International Journal of Statistics and Applied Mathematics 2024; 9(1): 139-144

International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452
Maths 2024; 9(1): 139-144

https://www.mathsjournal.com
Received: 28-12-2023
Accepted: 27-01-2024

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The impact of graph theory and geometry based on some topics to develop critical thinking of secondary school mathematics' students: A case of Mbulu district

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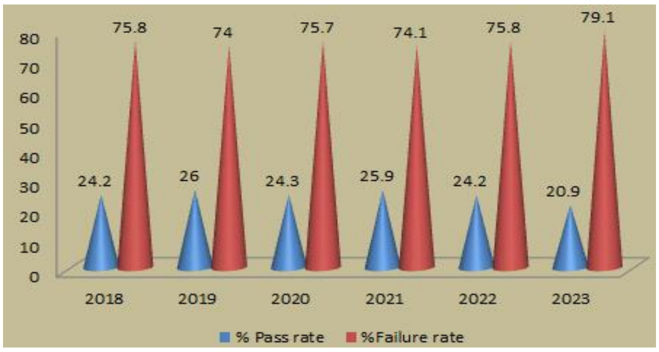
DOI: https://doi.org/10.22271/maths.2024.v9.i1b.1624

Abstract
This paper provides several mathematical classroom tasks based on the ideas of graph theory to encourage critical thinking in secondary school students. It is a theoretical investigation into the teaching of mathematics. Some graph theory concepts, including Complete graph, cycle, wheels, Shortest path, planar graph, tree, vertex and edge coloring, and Solid Geometry, can be used in secondary school to boost students' critical thinking abilities. The students should have fun doing these activities because the teacher may use them as a game in the classroom.

Keywords: Graph theory, geometry, critical thinking, secondary school students

Introduction
In Tanzania, many secondary school students think that mathematics is a difficult subject. Their Form Four examination results from the four years prior, particularly from Mbulu District, clearly demonstrate this. Moreover, a large number of students fail their form four national exams due to the topics' insufficient foundation (Mazana, Montero, & Casmir, 2020b) [7]. Courses like geometry and graph theory are essential to help students strengthen their reading, writing, and critical thinking skills. They need to learn geometry and graph theory in order to become more proficient in logical thinking and problem-solving (Mert Uyangör, 2019) [8]. Students who study these subjects will be more capable of solving problems, identifying patterns, and analyzing complicated problems. These problem-solving skills can benefit students in both their academic and professional activities and are valuable in many aspects of life. Students who study geometry comprehend shapes and spatial relationships better. (Fuys & Liebov, 1997) [4]. This knowledge is helpful in disciplines where spatial awareness is essential, such as architecture, engineering, and design. Students who study geometry improve their visualization skills and get a stronger understanding of concepts like area, volume, and symmetry, all of which are essential in a variety of real-world applications. The study of abstract networks and linkages is known as graph theory. (Pavlopoulos *et al.*, 2011) [10]. When teaching graph theory to secondary students, the goal is to foster critical and analytical thinking about connections and structures. Students learn about pattern recognition, graph analysis, and evidence-based decision making. This fosters the development of critical thinking skills and a deeper understanding of mathematical concepts, two things that are essential for success in both the workforce and postsecondary education. Geometry and graph theory are taught in STEM (Science, Technology, Engineering, and Mathematics) foundational courses (Wu & Rau, 2019). Introducing these topics to high school students could inspire them to work in STEM fields. In graph theory and geometry, students get a strong mathematical foundation that prepares them for further study in disciplines like computer science, physics, and engineering. Ultimately, this can help Tanzania's efforts to generate a labor force capable of driving technological innovation and economic progress. In Mbulu district, the performance has dropped a lot even though there are enough teachers and learning resources for the relevant subject. The results of the mathematics subject for 12 schools go down from year to year as shown in the figure below.



Source; Necta (CSEE) results from 2018 to 2023 statistics.

Fig 1: National form four Mathematics results in percentage for the 12 schools from Mbulu district

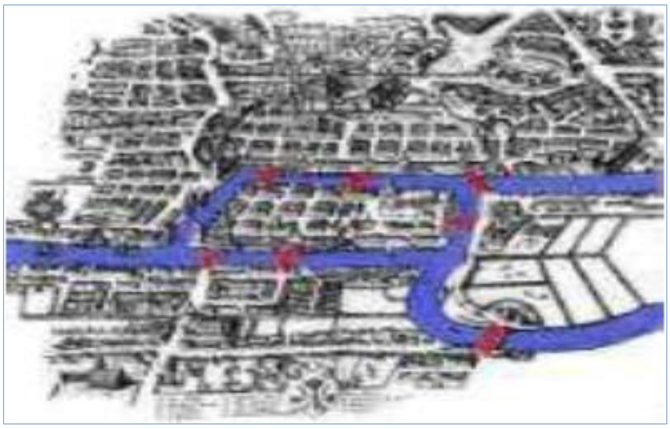
Related Work

Tanzanian secondary school does not include graph theory instruction. Smithers (2005) [1] conducted research for her master's thesis and discovered that the United States' Discrete Mathematics with Statistics and Probability course only covers a little bit of graph theory. Graph theory can be used as an activity to assist students strengthens their critical thinking skills, even if it hasn't been taught in the classroom. We won't have to spend too much time explaining graph theory to them, even though the students may become a little confused. Assume that fundamental instructions are provided in advance to students in secondary school. Then, without recognizing they are addressing graph theory problems, they may be instructed to finish activities based on those notions. Some graph theory concepts, like Complete graph, cycle, wheels, Shortest path, planar graph, tree, vertex and edge coloring can be utilized as teaching aids to support students' critical thinking abilities. In addition to reading, analyzing, leadership, and social skills, critical thinking should be taught to kids through a range of experiences, especially in the classroom. According to a recent guideline from Tanzania's Department of Education, one of the qualities that students should acquire through mathematics in the classroom is a critical attitude that they build during their learning process. Although secondary schools do not teach graph theory, this paper offers a number of classroom activities based on specific topics in graph theory that can be used to achieve that aim. The learner's ability to think critically and use all of the knowledge at their disposal to solve problems is the most important skill to have (Chukwuyenum, 2013) [2].

In this paper, we first review some basic principles linked to critical thinking. We will then review some basic concepts in graph theory that are needed for the in-class assignment. The main focus of this work is on a series of hands on teaching exercises based on the concepts of graph theory, such as Complete graph, cycle, wheels, Shortest path, planar graph, tree, vertex and edge coloring, and Solid Geometry. Most real-world issues are complicated and call for specialized knowledge to solve. We need to make an initial observation to collect the relevant data before we can begin to solve an issue. Once we have some useful information, we need to choose a tool that will assist us solve the problem. Sometimes a few trial and error sessions are necessary to choose the ideal tool for the task. One of the abilities needed for this kind of approach is critical thinking. In real life, most problems are difficult and require specialized knowledge to solve. To acquire the required information, we must first make an initial observation before beginning to solve a problem. We need to choose a tool that will help us solve the problem after gathering some useful information. Sometimes figuring out which tool is ideal for a given task requires some trial and error. For this kind of approach, critical thinking is one of the necessary talents. Dinuță (2015) [3] are researchers who have examined the relationship between critical thinking and mathematics in the classroom. Furthermore, he proposes that a curriculum that trains math teachers for secondary education include critical thinking. As a result, math performance among students will increase. Dinuță conducted research on the critical thinking skills required by children to comprehend geometric concepts. She asserts that "Learning fosters the growth of critical thinking, which is instructional strategies and methods to highlight the importance of understanding each geometric idea. (Dinuță, 2015) [3].

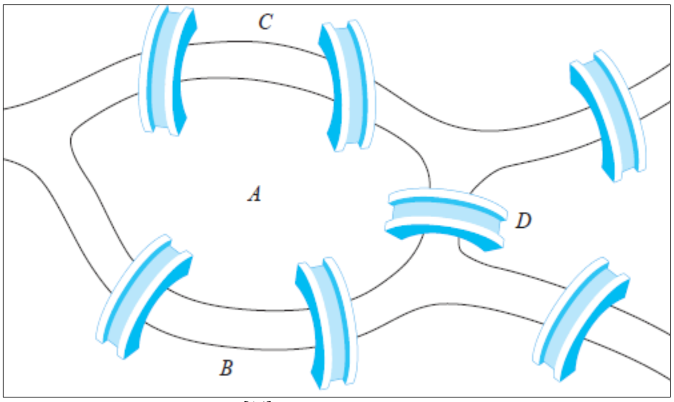
Graph Theory

Graph theory studies structures that have vertices and edges linking them Rosen, K. (2011) [13]. The earliest inspiration for graph theory came from mathematician Euler's Königsberg bridge issue (1736), which he later recognized as an Eulerian graph. Around the same time, Gustav Kirchhoff developed the idea of a tree, which is a connected graph free of cycles. The calculation of currents in electrical networks and circuits as well as the counting of chemical molecules were then done using this concept. The Königsberg city map from Euler's time is depicted in Figure 1 (Sachs, Stiebitz, & Wilson, 1988). As illustrated in Figure 2, we reduce the complexity of the map by focusing only on the pertinent regions, rivers, and bridges. Is it possible for us to start from any of the four land areas, go across each bridge precisely once, and then return to our starting point?



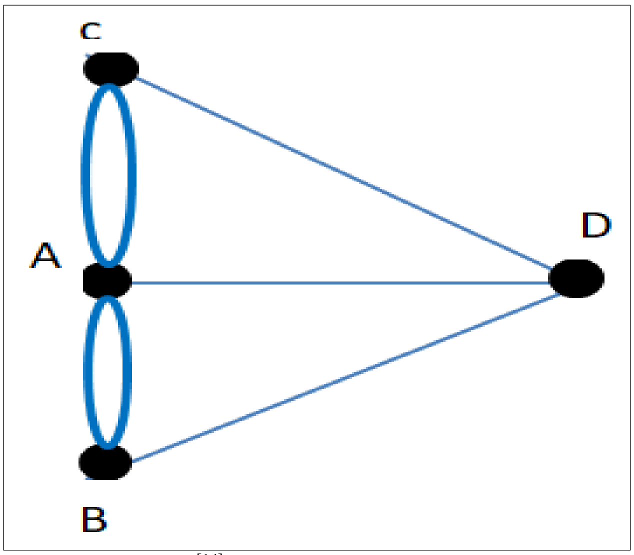
Source (Sachs, Stiebitz, & Wilson, 1988)

Fig 1: A city map of Königsberg during Euler's time



Source (Rosen, 2007) [14]

Fig 2: The Seven Bridges of Königsberg



Source (Rosen, 2007) [14]

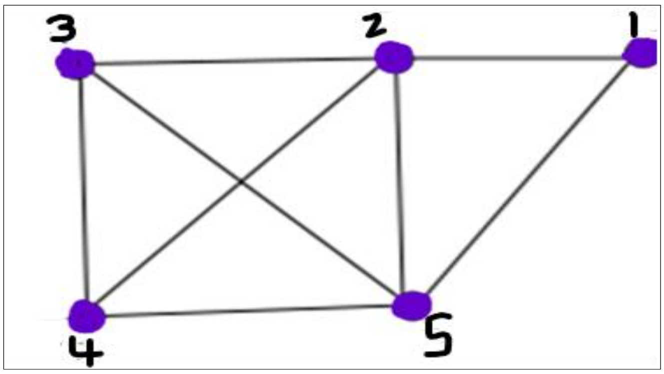
Fig 3: Multigraph Model of the Town of Königsberg.

Every piece of land was swapped out for a point, and every bridge for a line that connected the matching points. This means that, as Figure 3 shows, we have a graph. Euler asserts that the issue is geometric; nevertheless, the geometry he refers to is not the same as the geometry known to people in his day, which involves calculations and measurements. The geometry Leibniz mentioned was known as the "Geometry of Position". A graph is composed of a set of edges and a (finite) nonempty collection of vertices, or nodes. Each edge is connected to one or more vertices, also known as endpoints. There is an edge connecting its endpoints (Rosen, 2019a) [12].

Definition 1: Two vertices u and v in an undirected graph G are called adjacent (or neighbors) in G if u and v are endpoints of an edge e of G. Such an edge e is called incident with the vertices u and v and e is said to connect u and v (Rosen, 2007) [14].

Definition 2: The set of all neighbors of a vertex v of $\mathrm{G} = (\mathrm{V}, \mathrm{E})$, denoted by N(v), is called the neighborhood of v. If A is a subset of V, we denote by N(A) the set of all vertices in G that are adjacent to at least one vertex in A. So, N(A) = v ∈ A N(v) (Rosen, 2007) [14]

Theorem: Determine the neighborhoods of the vertices in the graph G displayed in Figure 4 below.



Source: Researcher's work

Fig 4: Simple undirected graph

Solution.

The neighborhoods of these vertices are

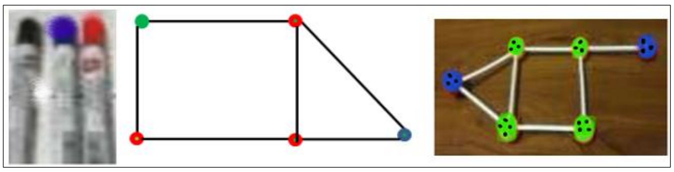
$N(1) = \{2,5\}, N(2) = \{1,3,4,5\}, N(3) = \{2,4,5\}, N(4) = \{2,3,5\}, N(5) = \{1,2,3,4\}$.

The graph is given in the figure. 3, the set of vertices is $V = \{1,2,3,4,5\}$ and the set of edges are

$E = \{\{1,2\},\{2,3\},\{1,5\},\{5,4\},\{2,5\},\{5,3\},\{2,4\},\{3,4\}\}$.

Methodology

This work is a theoretical investigation that offers lesson plans based on geometry and graph theory ideas**.** Before moving on to specific graph theory principles that we may apply to in class assignments, let's review some of the teaching tools for this session. It doesn't have to be expensive to teach graph theory exercises. students only need basic drawing equipment to create small circles and lines for this practice. Getting some readily available school supplies, like paper and pencils, colored pencils and paper, a whiteboard and marker, chalk, and a chalkboard, should be our first priority. We show vertices as little circles and edges as lines in a particular network (Capobianco & Molluzzo, 1978) [1]. If accessible, sticks and small colored balls can be utilized to symbolize vertices and edges. Any object that can represent vertices and edges can be used. It is not required to discuss graph theory principles in great detail in our experiment. Instead, all we need to do is provide a English description of the instructions for the task. The graph theory tasks that follow can be utilized in courses for secondary mathematics.



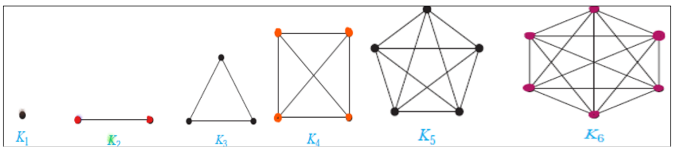
Source; Researcher's work

Fig 5: A variety of materials are employed to display a graph.

Discussions

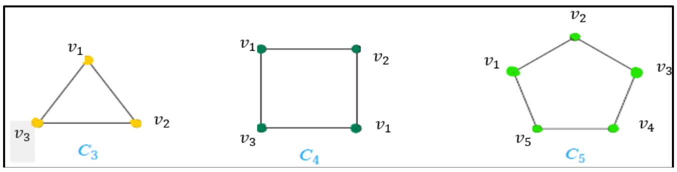
Complete Graphs

A simple graph with exactly one edge connecting each pair of different vertices is called a complete graph on n vertices, or K_n for short. Figure below shows the graphs K_n for n = 1, 2, 3, 4, 5, and 6. A Simple graph is considered non-complete if it contains at least one pair of unique vertices that are not connected by an edge.



Cycles

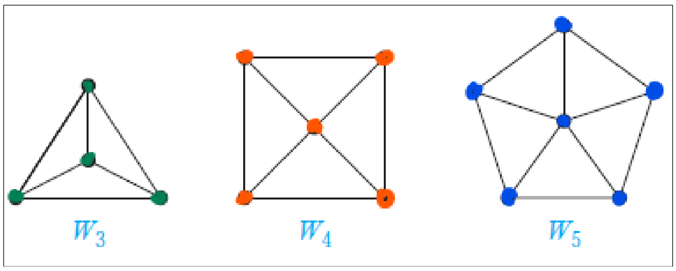
A **cycle** ***Cn***, $n \geq 3$, contains of n vertices $v_1, v_2, \ldots, v_n$ and edges $\{v_1, v_2\}, \{v_2, v_3\}, \ldots, \{v_{n-1}, v_n\}$, and $\{v_n, v_1\}$. The cycles C_3, C_4, and C_5 are shown in Figure below.



Source; Researcher's work

Wheels

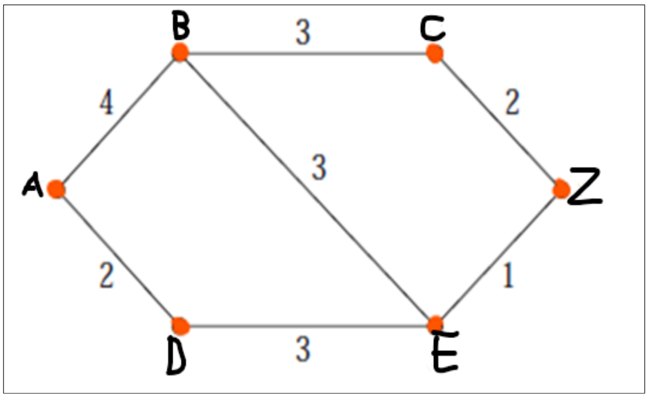
A wheel w_n is produced when a new vertex is added to a cycle C_n, for $n \geq 3$, and connected to each of the n vertices in C_n by new edges. The wheels w_3, w_4, and w_5 are displayed in Figure below.



Source: Field work

A Shortest-Path

One important difficulty with weighted graphs is to find the circuit with the least total length that touches every vertex of the entire graph exactly once. This is the well-known "traveling salesperson" problem, where the goal is to find an itinerary that minimizes the number of miles traveled by requiring a salesperson to visit each city exactly once. We'll talk about the traveling salesperson problem later in this section. There are several methods for determining the shortest path between two vertices in a weighted network.We will present a greedy algorithm that was devised in 1959 by the Dutch mathematician Edsger Dijkstra. The variation we shall show solves this problem in undirected weighted graphs with all positive weights. It is easily adaptable to handle directed graphs' shortest-path problems. For instance,



Source: Research work

Even if it is simple to find the shortest path through inspection, we shall build certain concepts that will help us comprehend Dijkstra's algorithm. Finding the length of the shortest path from A to each subsequent vertex, up until Z is reached. To generate the only paths starting at A that have only as a vertex, add an edge with as a single endpoint. These paths have a single edge. They are *{A, B}* have four lengths and $\{A, D\ \}$have two length. Consequently, D is the nearest vertex with the shortest path that connects A and D for a length of 2. We may identify which vertex is the second closest by examining all routes that begin with the shortest path from A to a vertex in the collection $\{A, D\}$ and continue with an edge that has one endpoint in $\{A, D\}$ and its other endpoint outside of this set. Two such paths exist to consider, *{A, D, E}* of length 7 and {*A*, *B*} of length 4 therefore, the shortest path between A and B is therefore four lengths, and B is the vertex that is second closest to A. To find the third vertex that is closest to A, all we have to do is examine the routes that begin with the shortest path from A to a vertex in the set $\{A, D, B\}$, then have an edge with one endpoint inside the set {A, D, B} and its other endpoint outside of it. There are three of these routes. {*A*, *B*, *C*} of length 7, {*A*, *B*, *E*} of length 7, and {*A*, *D*, *E*} of length 5. Since $\{A, D\}$, and E are the shortest paths, E is the third vertex closest to A. The shortest path from A to E is 5 in length.

………………………………………..

To determine the fourth nearest vertex to A, we need examine only the paths that begin with the shortest path from A to a vertex in the set {*A*, *D*, *B*, *E*}, followed by an edge that has one endpoint in the set {*A*, *D*, *B*, *E*} and its other endpoint not in this set. There are two such paths, {*A*, *B*, *C*} of length 7 and {*A*, *D*, *E*, *Z*} of length 6. Because the shorter of these paths is {*A*, *D*, *E*, *Z*} the fourth closest vertex to A is Z and the length of the shortest path from A to Z is 6.

Graph coloring

Edge and vertex coloring are two coloring subgenres related to graph coloring. The aim of both coloring techniques is to paint the entire graph consistently. As such, adjacent edges and vertices require the usage of different colors. The term "chromatic number" refers to the bare minimum of colors needed to solve the graph coloring problem (GCP). As displayed in the following Figures 5 and 6.
As shown in Figure 5 and Figure 6 below.

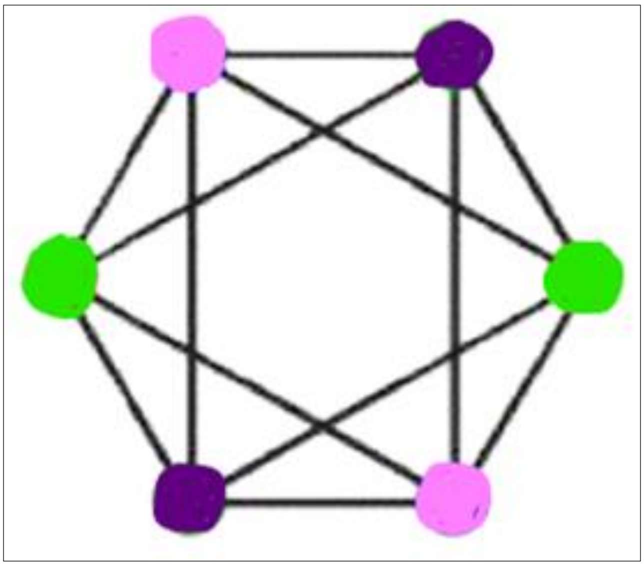
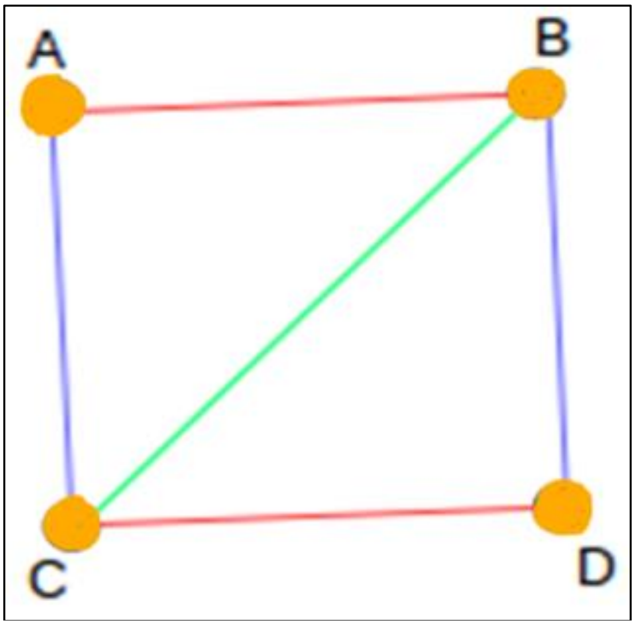


Fig 5: Vertex colouring



Source: Researcher's work

Fig 6: Edge colouring

In this study, we discussed graph coloring issues with vertices and edges.

The chromatic number for cycle C_n, $n \geq 3$

Definition 4: A **cycle** C_n, $n \geq 3$, consists of n vertices $v_1, v_2, \ldots, v_n$ and edges $\{v_1, v_2\}$,$\{v_2, v_3\}$, . . . , $\{v_{n-1}, v_n\}$, and $\{v_n, v_1\}$ (Arman, 2017), (Rosen, 2007) [14].

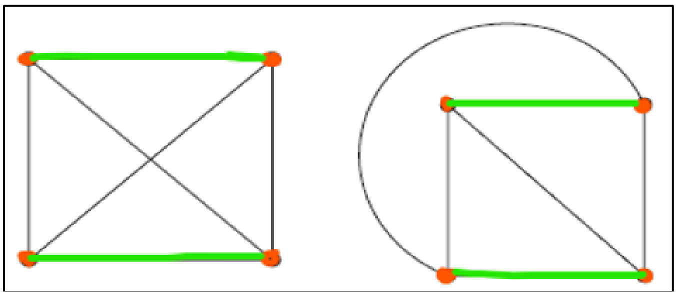
Theorem 1. We need to show that the chromatic number, $X(C_n) = 2$ if n is an even positive integer with $n \geq 4$ and $X(C_n) = 3$ if n is an odd positive integer with $n \geq 3$ (Rosen, 2007) [14].

Proof

When n is even, coloring C_n often requires two colors. All you have to do to create this coloring is choose a vertex and mark it red. Color the second vertex blue, the third vertex red, and so on as you move clockwise around the graph. Given that the first and $(n - 1)^{st}$ vertices, which are next to it, are both colored red, the n^{th} vertex can be colored blue. The chromatic number of C_n is three when n is odd and $n > 1$. Select a starting vertex to gain an understanding of this. To utilize just two colors, you must rotate the colors as you move clockwise around the graph. But the first and $(n - 1)^{st}$ found vertices are connected to the n^{th} vertex.

Planar graphs

If all of the edges of a graph can be drawn in a plane without crossing, the graph is said to be planar. We refer to this type of drawing as a planar representation of the graph. Even if a graph is typically represented with crossings, it may be feasible to draw it in a different way without crossings, in which case it would be planar.



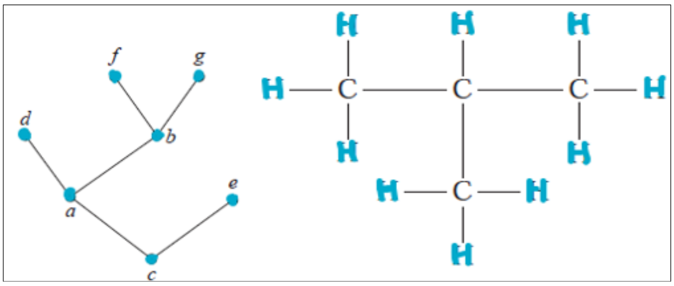
Source; Researcher's work

Trees

The English mathematician Arthur Cayley employed trees as early as 1857 to count particular kinds of chemical compounds. Since then, as the examples in this chapter will demonstrate, trees have been used to solve issues in a wide range of fields. In computer science, trees are especially helpful because they are used in many different kinds of algorithms. For example, efficient methods for finding things in a list are constructed using trees. They can be employed in algorithms that create effective codes and reduce data transmission and storage costs, such Huffman coding. Trees can be used to analyze and help develop winning strategies for games like chess and checkers. Trees are useful for modeling processes that involve making a series of decisions. Building these models can assist in figuring out the computational complexity of algorithms like sorting algorithms that rely on a series of judgments.

Definition 1: An undirected, linked graph without any simple circuits is called a tree.

A tree cannot have more than one edge or loop because it cannot have a simple circuit. For this reason, a tree can only be a simple graph.



Source; Researcher's work

Geometry

Geometry is the study of shape, sizes, and angles in a range of contexts, including everyday things. (Md. Mokter Hossain & Wiest, 2011) [9]. The Greek word "geo" means "Earth," and the word "metry" means "measurement," therefore one might also interpret "geometry" as "earth measurement." This phrases highlights geometry's beginnings as a investigation that examines the shapes and sizes of the Earth (Dunajski, 2022) [15]. There are two and three dimensional figures in Euclidean geometry. Two-dimensional shapes like triangles, squares, rectangles, and circles are frequently referred to as flat shapes in plane geometry. Solids are three dimensional objects such as cubes, cuboids, cones, etc. in solid geometry. Points, lines, and planes are geometry's fundamental building blocks (Ilka Agricola & Friedrich, 2008) [6]. Knowing the different geometric shapes may aid us in better understanding the shapes we come into contact with on a daily basis. We can compute a shape's area, perimeter, and volume using geometric ideas. It deals with the relative positions of basic geometric shapes such lines, points, circles, and triangles. On a plane, a point is an exact location or place. They are typically shown as a dot. It is crucial to realize that a point is a location as opposed to an object. It should ideally have no dimensions. In geometry, the terms "x-axis" and "y-axis" refer to the horizontal and vertical lines, respectively. If a line has a beginning and an end, it is called a line segment (Dunajski, 2022 [15] If a line has a beginning point but no endpoint, it is called a Ray line. An angle in planar geometry is a figure made up of two rays known as the angles sides and a common endpoint known as the angle's vertex. (Ilka Agricola & Friedrich, 2008) [6]

Solid geometry

Three-dimensional objects such as cubes, prisms, cylinders, and spheres are the subject of solid geometry. It addresses the figure's three dimensions, including height, breadth, and length. However, some solids lack faces. (Md. Mokter Hossain & Wiest, 2011) [9]. The investigation of three dimensions in Euclidean space is recognized as solid geometry. The things that surround us have three dimensions. Every three-dimensional shape is produced by rotating two-dimensional shapes. Three essential features of 3D shapes are faces, edges, and vertices. Look closely at these words for different geometric shapes. The segment of a boundary line connecting a vertex to another vertex is called an edge. It creates the 3D forms' skeleton. A solid vertex is the point at which all of its edges meet together. The point where two edges converge is called a vertex.

Conclusion

This work is a theoretical investigation that offers lesson plans based on geometry and graph theory ideas. We anticipate that these exercises will help students become more adept at critical thinking. It will take more experimental research to determine whether or not this in-class exercise will assist students in developing their critical thinking skills.

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