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Experimental study on flow characteristics of drop inlet spillway under varying slope & discharge conditions

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Abstract

The flow over a drop-inlet spillway has a complex nature, and its characteristics are remarkably different from other spillways. This study conducts experimental investigations and numerical simulations on the flow behaviour of water in drop-inlet spillways. The drop-inlet spillway is the most effective hydraulic structure for dissipating energy downstream of the spillway crest. This work evaluates the hydrologic and hydraulic functioning of a prototype hydraulic structure as used in natural channels of small agricultural watersheds, by generating some of the basic information & other relevant parameters for the area under study. The overall values of key flow parameters (h, v, E, n, Fr, Re) have been suitably worked out and analysed, which altogether gave a broad range under hydraulic structures with varied configurations in regards to discharges and the channel bed slopes. Physical model study plays a vital role in the planning and designing of hydraulic structures. A vast review was done on updated progress on the hydraulic structure and their relevant monitoring and evaluations in hydraulic flume as well as real channel flow conditions. The avenged values of v under low discharge conditions remains 0.192 m/s, moreover when the channel bed slope enhanced form 0.5 to 2% it enhanced to 0.317 m/s. Under high discharge conditions it was found to 0.534 to 0.682 m/s. Efforts are made to derive the real limits of flow conditions all along the channel with in-depth evaluations of flow parameters, flow regimes, and all associated flow conditions. Also, the values of C_d for specific Drop inlet spillway hydraulic structures are attempted here with an effort to further extend its applicability to similar ungauged situations. The C_d values for the drop inlet spillway were found the be in the range of 0.004 to 0.107 and alternate predictive equations in this regard too also synthesized in this study.

Keywords: Drope inlet, Froude no., Reynolds no., hydraulic flumes, potential head

1. Introduction

In many water related projects, water-retaining structures such as gates and dams are usually required to be built to meet the function of flow interception, flood control, reservoir storage adjustment. A significant head difference between upstream and downstream of the constructed project is thus formed, leading to huge energy contained in the water flowing over the weir or dam. Building a spillway is a commonly used engineer measure to safely release the flow from the reservoir to the down- stream river (Parsaie & Haghiabi 2019; Kocaer & Yarar 2020) [10, 8] and dissipate the tremendous flow energy generated by crossing weirs or dams (Khatibi *et al*. 2014; Rajaei *et al*. 2020; Wu *et al*. 2020) [7, 11]. The Drop inlet spillway functions, in variety of ways to reduce the flow velocity, to enhance the hydraulic roughness, and thus safely conveying the glows, with variety of flow transactions and transformations. In the domain of fluid mechanics and hydraulics there happens to be big array of hydraulic structures with enormous configurations both structural as well as flow related. Researchers have performed extensive experiments in controlled as well as natural conditions to come out with multiple results showing importance of channel conditions as well types of hydraulic structures adopted there in. The structural as well as hydraulic performance of such hydraulic structures often remained the key focal point for all such researches conducted by past researchers. The volume and extent of such research is so much wide that its full inclusion in this small piece of research work looks less feasible and hence not expressed in its full extent, though the detailed review were screened and visualized for in depth understandings. A limited and relevant references are being discussed here in this subsection reflecting the salient efforts and findings by previous researchers including historical efforts too. Fiorotto *et al* (1992a, 1992b, 2000,) [3-5] had attempted structural configurations of some hydraulic structures for their relative performance evaluations and accordingly ranked the effectiveness and suitability of these structures to dissipative the flow energy inside the channel domain. Salvetti *et al*. (1997) [12] authors conducted a variety of hydraulic studies by considering number of physical mode and evaluating the nature and extent of turbulence inflow. they also attempted simulation of energy profile in corpora ting eddy-current of flow. Pang (1998) [9] an important study on river flood flow along with associated energy losses was conducted to deliver important numerical interference which were practically impotent for deciding design and configuration of the hydraulic structures

The nature and extent of flow turbulences as well as the hydraulic jumps largely depend upon the nature of the channel bed surface. How much rough it is, used to be an important factor in governing the ultimate flow dynamics. Researchers like Carolo *et al*. (2007) [1] had well proved and amply documented this fact by publishing their results in this regard. Ead and Rajaratnam (2002) [2] evaluated the corrugated nature of stream beds and its total influences on the channel flows and other associated indicators to establish the real flow regimes. These flow regimes revealed their suitability or nonsuit ability towards associated hydraulic structures as installed therein. Managerial options as well as decision support to choose a best-suited hydraulic structure in accordance to flow regime categories was recommended.

Materials and Methods

The research was carried out at the hydraulic laboratory situated in the College of Agricultural Engineering & Technology, AAU, Godhra, Gujarat, India. The setup involved horizontal rectangular Thick Perspex sheets, each measuring 6 meters in length, securely fastened on either side of the MS Sheet bed. Two gates were installed, one at the upstream end and the other at the downstream, with adjustable features to regulate their positions. Transparent Perspex sheets, 6 mm thick and 6 meters long, were installed on both sides of the Test Section, facilitating direct observation of flow patterns, types, and general flow regimes within the flume. An extensive analysis was conducted on various significant flow parameters (h, v, E, n, Fr, Re) which provided a comprehensive range of data regarding different hydraulic structures with varying configurations in relation to discharge rates and channel bed slopes. The scientific evaluation of the mentioned hydraulic structure involved a visual characterization of hydraulic jumps, Froude number values within the channel, establishment of Stage Discharge relationships, evaluation of the hydraulic performance of scaled-down broad-crested rectangular weirs, determination of Discharge Coefficients and associated predictive equations, as well as an examination of composite influences and variability of Manning's roughness coefficient under diverse flow scenarios concerning both discharge rates and channel bed slope conditions. Notably, Manning's roughness coefficient was meticulously observed under various test conditions, encompassing multiple channel bed slopes (0.5, 1.0, 1.5, and 2 percent) and varying discharge rates (up to 6 measurements for a single structure within the specified ranges). While the study involved a total of approximately 100 test runs with various permutations and combinations, the findings presented herein are confined to about 75 test runs, including multiple replications.

Structural configurations of flume setup: The detailed configurations in regards to components/ constituents of tilting hydraulic flumes are provided below in abstract shape followed by a functional conceptual line diagram in, Upstream & Downstream Gates with Fig. **1** handle, Sump Tank: Made from fibre Sheet with MS Angle reinforcement (Size 140 cm (L) x 95 cm (B) x 35 cm (H) with capacity = 500 Liters, Monoblock Pump: 3 HP, Centrifugal, Size 80 x 65 mm, Discharge Head 2 m (range 1.5 to 3 m). The potential capacity of discharge = 10 L/s, RPM 2840, 3 Phase, 230 VAC, Make Kirloskar", Bye-pass arrangement with Gate Valve, "Gun Metal" for controlling the flow, Hook Gauge with Trolley. Range: 0 – 600 mm, Manometer: Acrylic Body Differential Type Range 0–500 mm, Orifice Plate: Made from MS Plate, Dia. 34.5mm, Piping: GI Material – 69 mm size, Starter: AC 3 phase, 7.5 hp.

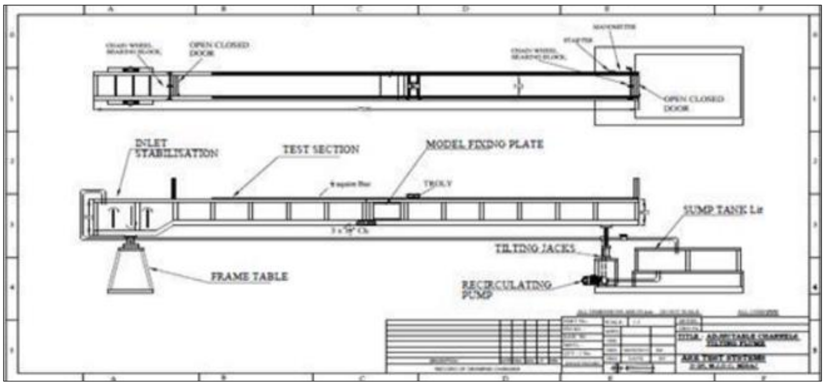


Fig 1: Conceptual line diagram for hydraulic flume studies under present study

The real pictorial view of a tilting hydraulic flume as adopted in the present study is illustrated in Fig. 2. The flume comprising a 500-liter main water tank had a provision to provide the system with adjustable flow rates with the help of a discharge regulator. The ranges of available discharge rates varied from 0.5 lit/sec to 5 lit/sec which were suitably utilized in the present study by incorporating different sloping conditions. The broad categories of channel bed slopes as adopted in present studies remain 0.5%, 1%, 1.5% & 2%. Systematic observations were undertaken inside the flume adopting standard protocols & delivering below given specified entities, Flow depths at varied close interval locations along the longitudinal section of the flume under various sets of discharge rates as well as channel bed slopes. Flume cross-sectional areas, Magnitudes of velocity heads, Water flow velocities, Total energy head, Values of Manning's roughness coefficients, Froude no., Reynolds no. & Type, location & extent of hydraulic jumps as observed during different test conditions under the above-cited combinations.

Fig 2: The real physical view of standard tilting hydraulic flume as adopted in present study

Description of Downscaled Physical Models of Hydraulic Structures

The structures planned and utilized in this study were down scaled model of a real drop inlet structure which was designed and exists at experimental research farm of the CAET Godhra. This model was made by a linear down scaling of 1 : 40 in major directions. Utilizing 6 mm Acrylic sheet followed by accurate fabrications. The box inlet of drop inlet structure was a straight box riser with 3 crest lengths being 8.5 cm, 13.cm, and 8.5 cm, making total equivalent crest length of rectangular box inlet broad crest as 20 cm. The exact dimensional configuration of this specific hydraulic structure is provided in Table 1, while the pictorial view of finally fabricated structure is illustrated in Figure 3, incorporating the real pictorial scenario of its installed location inside the tilting hydraulic flume adopted in this study. The exact details of placement of this hydraulic structure inside the tilting hydraulic flume is clearly depicting the varied nature for flows as well as the prevailing controlling conditions of flows at upstream as well as downstream ends with peculiar patterns of flows.

Table 1: Average range of experimental limits adopted in the tilting flume study

Types of Structures	Range of Channel bed slopes (%)	Range of Discharges Rates (Liters/Sec)	Dimensions of the structures
Drop inlet spillway	0.5 to 2.0 (0.5, 1.0, 1.5,2.0)	0.5 to 5.0 (Random Values – 3 to 4 numbers in each run)	slope tan50 i.e. 1 : 3.75; Height 20 cm Base Width 75 cm with 4 equal steps @ 5 cm

Fig 3: A Pictorial View of Down Scaled Drop Inlet Spillway with Straight Box Inlet-(Acrylic), Showing Physical Configurations as well as flow patterns while placing it In Flume Channel

Monitoring flow parameters

Monitoring & Recording of Flow Parameters The parameters that were monitored in this research work and the simplistic procedure/methodology adopted for their recording remained as flows, Flow depths, Flow Velocity Heads, Froude Number, Reynold's number, Manning's Roughness Coefficients, Patterns of Hydraulic Jumps.

Hydraulic Jump & Froude Number Inside the Existing Channel

The present study dealt with such objectives in mind, where the values of Froude numbers and Reynold’s numbers are critically observed and assessed in real flow conditions using a variety of down-scaled hydraulic structures in tilting flumes. The net variations all along the channel bed length are documented reflecting the significant variations in this regard under various options of hydraulic structures which is shown in Fig. 4.

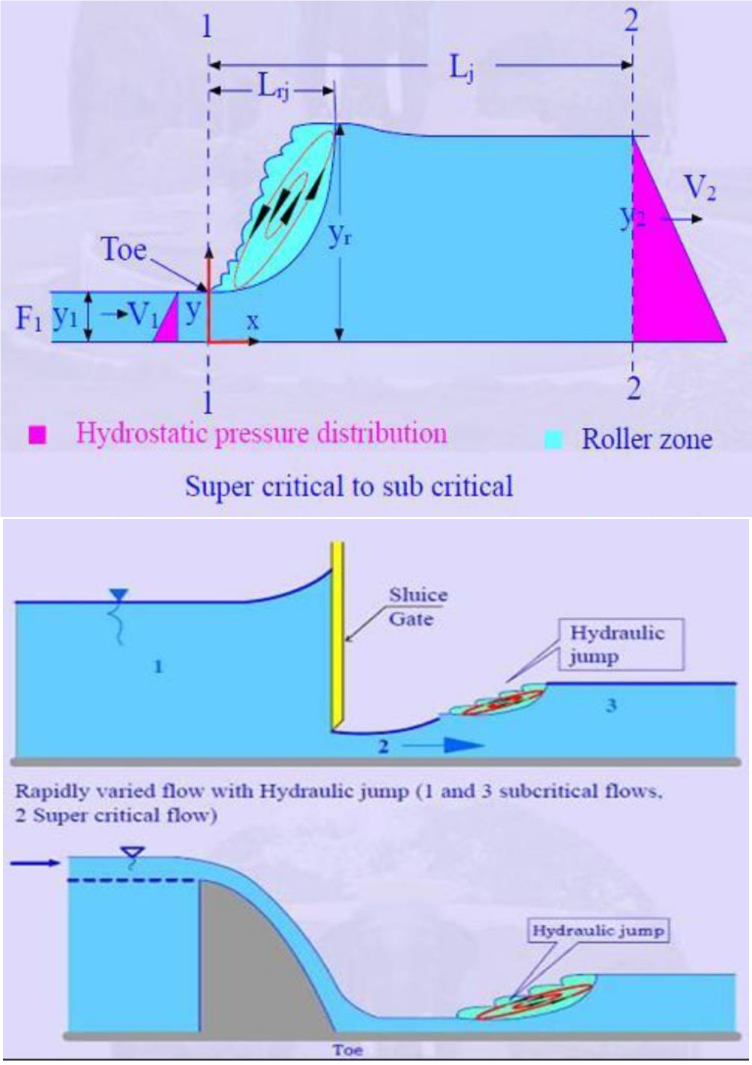


Fig 4: Conceptual Framework of any Hydraulic Jump during Flows in Open Channel

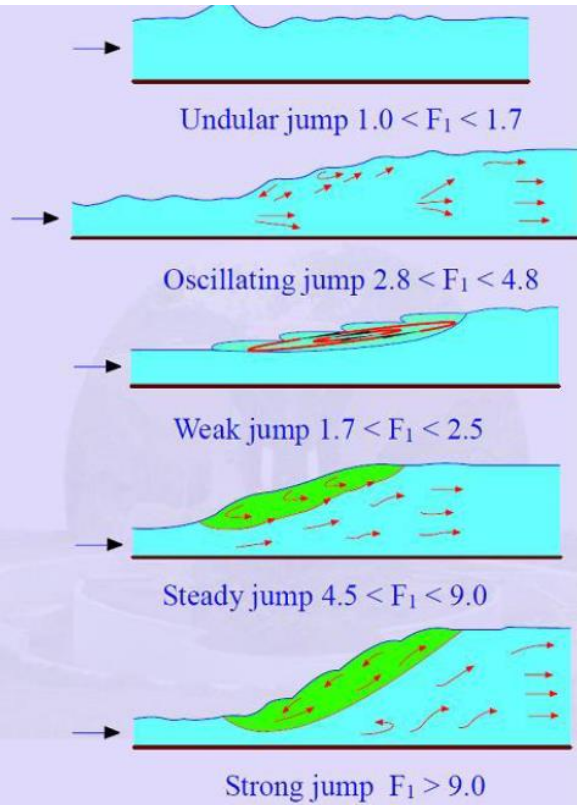


Fig 5: Classifications of Hydraulic Jumps from a Practical Point of View

The Froude number in the approach channel (Fr), which is the square root of the ratio of inertial to gravity forces, plays an important role in flume and weir design (Eq.1).

$$F_r = \frac{V}{\sqrt{gh}} \dots \qquad \dots\dots\dots\dots\dots\dots\text{(1)}.$$

In order for a hydraulic jump to occur, the flow must be supercritical. The jump becomes more turbulent and more energy dissipates, as Froude's number increases. A jump can occur only when the Froude's number is greater than 1.0. This number, representing the ratio of inertial and gravity forces, is expressed by the average flow velocity V and the celerity of gravity wave in shallow water.$\sqrt{g.y}$ Using the Froude number one can distinguish Critical flow when Fr = 1, Supercritical flow when Fr> 1, and Subcritical flow when Fr< 1. The pictorial view of the hydraulic jump (Moore, 1943) is shown in Fig. 5.

Composite Influences and Variability of Roughness

For the estimation of such roughness historically we have three basic models, namely Manning's formula, Chazy's formula, and Darcy's Weisbach formula. In the present study only Manning's roughness is taken into account whose standard equation is illustrated as follows in eq.(2),

$$V = \frac{1}{n} R^{2/3} S^{1/2} \dots \qquad \dots\dots\dots\dots\dots\dots\text{(2)}.$$

Where v is the mean flow velocity, n is Manning's roughness coefficient, R is hydraulic radius, and S is the energy grade slope which is sometimes assumed to be equivalent to channel bed slopes specifically under kinematic wave flows. Similarly, Reynold's number also plays a very important role in deciding the turbulences and the nature of flows and flow regimes under different conditions. For flows of water in open channels, Reynold's number can be given by below given generalized eq.3,

$$Re = \frac{vD}{10^{-5}} \ldots\ldots\ldots\ldots\ldots\ldots\text{(3).}$$

In a simplistic and direct format the 'v' is the velocity 'D' is hydraulic depth, and 10^{-5} is the value of kinematic viscosity of water placed directly here.

Table 4. Overall range of hydraulic parameters during flume test runs using Drop inlet Spillway (Downscaled Model - Acrylic)

Table 3: Net variations (reductions or enhancements) in different flow parameters owing to combinations of discharge & slopes under Drop inlet spillway structures as evaluated under controlled experimental conditions.

Treatments	**Flow parameters**												
	Q	**h**		**v**		**H**		**n**		**Fr**		**Re**	
Drop inlet downscaled model	Slop	0.5%	1.5%	0.5%	1.5%	0.5%	1.5%	0.5%	1.5%	0.5%	1.5%	0.5%	1.5%
	Low Q	1.629	1.029	0.192	0317	0.019	0.019	0.028	0.028	0.601	1.223	237.11	245.96
	High Q	2.7	1.986	0.534	0.682	0.048	0.046	0.015	0.019	1.431	1.864	949.17	989.59
	Diff.	40	48	64	54	60	65	46	32	58	34	75	75

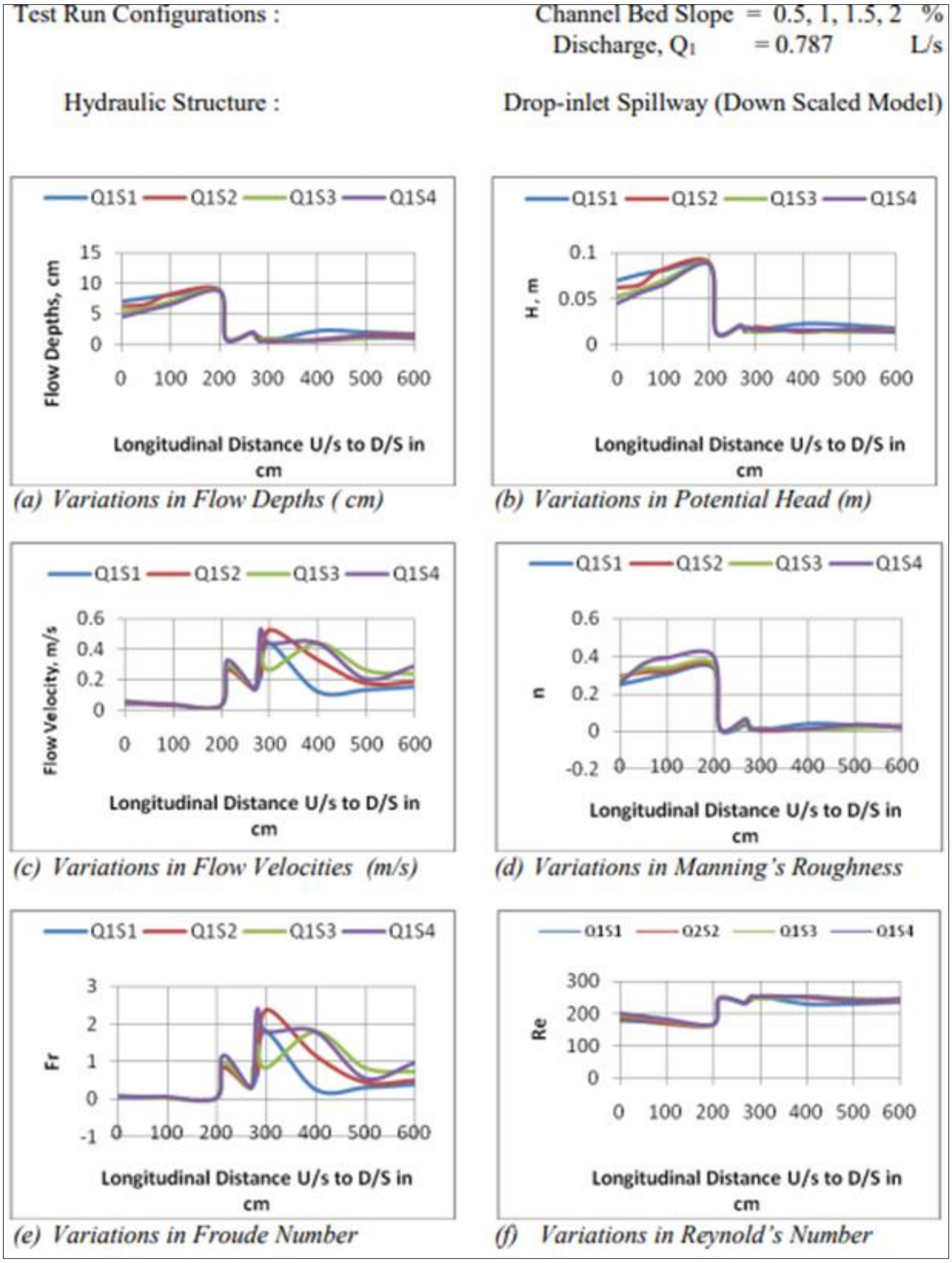
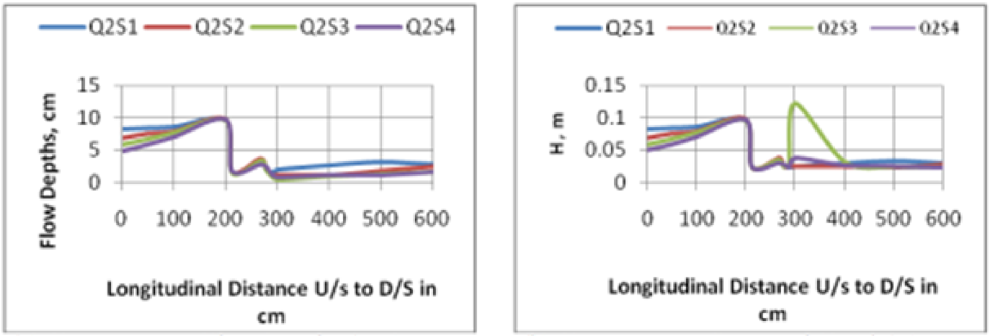


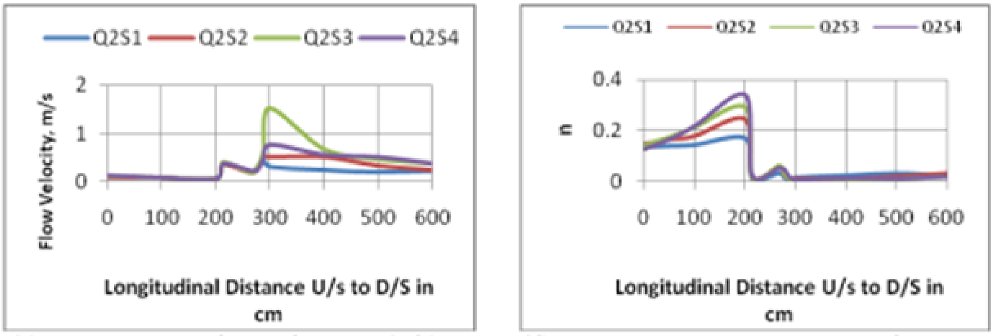
Fig 6: Observed Hydraulic Parameters for Flows in Channel having Drop Inlet Spillway with 0.787 L/s Discharge on 4 Different Channel Bed Slopes

Test Run Configurations : Channel Bed Slope = 0.5, 1, 1.5, 2 %
Discharge, Q_2 =1.8297 L/s

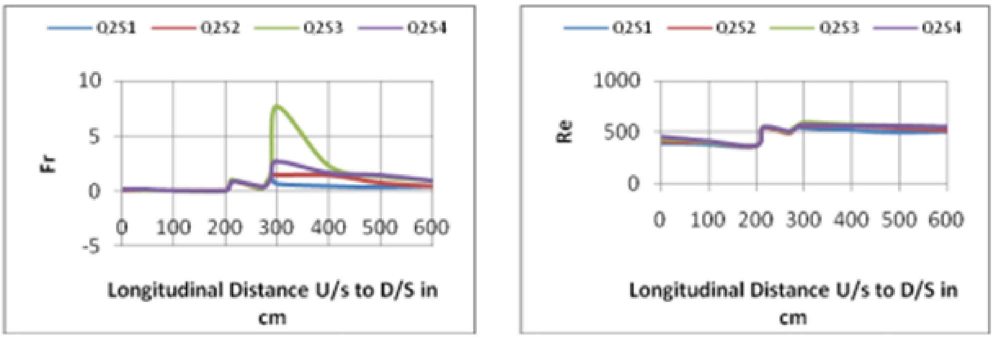
Hydraulic Structure : Drop-inlet Spillway (Down Scaled Model)



(a) *Variations in Flow Depths (cm)* (b) *Variations in Potential Head (m)*



(c) *Variations in Flow Velocities (m/s)* (d) *Variations in Manning's Roughness*

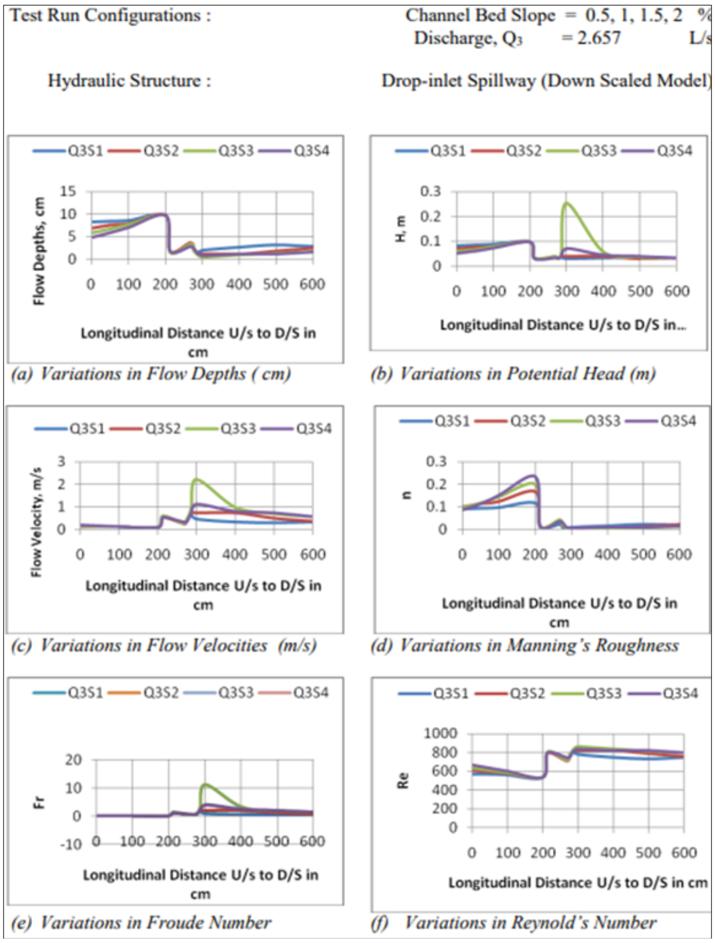


(e) *Variations in Froude Number* (f) *Variations in Reynold's Number*

Fig 7: Observed Hydraulic Parameters for Flows in Channel having Drop Inlet Spillway with 1.8297 L/s Discharge on 4 Different Channel Bed Slopes

Table 4: Overall range of hydraulic parameters during flume test runs using Drop inlet Spillway (Downscaled Model - Acrylic)

Flow Parameters	**Low Discharge Conditions *(0.79 l /s)***								**High Discharge Conditions *(3.35 l /s)***							
	Low Bed Slope (0.5%)				*High Bed Slope (2%)*				*Low Bed Slope (0.5%)*				*High Bed Slope (2%)*			
	Max	Min	Avg.	Deviation in %	Max	Min	Avg.	Deviation in %	Max	Min	Avg.	Deviation in %	Max	Min	Avg.	Deviation in %
h	8.7	0.6	1.629	93.1	8.7	0.5	1.029	94.3	10.8	0.8	2.700	92.6	10.5	1.2	1.986	88.60
v	0.44	0.03	0.192	93.1	0.53	0.03	0.317	94.3	1.40	0.10	0.534	92.6	0.93	0.11	0.682	88.6
H	0.09	0.01	0.019	84.5	0.09	0.013	0.016	84.7	0.11	0.04	0.048	67.7	0.11	0.04	0.046	66.5
n	0.34	0.01	0.028	98.5	0.68	0.008	0.028	98.9	0.11	0.00	0.015	98.2	0.21	0.008	0.019	96.4
Fr	1.84	0.04	0.601	97.8	2.4	0.041	1.223	98.3	5.11	0.13	1.431	97.4	2.82	0.14	1.864	95.2
Re	252.5	166.2	237.11	34.2	254.1	166.2	245.96	34.6	1059.5	648.9	949.17	38.8	1033.4	656.5	989.59	36.5



Test Run Configurations : Channel Bed Slope = 0.5, 1, 1.5, 2 %
Discharge, Q_3 = 2.657 L/s

Hydraulic Structure : Drop-inlet Spillway (Down Scaled Model)

(a) Variations in Flow Depths (cm)

(b) Variations in Potential Head (m)

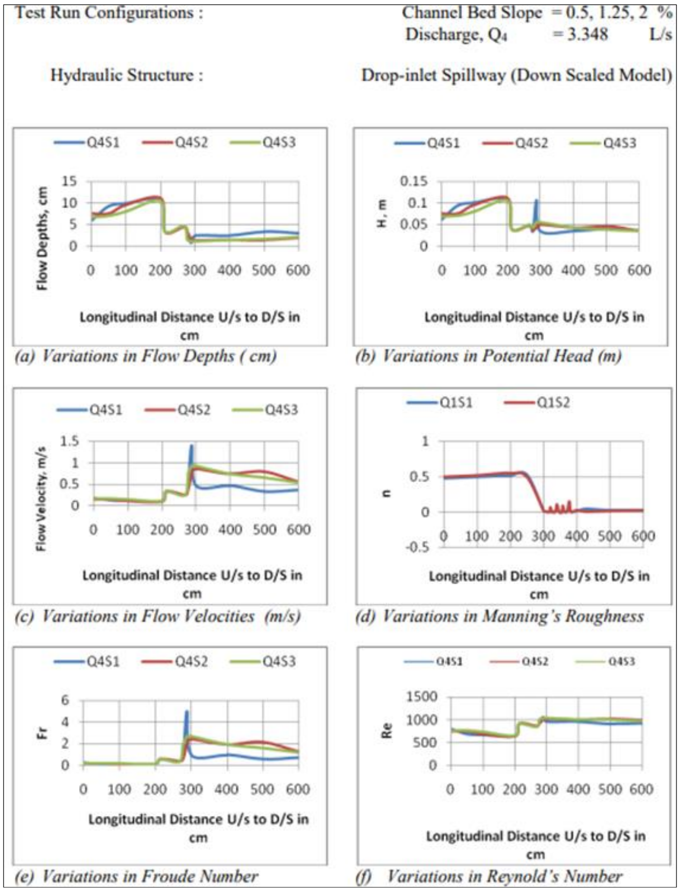
(c) Variations in Flow Velocities (m/s)

(d) Variations in Manning's Roughness

(e) Variations in Froude Number

(f) Variations in Reynold's Number

Fig 8: Observed Hydraulic Parameters for Flows in Channel having Drop Inlet Spillway with 2.657 L/s Discharge on 4 Different Channel Bed Slopes



Test Run Configurations : Channel Bed Slope = 0.5, 1.25, 2 %
Discharge, Q_4 = 3.348 L/s

Hydraulic Structure : Drop-inlet Spillway (Down Scaled Model)

(a) Variations in Flow Depths (cm)

(b) Variations in Potential Head (m)

(c) Variations in Flow Velocities (m/s)

(d) Variations in Manning's Roughness

(e) Variations in Froude Number

(f) Variations in Reynold's Number

Fig 9: Observed Hydraulic Parameters for Flows in Channel having Drop Inlet Spillway with 3.348 L/s Discharge on 4 Different Channel Bed Slopes

Influence of Hydraulic Structures on Variability Patterns of Flow Parameters

The overall net influence of types of various hydraulic structure as adopted in present study were found of significant state, giving ranges as well as percentage variations in varying magnitudes. The factors which influence the magnitudes of these flow parameters were the discharge rates as well as the channel bed slopes whatever adopted in this study. An in depth analysis is performed with categorized vision to reflect such influences for individual type of down scaled hydraulic structures under set of experimental conditions encompassing a pre determined discharge and slope combinations. The net variations in regards to various flow parameters as observed in the study are categorically tabulated and also graphically illustrated in Figures 6 to 9.The trend of these variations as observed in this case for different combinations of experimental discharges and slopes are numerically illustrated in Table 3, which is self-explanatory.

Results and Discussion

The averaged values of h under low discharge conditions changed from 1.63 to 1.03 cm when the channel bed slope enhanced form 0.5 to 2%. Similarly under high discharge conditions these values remained 2.7 & 1.99 cm respectively. The avenged values of v under low discharge conditions remains 0.192 m/s, moreover when the channel bed slope enhanced form 0.5 to 2% it enhanced to 0.317 m/s. Under high discharge conditions it was found to 0.534 to 0.682 m/s. Similar values and changes in regards to other flow parameters are well depicted in Table 4.5 which are self explanatory. The detailed values of all the flow parameters in respect of 4 specific discharge rates ranging from 0.79 to 3.35 L/s under 4 different slopes are well shown in Figures 6 to 9, where the trends of variations as well as the net values of 6 different flow parameters (as observed under present experimental set up) is clearly depicted showing significant variations and uniqueness with regards to respective hydraulic structures and associated flow conditions.

Coefficients of Discharges and Predictive Equations for Different Flow Conditions

The whole scenarios in regards to flow depths, flow velocities, roughness, Fr values, Re values, and over all flow regime during these additional runs is laborately depicted in Figures 6 to 9, giving a broad range of flow conditions. The C_d values for each categorized hydraulic structure under different sets of flow regimes/ conditions showed significant variations in accordance to type of structures as well as the flow conditions. The C_d values for the Drop inlet Spillway were found the be in the range of 0.004 to 0.107. Quadratic equation was found to be the easiest and most suitable function among the variety of alternate equations and models, hence the final shape of C_d predictive equations for hydraulic structures under study is shown in eq.(4), incorporating the numerical values of associated coefficients/parameters.

Drop Inlet Spillway- Straight Box Inlet (Acrylic);

$$C_d = -0.00566 + 8.041\ H - 159.39 \qquad \ldots\ldots\ldots\ldots(4)$$

Cd = Coefficient of Discharge; H = Total potential Head (m)

The Predictive performance of above cited equations were tested and a categorical comparisons of observed and predicted values of C_d values under different sets of conditions in regards to flows as well as hydraulic structures are illustrated in Figure 10, where the qualitative performance of these synthesized equations is well demonstrated.

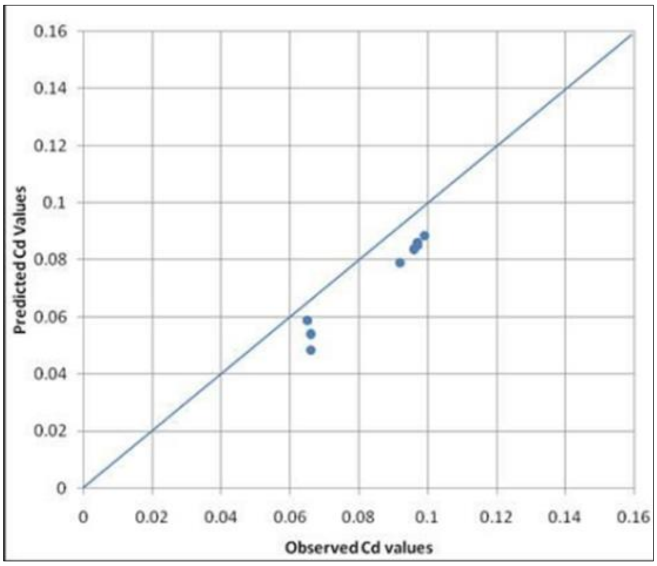


Fig 10: Validation Results of Cd Predictive Equations for Drop Inlet Spillway with Box Inlet

The predictive equations were derived for down-scaled models under generalized conditions with a pooled data set earmarked for calibration. Detailed predictive performances of these C_d equations are reported in the chapter reflecting their possible utilities in some a like ungauged situations.

Summary and Conclusions

The quantum of variations was highly dispersed at certain experimental configurations while it was a bit regular in some sets of conditions. Results of study revealed following key factual findings in regards to four specific hydraulic structures which were fabricated and installed inside the tilting hydraulic flume to derive numerical values of different flow parameters. In case where Down Scaled Drop Inlet Spillway was utilized in flume channel (pacing it at middle reach i.e. at about 3 meters from upstream end of flume) to control or regulate the flows all along the channel length of 6 meters, the observed averaged values of h under low discharge conditions changed from 1.63 to 1.03 cm when the channel bed slope enhanced from 0.5 to 2%. Similarly under high discharge conditions these values remained 2.7 & 1.99 cm respectively. The avenged values of v under low discharge conditions remains 0.192 m/s, moreover when the channel bed slope enhanced form 0.5 to 2% it enhanced to 0.317 m/s. Under high discharge conditions it was found to 0.534 to 0.682 m/s. Additionally, an attempt was made to utilize the observed set of flow data, as well the synthesized values of coefficient of discharges for their probable utilities to arrive at some of the generalized predictive equations for utility in similar or alike un gauged situations. A sizeable portion of the observed data and thus the Cd values were suitably screened and utilized for its curve fittings and arriving at some of the simplistic predictive equations where the Cd values can be directly computed by utilizing mere the flow heads/ flow depths over the crest of weir keeping strict adherence to experimental conditions as adopted in the present study. Utilizing the procedure as illustrated in the preceding chapter, the equations were developed for various hydraulic structures, which were in Quadratic Mathematical Shape. Quadratic equation was found most easy and suitable function among a variety of alternate equations and models, hence the final shape of Cd predictive equations for hydraulic structures under study is synthesized as flow.Drop Inlet Spillway-Straight Box Inlet (Acrylic);

$C_d = -0.00566 + 8.041\ H - 159.39H^2$.

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