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Inter relations between algebraic boundary and other boundary

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Abstract

In this study, various closed algebras on fuzzy rough algebraic TM systems are defined. The relationships between closed algebras and between algebraic boundaries and other boundaries are also provided. In order to emphasize the results, examples are provided.

Keywords: Fuzzy rough algebraic *pre* boundary, fuzzy rough algebraic α boundary, fuzzy rough algebraic β boundary, fuzzy rough algebraic γ boundary, fuzzy rough algebraic δ boundary

1. Introduction

In 1965, Zadeh ^[9] presented the concept of Fuzzy Set Theory. He presented a mathematical technique to create some imprecise explanations. A fuzzy set is one that has components of a particular degree but no clearly defined boundaries. The range of a fuzzy set is 0 to 1. Due to its flexibility with varying data, fuzzy logic has been shown to have applications in several aspects of our daily lives, particularly in the medical area. Some features of fuzzy rough sets in TM boundary algebra, initially established by K. Manimegalai and A. Tamilarasi ^[5], are discussed in this study. In this study, various closed algebras on fuzzy rough algebraic TM systems are defined. The relationships between closed algebras and between algebraic boundaries and other boundaries are also provided. In order to emphasize the results, examples are provided.

2. Preliminaries

2.1 Definition ^[2, 7]

Let X be the fuzzy topological space, and let λ be a fuzzy set. The boundary Bd of λ , is then determined as $Bd(\lambda) = cl(\lambda) \cap cl(\lambda)'$. Obviously, $Bd(\lambda)$ is a fuzzy closed set.

2.2 Definition ^[1]

A fuzzy set A in a fuzzy topological system (X, τ) is called fuzzy α open if $A \leq int(cl(int(A)))$. The compliment of fuzzy α open is fuzzy α closed.

2.3 Definition ^[3, 6]

A fuzzy set A in a fuzzy topological system (X, τ) is called fuzzy β open if $A \leq cl(int(cl(A)))$. The compliment of fuzzy β open is fuzzy β closed.

2.4 Definition ^[7]

A fuzzy set A in a fuzzy topological system (X, τ) is called fuzzy *pre* open if $A \leq int(cl(A))$. The compliment of fuzzy *pre* open is fuzzy *pre* closed.

2.5 Definition ^[4]

A fuzzy subset A of fuzzy topological space (X, τ) is said to be fuzzy γ open set if $A \leq (cl(int(A))) \cup (int(cl(A)))$. The compliment of fuzzy γ open set is fuzzy $\square\square$ closed set.

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2.6 Definition ^[8]

A fuzzy subset A of fuzzy topological space (X, τ) is said to be fuzzy δ closed if $A = cl(A)$.

3. Inter relations between algebraic boundary and other Boundary**3.1 Definition**

A non-empty set A is a family of fuzzy rough algebraic in X satisfies the following conditions:

- $\tilde{0}, \tilde{1} \in TM$
- If $A, B \in TM$ then $A \cap B \in TM$
- If $A_i \in TM$ for each $i \in J$ then $\cup_{i \in J} A_i \in TM$ where J is an indexed set.

Then A is said to be a fuzzy rough algebraic TM system. Any member of fuzzy rough algebraic TM system is called fuzzy rough algebraic TM open. The complement of fuzzy rough open algebraic is fuzzy rough algebraic TM closed.

3.2 Definition

Let (X, TM) be a fuzzy rough algebraic TM system and A be any fuzzy rough algebraic. Then the fuzzy rough algebraic boundary of A , is denoted and defined as $\mathcal{F}_{RTM}Bd(A) = \mathcal{F}_{RTM}cl(A) \cap \mathcal{F}_{RTM}cl(A')$. Obviously $\mathcal{F}_{RTM}Bd(A)$ is fuzzy rough algebraic TM closed.

3.3 Definition

Let (X, TM) be a fuzzy rough algebraic TM system. A fuzzy rough algebraic A in X is said to be fuzzy rough algebraic *pre* closed ($\mathcal{F}_{RTM}pC$ shortly) if $A \supseteq (\mathcal{F}_{RTM}cl(\mathcal{F}_{RTM}int(A)))$. The complement of every fuzzy rough algebraic *pre* closed is fuzzy rough algebraic *pre* open ($\mathcal{F}_{RTM}pO$ shortly).

3.4 Definition

Let (X, TM) be a fuzzy rough TM system and A be any fuzzy rough algebraic. Then A is said to be a fuzzy rough algebraic α closed ($\mathcal{F}_{RTM}\alpha C$ shortly) if $A \supseteq \mathcal{F}_{RTM}cl(\mathcal{F}_{RTM}int(\mathcal{F}_{RTM}cl(A)))$. The complement of fuzzy rough algebraic α closed is fuzzy rough algebraic α open ($\mathcal{F}_{RTM}\alpha O$ shortly).

3.5 Definition

Let (X, TM) be a fuzzy rough TM system and A be any fuzzy rough algebraic. Then A is said to be a fuzzy rough algebraic β closed ($\mathcal{F}_{RTM}\beta C$ shortly) if $A \supseteq \mathcal{F}_{RTM}int(\mathcal{F}_{RTM}cl(\mathcal{F}_{RTM}int(A)))$. The complement of fuzzy rough algebraic β closed is fuzzy rough algebraic β open ($\mathcal{F}_{RTM}\beta O$ shortly).

3.6 Definition

Let (X, TM) be a fuzzy rough TM system and A be any fuzzy rough algebraic. Then A is said to be a fuzzy rough algebraic γ closed ($\mathcal{F}_{RTM}\gamma C$ shortly) if $A \supseteq (\mathcal{F}_{RTM}int(\mathcal{F}_{RTM}cl(A))) \cap (\mathcal{F}_{RTM}cl(\mathcal{F}_{RTM}int(A)))$. The complement of fuzzy rough algebraic γ closed is fuzzy rough algebraic $\square \square$ open ($\mathcal{F}_{RTM}\gamma O$ shortly).

3.7 Definition

Let (X, TM) be a fuzzy rough TM system and A be any fuzzy rough algebraic. Then A is said to be a fuzzy rough algebraic δ closed ($\mathcal{F}_{RTM}\delta C$ shortly) if $A = \mathcal{F}_{RTM}cl(A)$. The complement of fuzzy rough algebraic δ closed is fuzzy rough algebraic δ open ($\mathcal{F}_{RTM}\delta O$ shortly).

3.8 Definition

Let (X, TM) be a fuzzy rough algebraic TM system and A be a fuzzy rough algebraic in X . Then the fuzzy rough algebraic pre interior ($\mathcal{F}_{RTM}pint$ shortly), fuzzy rough algebraic pre closure ($\mathcal{F}_{RTM}pcl$ shortly), fuzzy rough algebraic α interior ($\mathcal{F}_{RTM}\alpha int$ shortly), fuzzy rough algebraic α closure ($\mathcal{F}_{RTM}\alpha cl$ shortly), fuzzy rough algebraic β interior ($\mathcal{F}_{RTM}\beta int$ shortly), fuzzy rough algebraic β closure ($\mathcal{F}_{RTM}\beta cl$ shortly), fuzzy rough algebraic γ interior ($\mathcal{F}_{RTM}\gamma int$ shortly), fuzzy rough algebraic γ closure ($\mathcal{F}_{RTM}\gamma cl$ shortly), fuzzy rough algebraic δ interior ($\mathcal{F}_{RTM}\delta int$ shortly) and fuzzy rough algebraic δ closure ($\mathcal{F}_{RTM}\delta cl$ shortly) of A are defined by

1. $\mathcal{F}_{RTM}pcl(A) = \cap \{B : B \text{ is a fuzzy rough algebraic pre closed in } X \text{ and } A \subseteq B\}$
2. $\mathcal{F}_{RTM}pint(A) = \cup \{K : K \text{ is a fuzzy rough algebraic pre open in } X \text{ and } K \subseteq A\}$.
3. $\mathcal{F}_{RTM}\alpha cl(A) = \cap \{B : B \text{ is a fuzzy rough algebraic } \alpha \text{ closed in } X \text{ and } A \subseteq B\}$
4. $\mathcal{F}_{RTM}\alpha int(A) = \cup \{K : K \text{ is a fuzzy rough algebraic } \alpha \text{ open in } X \text{ and } K \subseteq A\}$.
5. $\mathcal{F}_{RTM}\beta cl(A) = \cap \{G : G \text{ is a fuzzy rough algebraic } \beta \text{ closed in } X \text{ and } A \subseteq G\}$
6. $\mathcal{F}_{RTM}\beta int(A) = \cup \{S : S \text{ is a fuzzy rough algebraic } \beta \text{ open in } X \text{ and } S \subseteq A\}$.

7. $\mathcal{F}_{\mathcal{RTM}}\gamma cl(A) = \bigcap\{N: N \text{ is a fuzzy rough algebraic } \gamma \text{ closed in } X \text{ and } A \subseteq N\}$
8. $\mathcal{F}_{\mathcal{RTM}}\gamma int(A) = \bigcup\{M: M \text{ is a fuzzy rough algebraic } \gamma \text{ open in } X \text{ and } M \subseteq A\}$.
9. $\mathcal{F}_{\mathcal{RTM}}\delta cl(A) = \bigcap\{P: P \text{ is a fuzzy rough algebraic } \delta \text{ closed in } X \text{ and } A \subseteq P\}$
10. $\mathcal{F}_{\mathcal{RTM}}\delta int(A) = \bigcup\{C: C \text{ is a fuzzy rough algebraic } \delta \text{ open in } X \text{ and } C \subseteq A\}$.

3.9 Definition

Let A be any fuzzy rough algebraic of fuzzy rough algebraic TM system (X, TM) . Then A is said to be

1. Algebraic α boundary ($\alpha_{TM}Bd(A)$ shortly) if

$$\alpha_{TM}Bd(A) = \mathcal{F}_{\mathcal{RTM}}\alpha cl(A) \cap \mathcal{F}_{\mathcal{RTM}}\alpha cl(A').$$

2. Algebraic β boundary ($\beta_{TM}Bd(A)$ shortly) if

$$\beta_{TM}Bd(A) = \mathcal{F}_{\mathcal{RTM}}\beta cl(A) \cap \mathcal{F}_{\mathcal{RTM}}\beta cl(A').$$

3. Algebraic γ boundary ($\gamma_{TM}Bd(A)$ shortly) if

$$\gamma_{TM}Bd(A) = \mathcal{F}_{\mathcal{RTM}}\gamma cl(A) \cap \mathcal{F}_{\mathcal{RTM}}\gamma cl(A').$$

4. Algebraic δ boundary ($\delta_{TM}Bd(A)$ shortly) if

$$\delta_{TM}Bd(A) = \mathcal{F}_{\mathcal{RTM}}\delta cl(A) \cap \mathcal{F}_{\mathcal{RTM}}\delta cl(A').$$

4.1 Proposition

Fuzzy rough TM closure of a fuzzy rough algebraic pre open is a fuzzy rough algebraic regular closed.

Proof

Let A be a fuzzy rough algebraic pre open. Therefore, by Definition 2.9, $A \subseteq (\mathcal{F}_{\mathcal{RTM}}int(\mathcal{F}_{\mathcal{RTM}}cl(A)))$. Hence, $\mathcal{F}_{\mathcal{RTM}}cl(A) \subseteq (\mathcal{F}_{\mathcal{RTM}}int(\mathcal{F}_{\mathcal{RTM}}cl(A)))$ and $\mathcal{F}_{\mathcal{RTM}}int(\mathcal{F}_{\mathcal{RTM}}cl(A)) \subseteq \mathcal{F}_{\mathcal{RTM}}cl(A)$. So, $\mathcal{F}_{\mathcal{RTM}}cl(\mathcal{F}_{\mathcal{RTM}}int(\mathcal{F}_{\mathcal{RTM}}cl(A))) = A$. Hence, $\mathcal{F}_{\mathcal{RTM}}cl(A)$ is a fuzzy rough algebraic regular closed?

3. Results

1. Any union (resp. intersection) of fuzzy rough algebraic α open (resp. fuzzy rough algebraic α closed) is a fuzzy rough algebraic α open (resp. fuzzy rough algebraic α closed)
2. Any finite intersection (resp. union) of fuzzy rough algebraic α open (resp. fuzzy rough algebraic α closed) is a fuzzy rough algebraic α open (resp. fuzzy rough algebraic α closed)

Proposition 3.1

The collection of all fuzzy rough algebraic α open is a fuzzy rough algebraic TM system on X.

Proof

By the Result (3.1) and since $\tilde{0}$ and $\tilde{1}$ is also fuzzy rough algebraic α open in X. Hence the proof.

Result 3.2

Every fuzzy rough algebraic α closed is fuzzy rough algebraic β closed.

Result 3.3

From the Definition 3.8 the following conclusions are made.

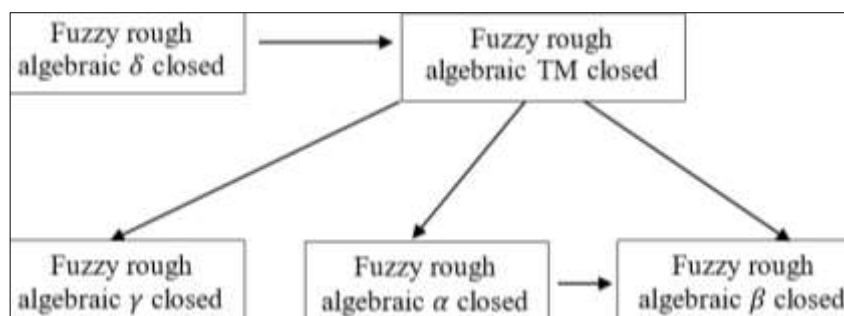


Fig A: Algebraic closed Inter-relation

Converse need not be true as shown in the following examples.

Example 3.3

Let $U = \{a, b, c\}$ and let $X_L = \{a\}$ and $X_U = \{a, c\}$ with $X_L \subset X_U$. Then the Boolean algebra is

$$\mathcal{B} = \{\tilde{0}, \{a\}, \{b\}, \{b, c\}, \{a, c\}, \{a, b\}, \{a, b, c\}\}.$$

$$\text{Define } A_L(x): X_L \rightarrow I \Rightarrow A_L(a) = 0.3$$

$$A_U(x): X_U \rightarrow I \Rightarrow A_U(a) = 0.5 \text{ and } A_U(c) = 0.3$$

$$\text{and define } B_L(x): X_L \rightarrow I \Rightarrow B_L(a) = 0.2$$

$$B_U(x): X_U \rightarrow I \Rightarrow B_U(a) = 0.2 \text{ and } B_U(c) = 0.3$$

Then $A = \left\{ \left(\frac{a}{0.3} \right), \left(\frac{a}{0.5}, \frac{c}{0.3} \right) \right\}$ and $B = \left\{ \left(\frac{a}{0.2} \right), \left(\frac{a}{0.2}, \frac{c}{0.3} \right) \right\}$ the sub-Boolean algebra of \mathcal{B} . The complement A' and B' is given by $A' = \left\{ \left(\frac{a}{0.5} \right), \left(\frac{a}{0.7}, \frac{c}{1} \right) \right\}$, $B' = \left\{ \left(\frac{a}{0.8} \right), \left(\frac{a}{0.8}, \frac{c}{1} \right) \right\}$. Therefore, the fuzzy rough algebraic TM system is $\{\tilde{0}, \tilde{1}, A, B\}$.

Now let us define a fuzzy rough algebraic $C = \left\{ \left(\frac{a}{0.6} \right), \left(\frac{a}{0.7}, \frac{c}{0.8} \right) \right\}$. The complement $C' = \left\{ \left(\frac{a}{0.3} \right), \left(\frac{a}{0.4}, \frac{c}{1} \right) \right\}$.

The closure of C and C' is obtained as $\mathcal{F}_{\mathcal{RTM}}cl(C) = \left\{ \left(\frac{a}{0.8} \right), \left(\frac{a}{0.8}, \frac{c}{1} \right) \right\}$ and $\mathcal{F}_{\mathcal{RTM}}cl(C') = \left\{ \left(\frac{a}{0.5} \right), \left(\frac{a}{0.7}, \frac{c}{1} \right) \right\}$. Since $C \subseteq \mathcal{F}_{\mathcal{RTM}}cl(C)$, C is fuzzy rough algebraic TM closed but not fuzzy rough algebraic δ closed as $C \neq \mathcal{F}_{\mathcal{RTM}}cl(C)$.

Example 3.2

Consider the fuzzy rough algebraic A and B as defined in Example 2.1.2.

$$\text{Define a fuzzy rough algebraic } D = \left\{ \left(\frac{a}{0.4} \right), \left(\frac{a}{0.5}, \frac{c}{0.3} \right) \right\}.$$

$$\text{Then } \mathcal{F}_{\mathcal{RTM}}cl(D) = \left\{ \left(\frac{a}{0.5} \right), \left(\frac{a}{0.7}, \frac{c}{1} \right) \right\} \text{ and } \mathcal{F}_{\mathcal{RTM}}int(D) = \left\{ \left(\frac{a}{0.3} \right), \left(\frac{a}{0.5}, \frac{c}{0.3} \right) \right\}$$

$$\text{So, } \mathcal{F}_{\mathcal{RTM}}int(\mathcal{F}_{\mathcal{RTM}}cl(D)) = \left\{ \left(\frac{a}{0.3} \right), \left(\frac{a}{0.5}, \frac{c}{0.3} \right) \right\} \text{ and } \mathcal{F}_{\mathcal{RTM}}cl(\mathcal{F}_{\mathcal{RTM}}int(D)) = \left\{ \left(\frac{a}{0.5} \right), \left(\frac{a}{0.7}, \frac{c}{1} \right) \right\}.$$

$$\text{Hence } \mathcal{F}_{\mathcal{RTM}}int(\mathcal{F}_{\mathcal{RTM}}cl(D)) \cap \mathcal{F}_{\mathcal{RTM}}cl(\mathcal{F}_{\mathcal{RTM}}int(D)) = \left\{ \left(\frac{a}{0.3} \right), \left(\frac{a}{0.5}, \frac{c}{0.3} \right) \right\}.$$

Therefore, $D \supseteq \mathcal{F}_{\mathcal{RTM}}int(\mathcal{F}_{\mathcal{RTM}}cl(D)) \cap \mathcal{F}_{\mathcal{RTM}}cl(\mathcal{F}_{\mathcal{RTM}}int(D))$ and so D is fuzzy rough algebraic γ closed. But D is not fuzzy rough algebraic TM closed as $D \neq \mathcal{F}_{\mathcal{RTM}}cl(D)$.

Example 3.3

Consider the fuzzy rough algebraic A and B as defined in Example 2.1.2.

$$\text{Let } E = \left\{ \left(\frac{a}{0.5} \right), \left(\frac{a}{0.8}, \frac{c}{0.3} \right) \right\}.$$

$$\text{Then } \mathcal{F}_{\mathcal{RTM}}cl(E) = \left\{ \left(\frac{a}{0.8} \right), \left(\frac{a}{0.8}, \frac{c}{1} \right) \right\}.$$

$$\Rightarrow \mathcal{F}_{\mathcal{RTM}}int(\mathcal{F}_{\mathcal{RTM}}cl(E)) = \left\{ \left(\frac{a}{0.3} \right), \left(\frac{a}{0.5}, \frac{c}{0.3} \right) \right\}.$$

$$\text{Now } \mathcal{F}_{\mathcal{RTM}}cl(\mathcal{F}_{\mathcal{RTM}}int(\mathcal{F}_{\mathcal{RTM}}cl(E))) = \left\{ \left(\frac{a}{0.5} \right), \left(\frac{a}{0.7}, \frac{c}{1} \right) \right\}.$$

Hence, $E \supseteq \mathcal{F}_{\mathcal{RTM}}cl(\mathcal{F}_{\mathcal{RTM}}int(\mathcal{F}_{\mathcal{RTM}}cl(E)))$ which implies that E is fuzzy rough algebraic α closed and not fuzzy rough algebraic TM closed since $E \neq \mathcal{F}_{\mathcal{RTM}}cl(E)$.

Example 3.4

Define the fuzzy rough algebraic A and B as defined in Example 2.1.2. Let $S = \left\{ \left(\frac{a}{0.2} \right), \left(\frac{a}{0.5}, \frac{c}{1} \right) \right\}$.

$$\text{Then } \mathcal{F}_{\mathcal{RTM}}int(S) = \tilde{0} \text{ and } \mathcal{F}_{\mathcal{RTM}}cl(\mathcal{F}_{\mathcal{RTM}}int(S)) = \tilde{0}.$$

$$\text{Hence } \mathcal{F}_{\mathcal{RTM}}int(\mathcal{F}_{\mathcal{RTM}}cl(\mathcal{F}_{\mathcal{RTM}}int(S))) = \tilde{0},$$

which implies $S \supseteq \mathcal{F}_{\mathcal{RTM}}\text{int}(\mathcal{F}_{\mathcal{RTM}}\text{cl}(\mathcal{F}_{\mathcal{RTM}}\text{int}(S)))$. So S is a fuzzy rough β closed. But $S \neq \mathcal{F}_{\mathcal{RTM}}\text{cl}(S)$. Hence S is not fuzzy rough TM closed.

Example 3.5

Let $A = \left\{ \left(\frac{a}{0.3}, \left(\frac{a}{0.4}, \frac{c}{0.2} \right) \right) \right\}$ and $B = \left\{ \left(\frac{a}{0.2}, \left(\frac{a}{0.5}, \frac{c}{0.3} \right) \right) \right\}$ the sub-Boolean algebra of \mathcal{B} . The complement A' and B' is given by $A' = \left\{ \left(\frac{a}{0.6}, \left(\frac{a}{0.7}, 1 \right) \right) \right\}$, $B' = \left\{ \left(\frac{a}{0.5}, \left(\frac{a}{0.8}, 1 \right) \right) \right\}$. Therefore, the fuzzy rough algebraic TM system is $\{\tilde{0}, \tilde{1}, A, B\}$. Now let us define a fuzzy rough algebraic $C = \left\{ \left(\frac{a}{0.4}, \left(\frac{a}{0.5}, \frac{c}{0.2} \right) \right) \right\}$. The complement of C is defined as $C' = \left\{ \left(\frac{a}{0.5}, \left(\frac{a}{0.6}, 1 \right) \right) \right\}$. Then by computation one can find that $C \supseteq \mathcal{F}_{\mathcal{RTM}}\text{int}(\mathcal{F}_{\mathcal{RTM}}\text{cl}(\mathcal{F}_{\mathcal{RTM}}\text{int}(C)))$. Hence C is fuzzy rough algebraic β closed. But $C \not\supseteq \mathcal{F}_{\mathcal{RTM}}\text{cl}(\mathcal{F}_{\mathcal{RTM}}\text{int}(\mathcal{F}_{\mathcal{RTM}}\text{cl}(C)))$. Hence C is not fuzzy rough algebraic α closed.

Result 3.4

From the Definition 3.2 and 3.9 the following conclusions are made.

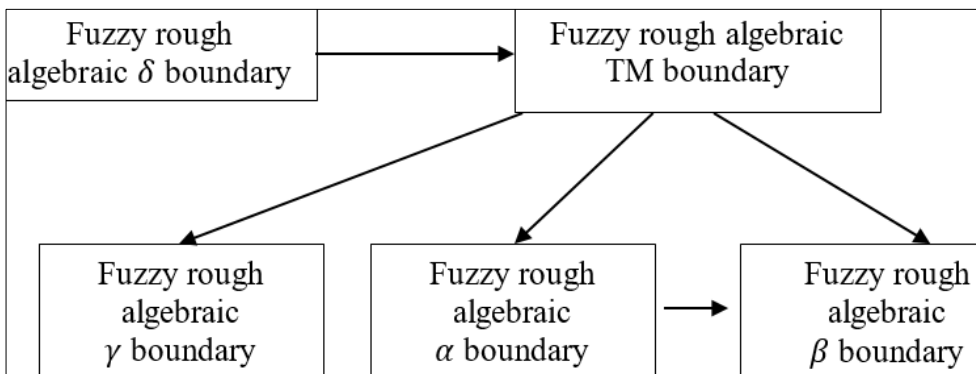


Fig 2: Inter relations between boundary

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