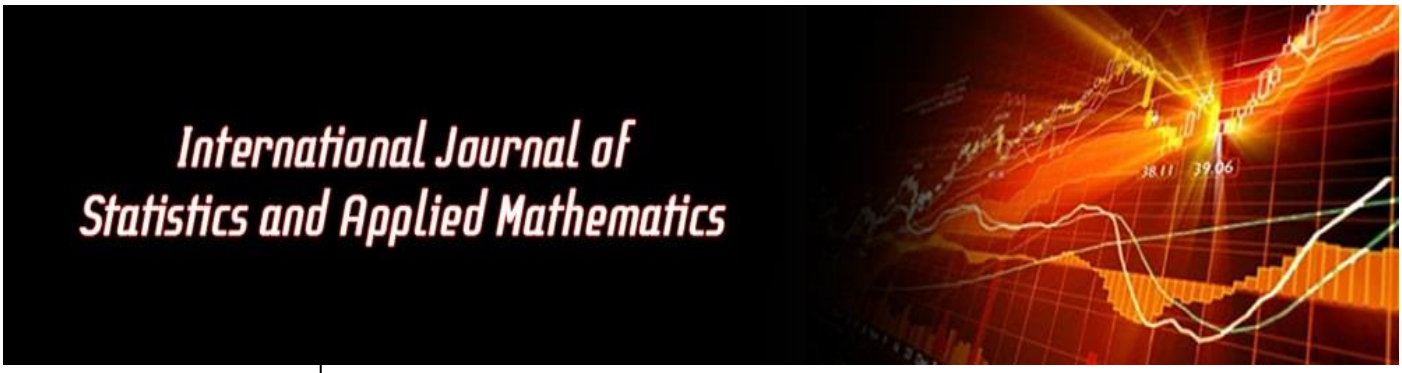

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S Sumithra
Assistant Professor, Department of Statistics, PSG College of Arts & Science, Coimbatore, Tamil Nadu, India

R Sampath Kumar
Associate Professor, Department of Statistics, Government Arts College, Coimbatore, Tamil Nadu, India

Corresponding Author:
S Sumithra
Assistant Professor, Department of Statistics, PSG College of Arts & Science, Coimbatore, Tamil Nadu, India

Designing of mixed sampling plan with chain sampling plan (ChSP-1) plan as attribute plan using MAPD and discriminant tangential angles

S Sumithra and R Sampath Kumar

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Abstract
In this paper a procedure for designing a mixed sampling plan with chain sampling plan (ChSP-1) plan as attribute plan indexed through maximum allowable percent defective (MAPD) along with tangential distance (Discriminant) is presented. It is discussed how the declination angle of the tangent at the inflection point of the OC curve discriminates the Mixed Sampling Plan (MSP). Tables are presented for the easy selection of plans based on MAPD with Discriminant or declination angle of the tangent.

Keywords: Maximum allowable percent defective, chain sampling plan, discriminant, tangent intercept, inflection point, operating characteristic curve, AMS (2000) Subject classification Number, Primary: 62P30, Secondary: 62D05.

1. Introduction
A variety of plans and procedures have been developed for special sampling situation involving both measurements and attributes. Each is tailored to do a specific job under prescribed circumstances. They range from a simplified variables approach to a more technically complicated combination of variables and attribute sampling called Mixed sampling plans. Mixed sampling plans are of two types, namely independent and dependent plans. Independent mixed plans do not incorporate first sample results in the assessment of the second sample. Dependent mixed plans combine the results of the first and second samples in making a decision if a second sample is necessary.

Mixed sampling plans consist of two stages of rather different nature. During the first stage the given lot is considered as a sample from the respective production process a criterion by variables is used to check process quality. If process quality is judged to be sufficiently good, the lot is accepted, otherwise the second stage of the sampling plan is entered and lot quality is checked directly by means of an attribute sampling plan.

MAPD is a key measure assessing to what degree the inflection point empowers the OC curve to discriminate between good and bad lots. The concept of MAPD (p_*) was introduced by Mayer (1967) [2] and further studied by Soundararajan (1975) [7] is the quality level corresponding to the inflection point on the OC curve. Mixed Sampling Plan (MSP) was first developed by Schilling (1967) [6] for the case of single sided specifications, standard deviation known by assuming an underlying normal distribution for measurements. Using Schilling's procedure, Devaarul (2003) has constructed tables for mixed sampling plans (independent case) having various sampling plans as attribute plans. Radhakrishnan and Sampath Kumar (2006) [3] have made contributions to mixed sampling plan with chain sampling plan as attribute plan for independent case. Sampath Kumar (2007) [8] has constructed mixed sampling plan with various sampling plans as attribute plans using Poisson distribution as a base line distribution. A tangent is drawn at the inflection point of an OC curve, where p_T is the tangent intercept to the p-axis and $L(p_T)$ is the tangent intercept to the L(p) axis. The discriminant 'D' is defined as the distance between $L(p_*)$ and $L(p_T)$ in the L(p) axis ie., $D = L(p_T) - L(p_*)$.

When D is large there is a high degree of discrimination power and when D is small the discrimination decreases. A procedure for designing a SSP indexed through MAPD and discriminant distance is developed by Ramkumar and Manjula (2007) [4]. They discuss a unique sampling plan at fixed MAPD and tangential distance and this distance is a powerful tool to identify the OC curve at the designed MAPD. Ramkumar (2009) [5] has designed a SSP indexed through MAPD and tangential distance (discriminant).

In this paper mixed sampling plan (independent case) with ChSP-1 plan as attribute plan is constructed using Poisson distribution as a base line distribution. The plans indexed through MAPD and tangential angles are constructed by fixing the values c and β_*'' . Tables are presented for easy selection of the plans based on MAPD with discriminant and tangential angles.

2. Glossary of symbols

The symbols used in this paper are as follows.

P: Submitted quality of lot or process.

P_*: Maximum allowable percent defective (MAPD)

β_j: Probability of acceptance for lot quality 'p_j'

β_j': Probability of acceptance assigned to first stage for percent defective 'p_j'

β_j'': Probability of acceptance assigned to second stage for percent defective 'p_j'

K: Variable factor such that a lot is accepted I $\bar{X} \le A = U - k\sigma$

n_1: Sample size for the variable sampling plan

n_2: Sample size for the attribute sampling plan

I: Acceptance number

D: Discriminant distance between $L(p_*)$ and $L(p_T)$

A: Declination angle

H_*: Relative slope at p_*

Pa(p): Probability of acceptance for given quality p

L(p): Probability of acceptance for given quality 'p'

$L(p_T)$: Tangent intercept to the L(p) axis

3. Operating procedure of mixed sampling plan having chsp-1 plan as attribute plan

Using Schilling (1967) [6] the operating procedure of Mixed Sampling plan using ChSP-1 as attribute plan with upper specification limit (U) and standard deviation (σ) is given as follows.

1. Determine the parameters of the mixed sampling plan n_1, n_2, k and c.
2. Take a random sample of size n_1 from the lot.
3. If the sample average $\bar{X}$ ≤ A = U - kσ , accept the lot
4. If the sample average $\bar{X}$ > A = U - kσ ,

i. Take a second sample of size n_2

ii. Count the number of defectives 'd' in the sample.

iii. If the number of defectives d = 0, accept the lot, If the number of defectives d ≥ 2, reject the lot.

iv. Accept the lot if d = 1 and if no defective units are found in the immediately preceding 'i' samples of size 'N'.

The OC function of the mixed sampling plan, suggested by Schilling (1967) [6] for the single sampling plan is

$$P_a(p) = P_{n_1}(\bar{X} \le A) + P_{n_1}(\bar{X} > A)\sum_{j=0}^{c} P(j : n_2) \quad (1)$$

Equation (1) can be expressed as $\beta_j = \beta_j' + (1-\beta_j')\beta_j''$. by taking the Single Sampling plan as attribute plan, equation (1) can be written as

$$P_a(p) = P_{n_1}(\bar{x} \le A) + P_{n_1}(\bar{x} > A)\left[e^{-np} + (e^{-np})^{i+1} np\right]$$

4. Construction of mixed sampling plan having chsp-1 plan as attribute plan

The detailed procedure adopted in this paper for the construction of mixed sampling plan having ChSP-1 plan as attribute plan indexed through MAPD is given below.

- Assume that the mixed sampling plan is independent
- Decide the sample size n_1 (for variable sampling plan) to be used
- Calculate the acceptance limit for the variable sampling plan as $A = U - k\sigma = U - [z(p_j) + \{z(\beta_j')/\sqrt{n_1}\}]\sigma,$ where z(t) is the standard normal variate corresponding to 't' such that $t = \int_{z(t)}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-u^2/2} du$.
- Determine the sample average $\bar{X}$. If a sample average $\bar{X} < A = L + K\sigma$, take a second stage sample of size 'n_2' using attribute sampling plan.
- Split the probability of acceptance β_* as β_*' and β_*'' such that $\beta_* = \beta_*' + (1-\beta_*'').$ Fix the value of β_*' .
- Determine β_*'' the probability of acceptance assigned to the attribute plan associated with the second stage sample as $\beta_*'' = \frac{\beta_* - \beta_*'}{1-\beta_*'}$.
- Determine the appropriate second stage sample of n_2 from the relation

$$\beta_*' = e^{-np} + (e^{-np})^{i+1} np$$

Using the above procedure tables can be constructed to facilitate easy selection of mixed sampling plan by using ChSP-1 plan as attribute plan indexed through MAPD.

5. Designing of chsp-1 with mapd and tangential angles

The equation of the tangent to the OC curve at the inflection point is given by Soundararajan (1975) [7] is

$$L(p_T) - L(p_*) = \left[\frac{dL(p)}{dp}\right]_{p=p_*} (p_T - p_*) \quad (5.1)$$

The tangent intercept to the L(p) axis is given by

$$L(p_T) = -p\frac{dL(p)}{dp} + L(p_*) \quad (5.2)$$

Which reduces to $L(p_T) = L(p_*)[h_* + 1]$

Then the discriminant $D = L(p_T) - L(p_*)$ (5.3)

(Or)

$$D = L(p_*)(h_*) \quad (5.4)$$

The declination angle

$$A = \tan^{-1}\left(\frac{p_*}{D}\right) \quad (5.5)$$

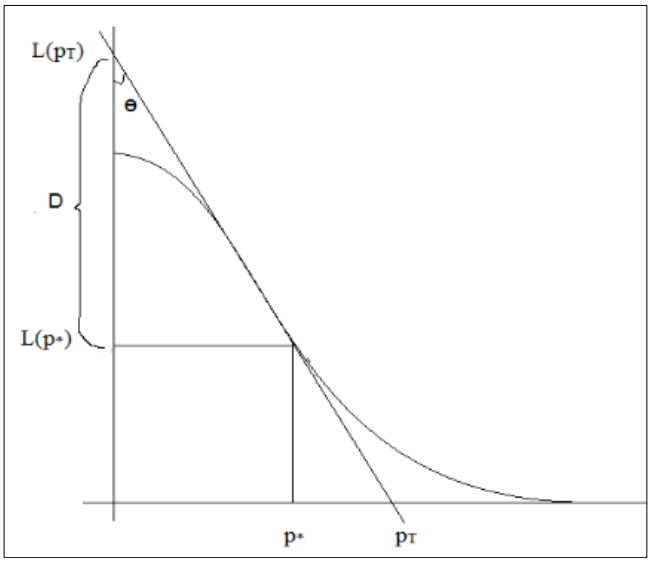

Fig 1: The OC curve, Distance, Tangent intercept and declination angle A for the sampling plan

6. Construction of tables

The probability of acceptance for ChSP-1 plan under Poisson model is used for determining the second stage probabilities and is given by

$$\beta'_* = e^{-np} + (e^{-np})^{i+1} np$$

Using the above equation, the inflection point (p_*) is obtained by using $\frac{d^2 p_a(p)}{dp^2} = 0$ and $\frac{d^3 p_a(p)}{dp^3} \neq 0$. The relative slope of the OC curve h_* is given by

$$h_* = \left[\frac{-p}{dp_a(p)}\right]\frac{dp_a(p)}{dp}$$

at $p = p_*$. For any values of c, the values of $n_2 p_*$, h_*, D and β''_* with $\beta'_* = 0.30$ are calculated using the Visual basic program and presented in the Table l. For any given values of p_* and D the angle A and n_2 values are calculated and presented in the Table 2.

7. Selection of the plan

Table 1 is used to calculate the discriminant distance D for various values of i and β'_*. For values of i, p_* and D, the values of the declination angle A and the sample size n_2 is calculated and are presented in the Table 2. Using Table 2 one can find the parameter of the Mixed sampling plan with ChSP-1 as attribute plan when (p_* and D) or (p_* and Angle) or (D and Angle) are given.

Example 1

A company fixes the MAPD at 11% and Discriminant of the OC curve D = 0.3654 Using Table 2, select the value of D = 0.3863, which is nearer to the given value D= 0.3654. Corresponding to the given p_* = 11% and D = 0.3863, one can find the second stage sample size n_2 = 4, i = 2, and A= 15.8945. Thus the parameters for the mixed sampling plan with ChSP-1 plan as attribute plan are i = 2 and n_2 = 4.

Example 2

A company claims that the OC curve for their product has MAPD = 5% and angle of declination of tangent at MAPD is 7°, D = p_*/ tan A i.e. D = 0.05/0.1228 = 0.4072. Using Table 2 select the nearest value of D = 0.4329. The value of D is 0.4072. Corresponding to p_* = 5% and D = 0.4329, one can find the second stage sample size n_2 = 11 and i = 1. Thus the parameters for the mixed sampling plan with ChSP-1 plan as attribute plan are i = 1 and n_2 = 11.

Example 3

Given D = 0.3795 and A = 8°. Since tan A = p_*/ D, p_* = D x tan A, i.e., p_* = 0.3795 x 0.1405 = 0.05 Using Table 2 corresponding to D = 0.3863, which is nearest to 0.3795 and p_* = 5%, one can find the second stage sample size n_2 = 8 and i = 2. Thus the parameters for the mixed sampling plan with ChSP-1 plan as attribute plan are i = 2 and n_2 = 8.

8. Construction of OC curves

The ChSP-1 plan for various Angles with fixed D and c are constructed and are presented in Table 1. The OC curves for the sampling plans are also drawn and are presented in Figure 2.

Table 1: The ChSP-1 for various Angles with a fixed D and c

S. No.	Angle	i	D	n_2
1.	6.5885	1	0.4329	15
2.	9.1852	1	0.4329	11
3.	11.7445	1	0.4329	8

* OC Curves are drawn

Table 2: The Discriminant, y- intercept of MAPD and tangent intercept for various i with $\beta'_* = 0.30$

i	β_*	β''_*	$n_2 p_*$	h_*	D
1.	0.7529	0.6470	0.7376	0.6692	0.4329
2.	0.7813	0.6876	0.5472	0.5618	0.3863
3.	0.8061	0.7230	0.4358	0.4682	0.3385
4.	0.8262	0.7517	0.3667	0.4031	0.3030
5.	0.8427	0.7753	0.3175	0.3533	0.2739
6.	0.8564	0.7949	0.2804	0.3142	0.2498
7.	0.8678	0.8111	0.2517	0.2833	0.2298
8.	0.8776	0.8251	0.2284	0.2577	0.2126
9.	0.8860	0.8374	0.2089	0.2361	0.1977
10.	0.8932	0.8474	0.1932	0.2186	0.1852

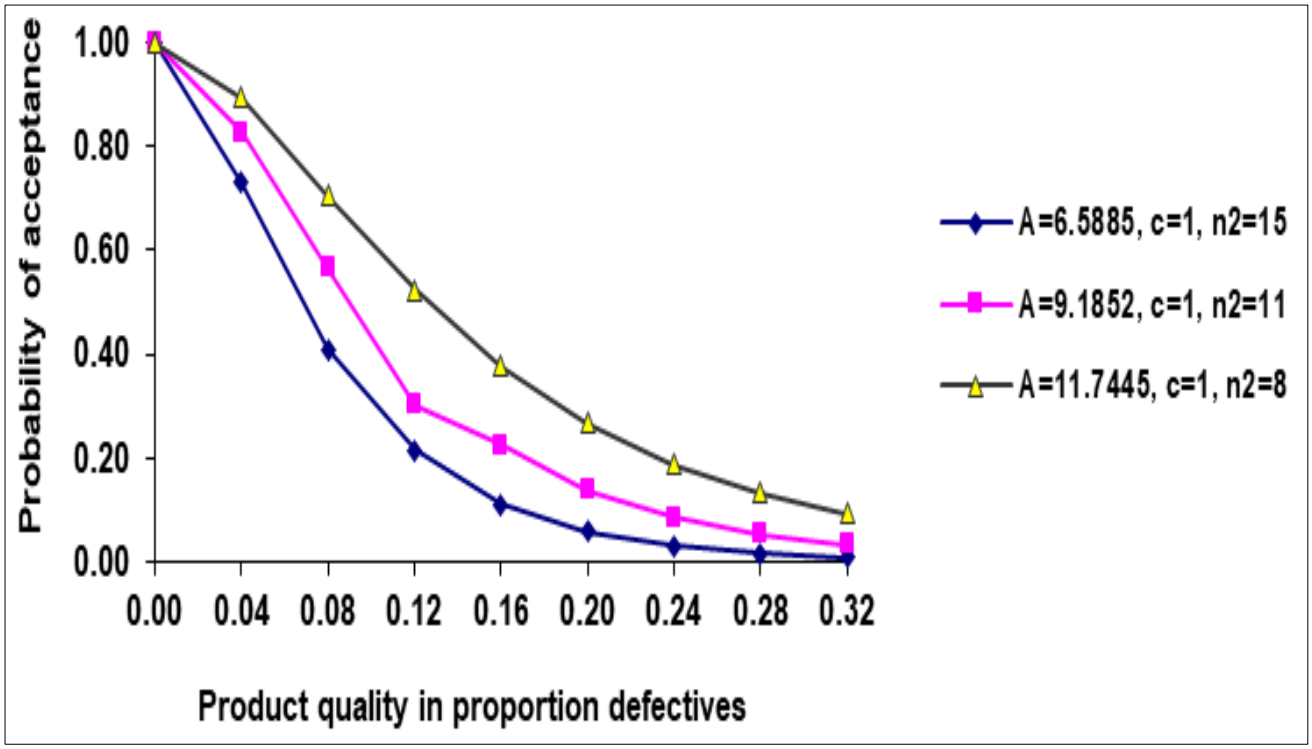

Fig 2: The OC curves for the sampling plans are also drawn and are presented

Table 3: Certain ChSP-1 for given discriminant D and declination angle A

i	P* / D	0.05	0.07	0.09	0.11	0.13	0.15	0.17	0.20
		A	A	A	A	A	A	A	A
		n_2	n_2	n_2	n_2	n_2	n_2	n_2	n_2
1.	0.4329	6.5885	9.1852	11.7445	14.2571	16.7150	19.1112	21.4399	24.7969
		11	8	6	5	4	4	3	3
2.	0.3863	7.3749	10.2709	131148	15.8945	18.5994	21.2212	23.7529	27.3721
		8	6	5	4	3	3	2	2
3.	0.3385	8.4024	11.6838	14.8893	18.0022	21.0090	23.8996	26.6665	30.5764
		7	5	4	3	3	2	2	2
4.	0.3030	9.3703	13.0084	16.5429	19.9527	23.2214	26.3377	29.2949	33.4274
		6	4	3	3	2	2	2	1
5.	0.2739	10.3453	14.3361	18.1899	21.8807	25.3902	28.7071	31.8264	36.1367
		5	3	3	2	2	2	1	1
6.	0.2498	11.3188	15.6542	19.8135	23.7664	27.4932	30.9839	34.2370	38.6822
		4	3	2	2	2	1	1	1
7.	0.2298	12.2751	16.9414	21.3875	25.5794	29.4972	33.1342	36.4931	41.0338
		4	3	2	2	1	1	1	1
8.	0.2126	13.2345	18.2245	22.9444	27.3572	31.4448	35.2049	38.6467	43.2508
		3	2	2	2	1	1	1	1
9.	0.1977	14.1929	19.4976	24.4767	29.0915	33.3274	37.1885	40.6919	45.3314
		3	2	2	1	1	1	1	1
10.	0.1852	15.1084	20.7051	25.9179	30.7083	35.0667	39.0052	42.5496	47.2003
		3	2	2	1	1	1	1	1

9. Conclusion

In this paper designing of mixed sampling plan with ChSP-1 plan as attribute sampling using MAPD and discriminant tangential angles with Poisson distribution as baseline distribution is presented. It is concluded from this study when angle of the tangent decreases, the discriminating power is increasing. Suitable tables are also provided for the easy selection of the plans for the engineers who are working on the floor of the assembly. Different sampling plans can also be constructed by changing the first stage probabilities (β_j').

10. References

1. Devarul S. Certain studies relating to mixed sampling plans and reliability based sampling plans [Ph.D. Thesis]. Tamil Nadu, India, Bharathiar University; 2003.
2. Mayer PL. A note on sum of poisson probabilities and an application. Industrial Quality Control. 1967;19(15):12-15.
3. Radhakrishnan R, Sampath Kumar R. Construction of mixed sampling plan indexed through MAPD and AQL with chain sampling plan as attribute plan. STARS, Interdisciplinary Journal. 2006;7(1):14-22.

4. Ramkumar TB, Manjula K. Design of single sampling Plan with MAPD and discriminant tangential distance. ISPS conference Thirupati, January 2007.
5. Ramkumar TB. Design of single sampling plan by discriminant at MAPD. Prob Stat Forum. 2009;2:104-114.
6. Schilling EG. A general method for determining the operating characteristics of mixed variables – 'Attributes Sampling Plans single sided specifications, S.D. known', PhD Dissertation – Rutgers - The State University, New Brunswick, New Jersy; c1967.
7. Soundararajan V. Maximum allowable percent defective (MAPD) single sampling inspection by attributes plam, Journal of Quality Technology. 1975;7(4):173-177.
8. Sampath Kumar R. Construction and selection of mixed variables-attribute sampling plans', Ph.D. Theses, Bharathiar University, Coimbatore, Tamil Nadu, India; c2007.