}_{lm}-\overline{\hat{\psi}_{lm}})^2}{r}$

Mean Absolute Deviation = $\frac{\sum_{i=1}^{r}Med(|\hat{\psi}_{lm}-\overline{\hat{\psi}_{lm}}|)}{r}$

Mean Square Error $(\hat{\psi}_m) = \frac{\sum_{i=1}^{r}(\hat{\psi}_{lm}-\psi_m)^2}{r}$

Relative Absolute Bias$(\hat{\psi}_m) = \frac{\sum_{i=1}^{r}|(\hat{\psi}_{lm}-\psi_m)|}{r\psi_m}$

Relative Error$(\hat{\psi}_m) = \frac{1}{r}\left(\frac{\sum_{i=1}^{r}MSE\sqrt{(\hat{\psi}_{lm})}}{\psi_m}\right)^2$

3. Observations from simulation results and conclusions

1. The Average Estimate (AE), Variance (VAR), Standard deviation (STD), Mean Square Error (MSE) and Relative Absolute Bias (RAB) are independent of true values of the parameters of Location (θ) and Scale (λ) by observing simulated data sets.
2. Average Estimate (AE) of Location parameter ($\tilde{\theta}$) and Scale parameter ($\tilde{\lambda}$) in Least Square Regression Method are decreasing when sample size (N) is increasing.
3. Variance (VAR) of Location parameter ($\tilde{\theta}$) and Scale parameter ($\tilde{\lambda}$) in Least Square Regression Method are increasing when sample size (N) is increasing.
4. Mean Absolute Deviation (MAD) of Location parameter ($\tilde{\theta}$) and Scale parameter ($\tilde{\lambda}$) in Least Square Regression Method and increasing when sample size (N) is increasing.
5. Mean Square Error (MSE) of Location parameter ($\tilde{\theta}$) and Scale paramete ($\tilde{\lambda}$) in Least Square Regression Method are decreasing when sample size (N) is increasing.
6. Relative Absolute Bias (RAB) of Location parameter ($\tilde{\theta}$) and Scale parameter ($\tilde{\lambda}$) Least Square Regression Method were increasing when sample size (N) is increasing.

Simulated datasets: We evaluate the performance of the Least Square Regression (LSR) method for estimating the

HLRD (θ, λ), Newton-Raphson simulation for a two parameter combinations and the process is repeated 10,000 times for different sample sizes n=50(50)500 are considered. The MRRs and their Average Estimate (AE), Variance (VAR), Mean Absolute Deviation (MAD), Mean Square Error (MSE), Relative Absolute Bias (RAB) and Relative Error (RE) of Least Square method to estimators of scale and Location parameters. Population parameters Scale = 2.5 and location = 3 in Table-3.4.

Table- 3.4

Least Square Regression-HLRD

Sample size	Parameters	Average estimation	Variance	MAD	MSE	RAB	RE
50	$\tilde{\theta}$	2.416037	0.163181	0.28466	0.3410125	0.0973271	0.2919814
	$\tilde{\lambda}$	3.87926	0.07679	0.294104	0.9023585	0.275852	0.6896301
100	$\tilde{\theta}$	2.353751	0.041262	0.142383	0.4176373	0.2154162	0.3231243
	$\tilde{\lambda}$	3.743336	0.070899	0.338597	0.545884	0.4973343	0.6216679
150	$\tilde{\theta}$	2.9431	0.520277	0.862913	0.0032376	0.02845	0.02845
	$\tilde{\lambda}$	3.587492	0.218549	0.607039	0.1826398	0.6524955	0.5437462
200	$\tilde{\theta}$	3.050192	0.794151	0.162325	0.0025193	0.0334616	0.0250962
	$\tilde{\lambda}$	3.02951	0.923774	0.232616	0.280381	0.4236081	0.2647551
250	$\tilde{\theta}$	3.556159	0.462213	0.942249	0.3093123	0.4634654	0.2780793
	$\tilde{\lambda}$	3.127421	0.955334	0.305475	0.393657	0.6274209	0.3137105
300	$\tilde{\theta}$	3.653171	0.458196	0.88823	0.426633	0.6531715	0.3265857
	$\tilde{\lambda}$	3.167273	0.909474	0.263539	0.445253	0.8007274	0.3336364
350	$\tilde{\theta}$	3.612437	0.430732	0.846908	0.3750793	0.71451	0.3062186
	$\tilde{\lambda}$	3.036132	0.89958	0.129998	0.2874373	0.7505846	0.2680659
400	$\tilde{\theta}$	3.814314	0.24649	0.640808	0.663108	0.0857526	0.4071572
	$\tilde{\lambda}$	3.348839	0.690315	0.070497	0.7205277	0.3581425	0.4244195
450	$\tilde{\theta}$	3.442474	0.569817	0.915029	0.195783	0.6637107	0.2212369
	$\tilde{\lambda}$	2.635621	0.873551	0.220174	0.0183931	0.01	0.0678105
500	$\tilde{\theta}$	3.34759	0.58741	0.959337	0.1208191	0.5793175	0.1737952
	$\tilde{\lambda}$	2.851896	0.186118	0.412921	0.1238311	0.7037928	0.1759482

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