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Availability Analysis and Optimization of a Steam Generating System in a Thermal Power Station

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Abstract
Thermal power stations are an important source of electricity in which thermal energy is converted into mechanical energy which is further converted into electrical energy. Although, there are many functional units in a thermal power station. The present paper is concerned with the main part i.e. Steam-generation part containing gas classifier, crushing mills and boilers working in series. The system is analyzed for availability and optimization. Gas Classifier has units in series, failure of a unit causes complete failure of the system. The crushing mills has two units in operating state, one unit in standby mode and further supported by fuel subsystem which help the system to survive in case of failure of operating units. Boiler is composed of several components in parallel. There may be partial failures but it never fails completely. Taking constant failure and repair rates for each subsystem, the mathematical formulation of the problem has been done using Markov Method. The governing differential equations are solved recursively for a steady state. Table and graph is shown followed by discussion and special cases. Availability optimization is also discussed.

Keywords: Reliability, availability, markov method

Introduction

Competition is the life blood of our present day industrial civilization. This fact is apparent everywhere ranging from user of smallest domestic appliance to those responsible for the management of largest industrial concerns, technological projects or process industries. Panipat Thermal Power Station was installed by Haryana Power Generation Corporation limited (HPGCL) in 1979 for generation of adequate power to feed the factories in the city. At present it has a capacity of 710 MW.

In thermal Power station, thermal energy is first converted into mechanical energy which is further converted into electrical energy. Heat is obtained by burning the coal in the furnace. Firstly, coal is crushed and pulverized and then fed into furnace where it burns to produce heat. Heat thus generated is transferred to boiler tubes which are full of water. After getting heat, the water is converted into steam-water mixture. The liquid thus separated comes back to boiler tubes and the steam goes to super-heater, where it is superheated by heat of gases. The superheated steam from super-heater goes to steam turbine where thermal energy is converted into mechanical energy. The mechanical energy is converted into electrical energy by means of generator coupled with steam turbine.

Before entering into low-pressure section of steam turbine, the steam is reheated by the furnace. The steam leaving the low pressure section is sent to the condenser. In the condenser, the steam is completely converted into water. The water is again sent to boiler. But, before this, the water is passed through regenerative-feed-water-heater and the economizer, so that by the time water reaches the boiler, saturation temperature is attained. The air, before entering the furnace, is pre-heated by gases so that minimum coal is consumed.

For better understanding of the whole plant, we divide it into the following parts:

1. Coal Handling System
2. Steam Generating Section
3. Steam Turbine Section
4. Cooling water system
5. Ash handling system

In thermal power station, the functions of the coal handling system are to unload the coal, to handle coal in storage yards, the preparation of the coal for burning in the furnace. Among the above mentioned functions, the most important process is the preparation of pulverized coal. The coal arriving from colliery by train is unloaded by wagon tipplers. After the unloading, the coal is crushed into uniform size. The crushed coal is stored in bunkers from where it is sent for pulverizing. The coal is fired immediately after the pulverization.

Coal handling plant sends the prepared coal to steam generation section. In this section, coal is burnt and heat thus produced is transferred to water. The unit in which water is contained is called steam-generator. Here water is converted into super-heated steam. This system is comprised of various units like fuel furnace, water walls, boiler drum and tubes, super-heater coils, re heater coils and heat reclaiming devices.

The steam produced in the boiler is sent to the steam turbine where thermal energy is converted into mechanical energy. The steam turbine consists of a rotor having a series of blades on its circumference. The blades are made to withstand the action of steam and the centrifugal force. The rotor is rotated by the steam striking against the blades. As the steam expands in nozzle, it gains high velocity that is converted into mechanical energy. The principle of the turbine depends on Newton’s Second Law of motion. The power in the turbine is obtained by change in momentum of a high velocity jet impinging on a curved blade.

The water required to condense the steam at the exit of turbine is supplied from lake, river or sea. From the upper side of river the cooling water is passed through the condenser and heated water is sent to the lower side of river. But if there is no source of fresh water, the same heated water coming out from condenser is cooled again and again by means of cooling towers.

A large amount of ash is produced in thermal power station. It is about 10-20 percent of total coal burnt. This ash is first quenched and then disposed with the help of ash handling system and used for cement manufacturing.

Although there are many functional units in a thermal power station, here we consider the main part i.e. Steam generation part containing gas classifier, crushing mills and boilers, working in series. The system is analyzed for availability and optimization.

Gas classifier subsystem supplying air to the furnaces has units in series. Failure of a unit in the subsystem causes its failure and hence failure of whole station. The crushing system has two units in operating state and one in standby mode. The system is further supported by a fuel subsystem. In case both the units (Online and standby) of the subsystem fail, then fuel subsystem runs the system. As soon as unit (s) of subsystem is (Are) repaired, the fuel system is replaced by it. The standby in the subsystem and the fuel system are operated upon by two different imperfect switch-over devices. When one unit of subsystem fails and standby unit is available in the system, failed unit is switched out and standby unit is switched in with the help of imperfect switch-over device. Also both the operating units of subsystem may fail causing the system failure, but the system is survived by switching the fuel system through an imperfect switch-over device. The boiling subsystem is composed of several components in parallel. There may be partial failures in the subsystem, but it never fails completely. Failure of unit (s) in subsystem reduces its capacity and hence the efficiency of the plant.

Taking constant failure and repair rates of each subsystem, the mathematical formulation of the problem is done using Markov-Method. The governing differential equations are solved recursively for a steady state. Table, graph is shown followed by discussion and special cases. Availability optimization is also discussed.

Literature Review

Today with increasing use of highly complex systems, increasing automation, the importance of obtaining highly reliable system has been recognized. A brief look back to the historical beginning of a systematic approach to the reliability problem is revealing. In the expansion of the aircraft industry after First World War, the fact that an engine might fail was partially instrumented to the development of multi-engine aircraft. This led, in 1930, to the concept of expressing reliability or unreliability in the form of an average number of failures or as a mean failure rate for aircrafts. A further field of interest in this work was done in Germany during the Second World War. The development started in 1942 and the original concept for this reliability was that ‘a chain cannot be stronger than its weakest link’. After realizing the seriousness of this matter, it was seen that in addition of designing new product, there was a next problem: on of making the product reliable. Since mid 1950, much work had been done on reliability analysis. Availability is the combination of two elements: reliability and maintainability. This means that poor reliability can be offset by improved maintainability. Reliability is a particular case of availability in which no maintenance activity is practiced. Before going in detail, one must have some idea about Markov process. It is based on the assumption that only the last state occupied by the process is relevant in determining the future behavior. This assumption is very strong. If we turn to a process which is no longer strictly Markovian but retain enough of Markovian properties to deserve the name of Semi Markov Process in which transition from one state to another is governed by the transition probabilities of a Markov process but the time spent in each state, before a transition occurs, is a random variable depending upon the last transition made. Thus, at transition instants, the semi Markov process behaves just like a Markov process. Singh (1980) considered the Semi-Markov process generated by the system with imperfect switch over devices. Dhillon and Natesan [1] discussed power system in fluctuating environment.

When the system consists of more than one unit, then there is a chance for complete failure of the system due to single cause. Such failures are termed as common cause failures and these are highly effective as far as reliability analysis of the system is considered. Kumar *et al.* [2, 3] discussed feeding systems in the sugar industry and paper industry. Dayal and Singh [4] studied reliability analysis of a system in a fluctuating environment. Goel and Singh [5, 6] presented reliability analysis of a standby complex system having imperfect switch over device and availability analysis of butter manufacturing system in a dairy station. Mahajan and Singh [7] discussed the reliability analysis of utensils manufacturing station. Habchi [8] discussed an improved method of reliability assessment for suspended tests and Gunes and Deveci [9] have studied the reliability of service systems and its application in student office. While Gupta *et al.* [10] analyzed the reliability and availability of the serial processes in butter-oil processing plant. Sarhan [11] studied the reliability equivalences of a series system consisting of ’n’ Independent and Non-identical components. Bansal and Goel [12] studied the availability analysis of poultry, cattle and fish feed station by taking various probability considerations. Ram and Nagiya [13] discussed the gas turbine power station performance evaluation under key failures by assuming the different types of component failure by using supplementary variable techniques, Laplace transformation and Markov process. Ghamry *et al.* [14] *a*vailability and reliability analysis of a k-Out-of-n Warm Standby System with common-cause failure and fuzzy failure and repair rates by assuming that the failure time of each operating unit or warm standby unit follows Weibull distribution with two fuzzy parameters and the repair time of any failed unit follows exponential distribution with one fuzzy parameter. Each fuzzy parameter is represented by triangular membership function estimated from statistical data taken from random samples of each unit. Saini *et al*. [15] studied the availability and performance analysis of primary treatment unit of sewage station by introducing the concept of redundancy and constant failure rates. Bala [16] studied the effect of switch-over devices on availability of steam generating system. In last many years, several research papers and books have been published that discuss various facts of reliability technology. The concept of reliability can also be applied in other fields using different techniques. In these papers, authors used either Laplace transforms method or Lagrange's method to solve Chapman-Kolmogorov differential equations associated with a particular problem.

It has been observed that reliability and availability of a system in different industries have been discussed so far. This has motivated the author to consider the case of thermal power station, Panipat. Panipat Thermal Power station is a coal fired power station located in Assan village, Panipat [17]. It is owned by Haryana Power Generation Corporation. This Power station consists of eight units. Units 1-4 are known as Panipat I, and Units 5-8 are known as Panipat II. Units 1-4 were 110MW each and commissioned between 1979-1987. They reportedly retired in 2016. Units 5-6 are 210MW each and were commissioned in 1989 and 2001 respectively. Units 7-8 are 250 MW each and commissioned in 2004-2005. Unit 5 was also retired in March 2020. At present, three units (Unit 6-8) of PTPS (Panipat Thermal Power Station) are functional having capacity of 710MW.

System, Notations and Assumptions

The steam generating system of thermal power station, discussed in this paper consists of three subsystems A, B, D and two switch-over devices S_1 and S_2, working as follows.

Subsystem A: (Gas Classifier) provides air to furnace. It consists of components in series. Failure of any component in it causes the complete failure of A and hence the complete failure of the station.

Subsystem B: (Crushing Mills), from where powdered form of the coal is sent to boiler furnace with the help of compressed air consisting of two units in operating state and one in cold standby mode each composed of several components in series. The subsystem B is further supported by a fuel subsystem B_F. If two units of subsystem B fail, then fuel system runs the system. It is assumed that fuel subsystem B_F never fails because it is used only when both the main units fail. As soon as the unit of B are repaired it is switched in and the full subsystem B_F is switched out. This subsystem has units in series. The switches detect and disconnect failed units in B subsystem and engage the standby units to keep the system working, separate repair facilities are available for each subsystem and switchover devices so that there is no waiting time in the system.

Subsystem D: (Boilers), Steam is generated in it. It is composed of several components in parallel. Failure of a unit(s) in D reduces the working capacity of D and hence the efficiency of the station. It is assumed that subsystem D never fails completely.

Switch-over device S_1

It is imperfect. Whenever the unit of subsystem B fails, it is switched out and standby unit if available is switched in by S_1 successfully with probability u. Failure of S_1, when online unit has already failed causes the complete failure of the system.

Switch-over device S_2

It is also imperfect. Whenever two units fail in subsystem B, the fuel subsystem B_F is switched in by Switch-over device $\mathbf{S_2}$ successfully with probability v. Failure of S_2 when online unit (s) as in subsystem B has already failed causes complete failure of the system.

In addition to the notations used for sub-systems, i.e. A; B and D, we have also used the following assumptions and notations:

Assumptions

1. Failures and repairs are System-independent.
2. Separate repair facilities are available for each subsystem and switch over devices.
3. Upon failure, if all repair facilities are busy, the failed unit joins the end of the queue of respective non- operating units.
4. A repaired unit is as good as new and after repair it is immediately reconnected to the system.
5. Nothing can fail when the system is in failed state.
6. System comes in field state if the switch (Es) cannot detect and disconnect a failed unit.
7. Switchover is instantaneous.
8. The repair of a failed unit starts at once.
9. The failure and repair rates of all units are constant.

Notations

A, B, D denote that subsystems are in full operating state.

B_S denotes that subsystem B is working on standby unit.

B_F denotes that subsystem B is working on fuel system when all units in system B have failed.

B_F' denotes that subsystem B is working on fuel system when one unit in subsystem B is still in good state.

$\overline{D}$ denotes that subsystem D is working in reduced-state.

a denotes that system is in failed state.
b_1, b_2, b_3 denote that one, two and three units in subsystem B are in failed state.
α_1 denotes the failure rate of sub-system A from good to failed state.
α_2 denotes the transition rate of the one unit of subsystem B from good to failed state.
α_3 denotes the transition rate of two units of subsystem B from good to failed state.
α_4 denotes the failure rate of subsystem D from good to reduce state.
β_1 denotes the constant transition rate of the subsystem A from failed state to good state.
β_2 denotes the constant transition rate of subsystem B from failed state b_1 to good state.
β_3 denotes the constant transition rate of subsystem B from their failed state to good state.
β_4 denotes the constant transition rate of subsystem D from reduced state to good state.
β_5, β_6 denotes the respective mean constant repair rates of switch-over devices S_1, S_2 from failed states to good states.
u, v denotes the respective probabilities of successful working of switches S_1, S_2 for each failure event.
$\bar{u}$ = 1-u, $\bar{v}$ =1- v
$P_n(t)$ is the probability that the system is in n^{th} state at the time t, $(1 \le n \le 22)$
$P_n = \lim_{t \to \infty} P_n(t)$

Dash (′) denotes the derivative with respect to time t.
Following the above assumptions and notations, the block diagram and transition diagram of the system as shown in the figure 1 and 2 respectively.

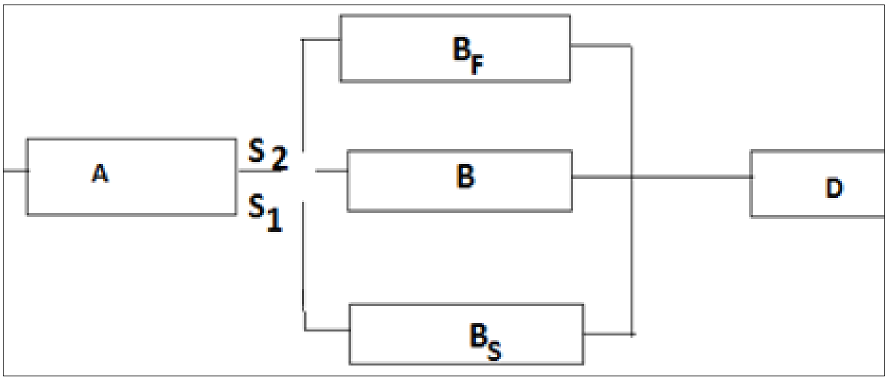


Fig 1: Block diagram of the system

Mathematical Formulation

To determine long run availability of steam generating system, the mathematical formulation of the model is carried out using Markov-Method. In this method, the state of the system is probability based. Probability considerations give the following differential-difference equations associated with transition diagram of the system at time t:

$$P_1'(t) + (u\,\alpha_2 + \bar{u}\,\alpha_2 + \bar{v}\,\alpha_3 + v\,\alpha_3 + \alpha_4 + \alpha_1)\,P_1(t) = u\,\beta_2\,P_2(t) + v\,\beta_3\,P_4(t) + \beta_4\,P_5(t) + \beta_1\,P_{15}(t) \quad (1)$$

$$P_2'(t) + (u\,\beta_2 + v\,\alpha_2 + \bar{v}\,\alpha_2 + v\,\alpha_3 + \bar{v}\,\alpha_3 + \alpha_4 + \alpha_1)\,P_2(t) = u\,\alpha_2\,P_1(t) + v\,\beta_3\,P_3(t) + v\,\beta_2\,P_4(t) + \beta_4\,P_6(t) + \beta_5\,P_{10}(t) + \beta_1\,P_{16}(t) \quad (2)$$

$$P_3'(t) + (v\,\beta_3 + \alpha_4 + \alpha_1)\,P_3(t) = v\,\alpha_3\,P_2(t) + \beta_4\,P_7(t) + \beta_6\,P_9(t) + \beta_1\,P_{17}(t) \quad (3)$$

$$P_4'(t) + (v\,\beta_3 + v\,\beta_2 + \alpha_4 + \alpha_1)\,P_4(t) = v\,\alpha_3\,P_1(t) + v\,\alpha_2 P_2(t) + \beta_4\,P_8(t) + \beta_6\,P_{11}(t) + \beta_1\,P_{18}(t) \quad (4)$$

$$P_5'(t) + (u\,\alpha_2 + \bar{u}\,\alpha_2 + \bar{v}\alpha_3 + v\,\alpha_3 + \beta_4 + \alpha_1)\,P_5(t) = \alpha_4 P_1(t) + u\,\beta_2 P_6(t) + v\,\beta_3 P_8(t) + \beta_1 P_{19}(t) \quad (5)$$

$$P_6'(t) + (u\,\beta_2 + \bar{v}\,\alpha_2 + v\alpha_3 + \bar{v}\alpha_3 + v\alpha_3 + \alpha_1 + \beta_4)\,P_6(t) = \alpha_4\,P_2(t) + u\,\alpha_2\,P_5(t) + v\,\beta_3\,P_7(t) + v\beta_2\,P_8(t) + \beta_5\,P_{14}(t) + \beta_1\,P_{20}(t) \quad (6)$$

$$P_7'(t) + (\beta_4 + v\,\beta_3 + \alpha_1)\,P_7(t) = \alpha_4\,P_3(t) + v\,\alpha_3 P_6(t) + \beta_6\,P_{13}(t) + \beta_1\,P_{21}(t) \quad (7)$$

$$P_8'(t) + (v\,\beta_3 + v\,\beta_2 + \beta_4 + \alpha_1)\,P_8(t) = \alpha_4\,P_4(t) + v\,\alpha_3 P_5(t) + v\,\alpha_2 P_6(t) + \beta_6\,P_{12}(t) + \beta_1\,P_{22}(t) \quad (8)$$

$$\beta_6\,P_{9+i}(t) = \bar{v}\alpha_3 P_{2+i}(t);\ i=0,4 \quad (9)$$

$$\beta_5 P_{10+i}(t) = \bar{u}\alpha_2\,P_{1+i}(t);\ i=0,4 \quad (10)$$

$$\beta_6 P_{11}(t) = \bar{v}\alpha_3\,P_1(t) + \bar{v}\alpha_2\,P_2(t) \quad (11)$$

$$\beta_6 P_{12}(t) = \bar{v}\alpha_3\,P_5(t) + \bar{v}\alpha_2\,P_6(t) \quad (12)$$

$$\beta_1\,P_{14+i}(t) = \alpha_1\,P_i(t);\ 1 \le i \le 8 \quad (13)$$

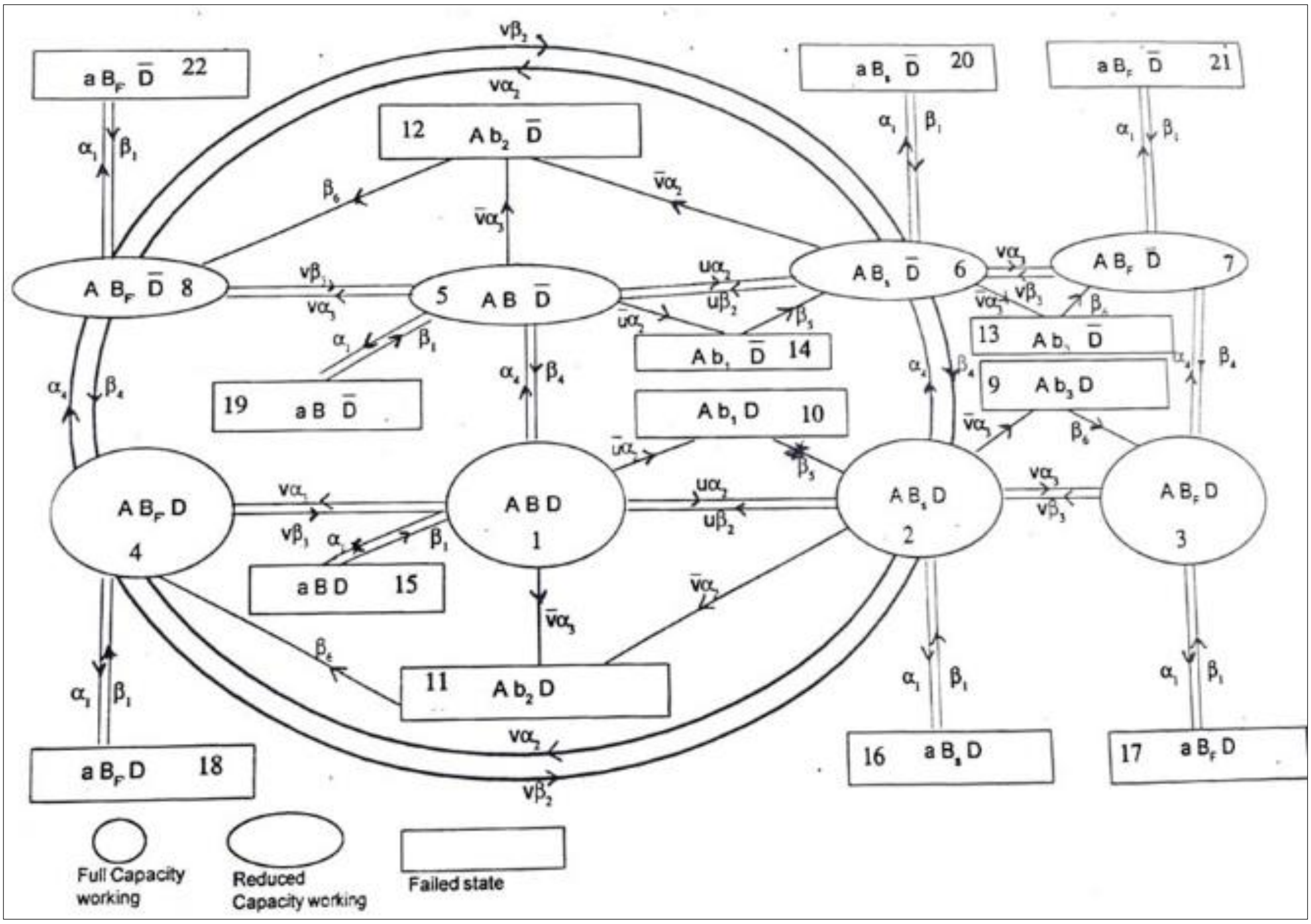


Fig 2: Transition diagram of the steam generating system

With initial conditions
P_1 (0)=1 and 0 otherwise.
The system of differential equations (1) to (13) with above initial conditions have been Solved for steady state.

Steady state
Since management is generally interested in the long run availability of the system, the system is required to run satisfactorily for a long time. We need the steady state probabilities of the system in order to calculate its long run availability. Steady state probabilities of the system are obtained by imposing the following restrictions:

As $t \rightarrow \infty, \frac{dP}{dt} \rightarrow 0$

Putting $\frac{dP}{dt} \rightarrow 0$ in equations (3.1) to (3.13) and solving recursively, we get the steady state probabilities as $p_n = r_n\, p_1;\ 1 \leq n \leq 22$ (14)

Using the normalizing condition $\sum_{n=1}^{n=22} p_n = 1$,
p_1 may be obtained and is evaluated as

$p_1 = [\sum_{i=1}^{i=22} r_i]^{-1}$

The steady state availability (A_v) of the system is given by

$$A_v = \sum_{n=1}^{n=8} p_n = [\sum_{n=1}^{n=8} r_n]\, [\sum_{n=1}^{n=8} r_n]^{-1} \tag{15}$$

Where $k_1 = \beta_4 + v\,\beta_3,\ k_2 = k_1 + v\,\beta_2,$

$l_1 = k_1\, [(\alpha_2 + \alpha_3 + u\,\beta_2 + \beta_4)\, k_2 - \alpha_2 v \beta_2] - v\,\beta_3\, \alpha_2\, k_2,$

$l_2 = k_1\, k_2\, \alpha_4,$

$l_3 = k_2 v \alpha_4\, \beta_4,$

$l_4 = k_1 v \beta_2 \alpha_4,$

$l_5 = k_1 (k_2\alpha_2 + v\,\beta_2\alpha_3,$

$m_1 = k_2 l_1\,\alpha_4,$

$m_2 = (k_2 u\beta_2 + v\,\beta_3\,\alpha_2)\,l_2,$

$m_3 = m_2 l_3\,(l_2)^{-1},$

$m_4 = m_2 l_4 (l_2)^{-1} + vb_3\,\alpha_4 l_1,$

$m_5 = l_1\,[(\alpha_2 + \alpha_3 + \beta_4)\,k_2 - v\,\beta_3\alpha_3] - l_5\,(k_2 u\,\beta_2 + v\,\beta_3\alpha_2),$

$m_6 = [(\alpha_2 + \alpha_4 + \alpha_3 + u\,\beta_2)\,l_1 - \beta_4\,l_2]m_5 - \beta_4\,l_5\,m_2,$

$m_7 = m_5\,l_1\,\alpha_2 + \beta_4\,l_5\,m_1,$

$m_8 = (v\,\beta_3\,l_1 + \beta_4\,l_3)\,m_5 - \beta_4\,l_5\,m_3,$

$m_9 = (v\,\beta_2\,l_1 + \beta_4\,l_4)\,m_5 + \beta_4\,l_5\,m_4,$

$n_1 = m_5\,\alpha_3\,l_1 k_2 + (l_1\,\alpha_3 + \alpha_2\,l_5)\,m_1\,\beta_4,$

$n_2 = m_5\,\alpha_2\,(l_1 k_2 + \beta_4\,l_2) + (l_1\,\alpha_3 + \alpha_2\,l_5)\,m_2\,\beta_4,$

$n_3 = \beta_4[m_5\,\alpha_2\,l_3 k_2 + (l_1\,\alpha_3 + \alpha_2\,l_5)\,m_3],$

$n_4 = m_5[l_1 k_2(\alpha_4 + v\,\beta_2 + v\,\beta_3) - \alpha_4\,\beta_4\,l_1 - \alpha_2\,\beta_4\,l_4] - (l_1\,\alpha_3 + \alpha_2\,l_5)\,m_4\,\beta_4,$

$n_5 = m_5\,l_1\,[k_1(\alpha_4 + v\,\beta_3) - \alpha_4\,\beta_4] - \alpha_3\,\beta_4\,(m_5\,l_3 + m_3\,l_5),$

$n_6 = m_1\,l_5\,\alpha_3\,\beta_4,$

$n_7 = m_5\,\alpha_3\,(l_1 k_1 + \beta_4\,l_2) + l_5\,m_1\,\alpha_3\,\beta_4,$

$n_8 = \alpha_3\,\beta_4\,(l_4 m_5 + l_5\,m_4),$

$n_9 = n_4 n_5 - n_8\,n_3,$

$n_{10} = m_8 n_4 + m_9\,n_3,$

$n_{11} = n_4 n_6 + n_8\,n_1,$

$n_{12} = m_6 n_4 - m_9\,n_2,$

$n_{13} = n_4 n_7 + n_8\,n_2,$

$r_1 = 1,$

$r_2 = [(n_4 m_7 + m_9\,n_1)\,n_9 + n_{10}\,n_{11}][n_9 n_{12} - n_{13} n_{10}]^{-1},$

$r_3 = [(n_{11} + n_{13}\,r_2)\,(n_9)^{-1},$

$r_4 = [(n_1 + n_2\,r_2 + n_3\,r_3)\,(n_4)^{-1},$

$r_5 = [(m_1 + m_2\,r_2 + m_3\,r_3 + m_4\,r_4)\,(m_5)^{-1},$

$r_6 = [(l_2 r_2 + l_3\,r_3 + l_4\,r_4 + l_5\,r_5)\,(l_1)^{-1},$

$r_7 = (\alpha_3\,r_3 + \alpha_3\,r_6)\,k_2^{-1},$

$r_8 = (\alpha_4\,r_4 + \alpha_3\,r_5 + \alpha_2\,r_6)\,k_1^{-1},$

$r_9 = r_2\bar{v}\alpha_3\,\beta_6^{-1},$
$r_{10} = \bar{u}\alpha_2\,\beta_5^{-1},$

$r_{11} = \bar{v}\ (\alpha_2\ r_2 + \alpha_3)\ \beta_6^{-1}$,

$r_{12} = \bar{v}\ (\alpha_2\ r_6 + \alpha_3 r_5)\ \beta_6^{-1}$,

$r_{13} = \bar{v}\ \alpha_3\ r_6\ \beta_6^{-1}$,

$r_{14} = \bar{u}\ \alpha_2\ r_5\ \beta_5^{-1}$,
$r_{15} = \alpha_1\ \beta_1^{-1}$,

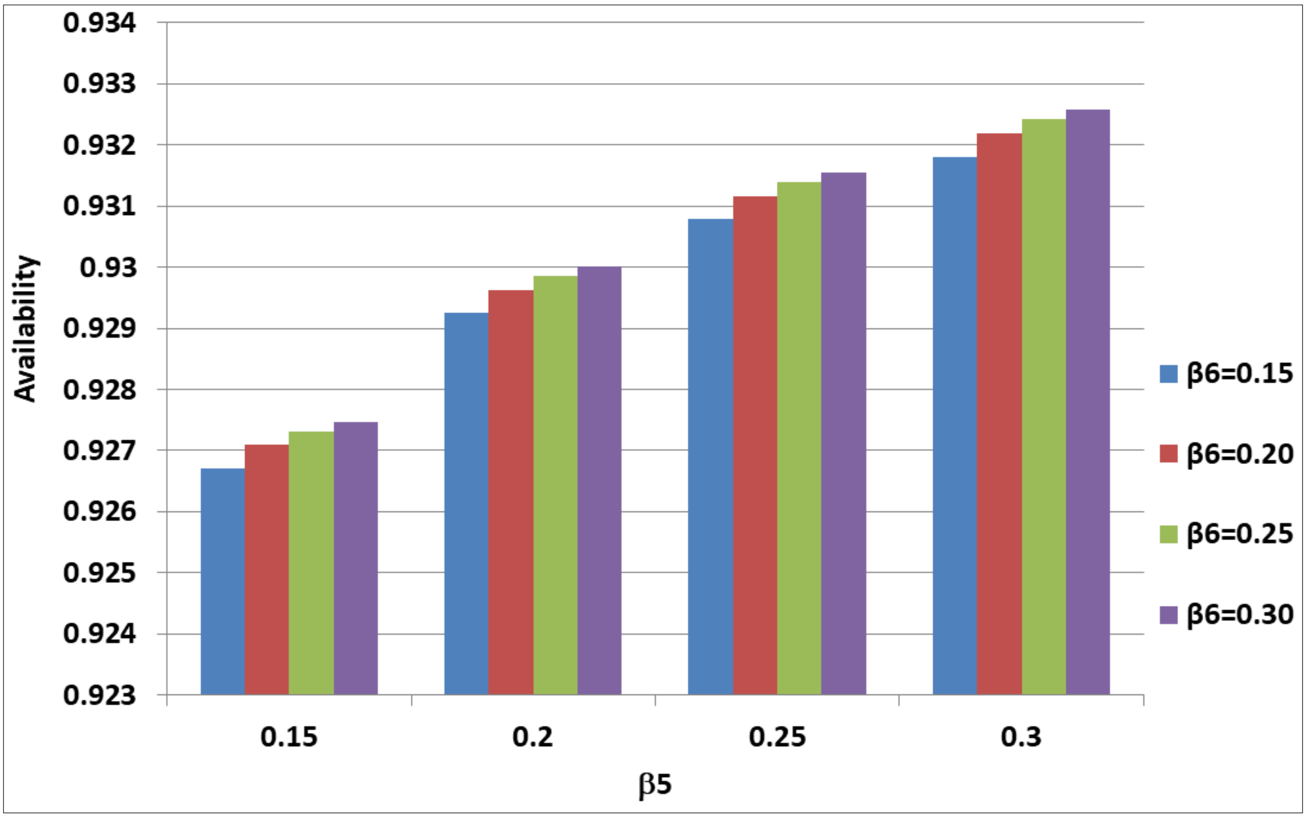
$r_{14+i} = r_{15}\ r_i;\ 2 \leq i \leq 8$

Numerical Illustration
To study the effect of switch -over device over the availability, we evaluate availability of the system by taking u = v = 0.9, $\alpha_1 = \alpha_2 = 0.02$, $\alpha_4 = 0.01$, $\alpha_3 = 0.001$, $\beta_2 = \beta_4 = 0.2$, $\beta_1 = 0.3$, $\beta_3 = 0.15$

The Availability Table and graph is given below:

Table 1: Availability Table

$\beta_5\ \beta_6$	0.15	0.20	0.25	0.30
0.15	0.92671533	0.929253191	0.930782538	0.93180489
0.20	0.92709225	0.92963207	0.93116266	0.93218585
0.25	0.92731848	0.92985955	0.93139089	0.93241458
0.30	0.92746937	0.93001127	0.93154310	0.93257265



Graph 1: Availability Graph

Special Cases
Case 1. Non Reparable from failed state
When there is no repair from failed state, then probability considerations give the following steady-state difference equations associated with the transition diagram of the system.

$(u\ \alpha_2 + \bar{u}\ \alpha_2 + \bar{v}\ \alpha_3 + v\ \alpha_3 + \alpha_4 + \alpha_1)\ p_1 = u\ \beta_2\ p_2 + v\ \beta_3\ p_4 + \beta_4\ p_5$ (16)

$(v\ \alpha_2 + \bar{v}\ \alpha_2 + \bar{v}\ \alpha_3 + v\ \alpha_3 + u\ \beta_2 + \alpha_4 + \alpha_1)\ p_2 = u\ \alpha_2\ p_1 + v\ \beta_3\ p_3 + p_4\ v\ \beta 2 + \beta_4\ p_6$ (17)

$(\alpha_1 + \alpha_4 + v\ \beta_{3)}\ p = v\ \alpha_3\ p_2 + \beta_4\ p_7$ (18)
$(\alpha_1 + \alpha_4 + v\ \beta_2 + v\ \beta_{3)}\ p_4 = v\ \alpha_3\ p_1 + v\ \alpha_2\ p_2 + \beta_4\ p_8$ (19)

$(u\,\alpha_2 + \bar{u}\,\alpha_2 + \bar{v}\,\alpha_3 + v\,\alpha_3 + \beta_4 + \alpha_1)\,p_5 = \alpha_4\,p_1 + u\,\beta_2\,p_6 + v\,\beta_3\,p_8$ (20)

$(\alpha_1 + v\,\alpha_3 + \bar{v}\,\alpha_3 + \bar{u}\,\alpha_3 + u\,\alpha_2 + \beta_4)\,p_6 = \alpha_4\,p_2 + u\,\alpha_2\,p_5 + v\,\beta_3\,p_7 + v\,\beta_2\,p_8$ (21)

$(\alpha_1 + v\,\beta_3 + \beta_4)\,p_7 = \alpha_4\,p_3 + v\,\alpha_3\,p_6$ (22)

$(\alpha_1 + v\,\beta_2 + v\,\beta_3 + \beta_4)\,p_8 = \alpha_4\,p_3 + v\,\alpha_3\,p_5 + v\,\alpha_2\,p_6$ (23)

$p_{9+i} = \bar{v}\,\alpha_3\,p_{2+i}$, i = 0, 4 (24)

$p_{10+i} = \bar{u}\,\alpha_2\,p_{1+i}$, i = 0, 4 (25)

$p_{11} = \bar{v}\,\alpha_3\,p_1 + \bar{v}\,\alpha_2\,p_2$ (26)

$p_{12} = \bar{v}\,\alpha_3\,p_5 + \bar{v}\,\alpha_2\,p_6$ (27)

$p_{15+i} = \alpha_1\,p_{1+i};\ 0 \le i \le 7$ (28)

Solving equations (4.1) to (4.13) recursively, we get various steady -state probabilities as

$p_n = s_n\,p_1,\ 1 \le n \le 22$ (29)

Where,

$a_1 = u\,\alpha_2 + \bar{u}\,\alpha_2 + \bar{v}\,\alpha_3 + v\,\alpha_3 + \alpha_4 + \alpha_1,$

$a_2 = v\,\alpha_2 + \bar{v}\,\alpha_2 + \bar{v}\,\alpha_3 + v\,\alpha_3 + u\,\beta_2 + \alpha_4 + \alpha_1,$

$a_3 = \alpha_1 + \alpha_4 + v\,\beta_3,$

$a_4 = \alpha_1 + \alpha_4 + v\,\beta_2 + v\,\beta_3,$

$a_5 = u\,\alpha_2 + \bar{u}\,\alpha_2 + \bar{v}\,\alpha_3 + v\,\alpha_3 + \beta_4 + \alpha_1,$

$a_6 = \alpha_1 + v\,\alpha_3 + \bar{v}\,\alpha_3 + \bar{u}\,\alpha_3 + u\,\alpha_2 + \beta_4,$

$a_7 = \alpha_1 + v\,\beta_3 + \beta_4,$

$a_8 = \alpha_1 + v\,\beta_2 + v\,\beta_3 + \beta_4,$

$a_9 = a_7\,(a_6\,a_8 - v^2\,\alpha_2\,\beta_2) - v^2\,\alpha_2\,\beta_2\,a_8,$

$e_1 = (a_5\,a_8 - v^2\,\alpha_3\,\beta_3)\,a_9 - (u\,\beta_2\,a_8 + v^2\,\alpha_3\,\beta_3)\,(u\,\alpha_2\,a_8 + v^2\,\alpha_3\,\beta_2)\,a_7,$

$e_2 = (a_9 - \alpha_4\,\beta_4\,a_7\,a_8)\,e_1 - (a_7)^2\,\alpha_4\,\beta_4\,a_8\,(u\,\alpha_2\,a_8 + v^2\,\alpha_3\,\beta_2)\,(u\,\beta_2\,a_8 + v^2\,\alpha_3\,\beta_3),$

$e_3 = u\,\alpha_2\,\alpha_9\,e_1 + \alpha_4\,\beta_4\,a_7\,a_9\,(u\,\alpha_2\,a_8 + v^2\,\alpha_3\,\beta_2),$

$e_4 = v\,\beta_3\,[e_1\,(a_9 + \alpha_4\,\beta_4\,a_8) + a_7\,\alpha_4\,\beta_4\,a_8\,(u\,\alpha_2\,a_8 + v^2\,\alpha_3\,\beta_2)\,(u\,\beta_2\,a_8 + v^2\,\alpha_3\,\beta_3)],$

$e_5 = v\,\beta_3\,e_1\,(a_9 + \alpha_4\,\beta_4\,a_7) + a_7\,v\,\alpha_4\,\beta_4\,(u\,\alpha_2\,a_8 + v^2\,\alpha_3\,\beta_2)\,[\beta_3\,a_9 + (u\,\beta_2\,a_8 + v^2\,\alpha_3\,\beta_3)\,\beta_2\,a_7],$

$e_6 = [a_9\,(a_3 a_7 - \alpha_4\,\beta_4) - v^2\,\alpha_3\,\alpha_4\,\beta_3\,\beta_4 a_7\,a_8,$

$e_7 = v\,\alpha_3 \alpha_4\,\beta_4\,a_7\,a_8\,a_9\,(u\,\alpha_2\,a_8 + v^2\,\alpha_3\,\beta_2),$

$e_8 = v\,a_7\,[e_1\,(\alpha_2\,a_9 + \alpha_3\,\alpha_4\,\beta_4\,a_8) + \alpha_3\,\alpha_4\,\beta_4\,a_7\,a_8\,(u\,\alpha_2\,a_8 + v^2\,\alpha_3\,\beta_2)\,(u\,\beta_2\,a_8 + v^2\,\alpha_3\,\beta_3)],$

$e_9 = v^2\,\alpha_3\,\alpha_4\,\beta_2\,a_7\,[\beta_4\,e_1 + (u\,\alpha_2\,a_8 + v^2\,\alpha_3\,\beta_2)\,\{\beta_3\,a_9 + (u\,\beta_2\,a_8 + v^2\,\alpha_3\,\beta_3)\,\beta_3\,a_7\}],$

$f_1 = [(a_4 a_9 - \alpha_4\,\beta_4)\,a_9 - v^2\,\alpha_2\,\alpha_4\,\beta_2\,a_7]\,e_1 - v^2\,\alpha_4\,\beta_4\,[\alpha_3\,a_9 + a_7\,\beta_4\,(u\,\alpha_2\,a_8 + v^2\,\alpha_3\,\beta_2)]\,[\beta_3\,a_9 + (u\,\beta_2\,a_8 + v^2\,\alpha_3\,\beta_3)\,\beta_2\,a_7],$

$f_2 = v\,a_8\,a_9\,[\alpha_3\,e_1 + \alpha_4\,\beta_4\,\{\alpha_3\,a_9 + a_7\,\beta_4\,(u\,\alpha_2\,a_8 + v^2\,\alpha_3\,\beta_2)\}],$

$f_3 = v\,a_8\,(u\,\beta_2\,a_8 + v^2\,\alpha_3\,\beta_3)\,[\alpha_2\,e_1(a_9 + \beta_4\,a_4\,a_7) + \alpha_4\,\beta_4\,a_7\,\{\alpha_3\,a_9 + a_7\,\beta_4\,(u\,\alpha_2\,a_8 + v^2\alpha_3\,\beta_4)\}],$

$f_4 = v^2 a_8 \beta_3 \alpha_4 \beta_4 [\alpha_2 e_1 + (u \beta_2 a_8 + v^2 \alpha_3 \beta_3) + \{\alpha_3 a_9 + a_7 \beta_4 (u \alpha_2 a_8 + v^2 \alpha_3 \beta_4)\}]$,

$s_1 = 1$,

$s_2 = [(e_6f_1 - e_9f_4)(e_3f_1 + e_5f_2) + (e_4f_1 + e_5f_4)(e_7f_1 + e_9f_2)] [(e_6f_1 - e_9f_4)(e_2f_1 - e_5f_3) - (e_4f_1 + e_5f_4)(e_8f_1 + e_9f_3)]$,

$s_3 = [(e_7f_1 + e_9f_2) + (e_8f_1 + e_9f_3) s_2] (e_6f_1 - e_9f_4)^{-1}$,

$s_4 = (f_2 + f_3 s_2 + f_4 s_3) f_1^{-1}$,

$s_5 = \alpha_4 [a_8 a_9 + a_7 a_8 (u \beta_2 a_8 + v^2 \alpha_3 \beta_3) s_2 + (u \beta_2 a_8 + v^2 \alpha_3 \beta_3) v \beta_3 a_8 s_3 + v \alpha_4 (\beta_3 a_9 + (u \beta_2 a_8 + v^2 \alpha_3 \beta_3) \beta_3 a_7) s_4]$,

$s_6 = [\alpha_4 a_7 a_8 s_2 + v \beta_2 \beta_4 a_7 s_4 + v \alpha_4 \beta_3 a_8 s_3 + a_7 s_5 (u \alpha_2 a_8 + v^2 \alpha_3 \beta_2)] a_9^{-1}$,
$s_7 = (\alpha_4 s_3 + v \alpha_3 s_6) a_7^{-1}$,

$s_8 = (\alpha_4 s_4 + v \alpha_3 s_5 + v \alpha_3 s_6) a_8^{-1}$,

$s_9 = \bar{v} \alpha_3 s_2$,

$s_{10} = \bar{u} \alpha_2$,

$s_{11} = \bar{v} (\alpha_3 + \alpha_2 s_2)$,

$s_{12} = \bar{v} (\alpha_3 s_5 + \alpha_2 s_6)$,

$s_{13} = \bar{v} \alpha_2 s_5$,

$s_{14} = \bar{u} \alpha_2 s_5$,

$s_{15+i} = \alpha_1 s_{1+i}, 0 \le i \le 7$.

Using normalizing condition

$\sum_{n=1}^{n=22} p_n = 1$,

p_1 may be obtained and is evaluated as
$p_1 = [\sum_{i=1}^{i=22} s_i]^{-1}$

The steady state availability (A_v) of the system is given by

$$A_v = \sum_{n=1}^{n=8} p_n = [\sum_{n=1}^{n=8} s_n] [\sum_{n=1}^{n=22} s_n]^{-1} \quad (30)$$

Case 2: Perfect Switch-over Devices
When the switch-over device is perfect, the results are obtained by taking u=v=1 in the forgoing analysis. By taking $\alpha_1 = \alpha_2 = 0.02$, $\alpha_4 = 0.01$, $\alpha_3 = 0.001$, $\beta_2 = \beta_4 = 0.2$, $\beta_1 = 0.3$, $\beta_3 = 0.15$
Availability of the system is evaluated to be 0.938493205.

Optimum Availability
When repair rates of switches are not given**.** Then on differentiating A_v with respect to β_5 and β_6 and using the condition of maxima, then the optimum availability of the system will be given by:

$Av = \frac{p\beta 5\beta 6}{q\beta 5\beta 6 + r\beta 5 + S\beta 6}$, where p, q, r, s are

$p = \sum_{i=1}^{i=8} r_i$,

$q = \sum_{i=15}^{i=22} r_i$,

$r = v [\alpha_3 (1 + r_2 + r_5 + r_6) + \alpha_2 (r_2 + r_6)]$,

$s = u \alpha_2 (1 + r_5)$.

Analysis of results
Analysis of availability and optimization of steam generating system in thermal power station can help in increasing the production of electricity. Study of above Availability table and graph reveals that β_5(mean constant repair rates of switch-over

devices S_1 from failed states to good states) increases the availability of the system more effectively than β_6 (mean constant repair rates of switch-over devices S_2 from failed states to good states). So, switch S_1 requires utmost care as compared to switch S_2. To obtain the value of β_5 and β_6 for optimum value of Av, we can fix the target of maximum availability under prevailing conditions of the system and financial constraints. Similarly, other repair/failure rates (instead of β_5 and β_6) may be evaluated for the maximum target of availability. Thus, a set of values is chosen and maintained for most economic gain. Similar comparative tables and graphs can be prepared by taking repair/failure rates for various components. As controlling the failure of subsystems/units is more difficult than controlling the repair. Table for repair rates provides good information about the effectiveness of the system components. Since, in practice, the management is always interested in long run availability, the system is analyzed for the same. Hence, it is suggested that management should try to keep the switches good to increase the overall performance of the station.

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