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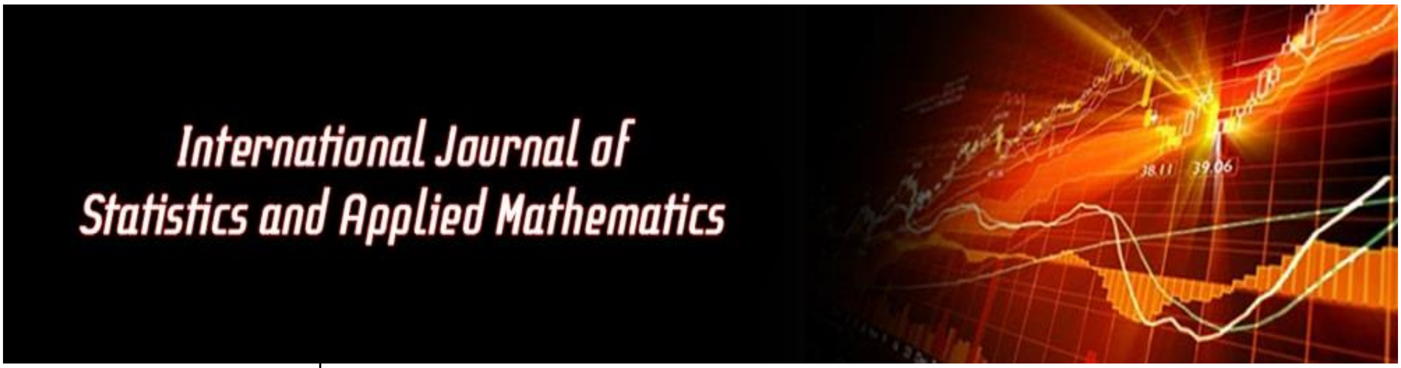

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An Application of multivariate time series models for forecasting the prices of tomato in Haryana

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Abstract
Time series forecasting of agricultural products has the basic importance in maintaining the sustainability of agricultural production. In this study, multivariate time series models: vector autoregressive (VAR) and vector autoregressive integrated moving average (VARMA) have been used for modelling the interdependence among the price series of tomato in selected APMC markets of Haryana. The forecasting performance of VAR and VARMA models have also been compared using percentage relative deviation (RD (%)), standard error of prediction (SEP) and mean absolute percent error (MAPE) for different forecast horizons 1, 3, 6, 9 and 12 months. The results suggest that forecast performance of VARMA models is more accurate than VAR models for forecasting the price series of tomato. The findings of the study would help in decision making for managing agricultural supplies and helping to improve the purchasing behavior of consumers.

Keywords: VAR, VARMA, tomato prices, multivariate time series and forecasting

Introduction

India is primarily an agriculture-based country and its economy largely depends upon agriculture. Agricultural price movements have been a matter of serious concern for policy makers in our country as the behaviors of agricultural prices is affecting the economic development. Among other things, price plays a strategic role in influencing the cultivation of food grains. Indeed, the price analysis of agricultural commodities assumes greater significant not only to the policy makers in formulating developmental plans but to both producers and consumers as well.

Time series forecasting is an important area in which prediction of future values of a variable is made based on past values of the variable first formulated the concept of Autoregressive (AR) and Moving Average (MA) models. Box and Jenkins (1970) integrated the existing knowledge in the book entitled "Time Series Analysis: Forecasting and Control" which has an enormous impact on the theory and practice of modern time series analysis and forecasting.

The modeling of multiple time series at the same time is known as multivariate time series or vector time series, which is used to examine the relationship between the time series as well as the structure responsible for their dynamic movement. There are two main reasons for using multivariate time series models: (i) to explain the dynamic relationships among the various time series, and (ii) to improve the accuracy of forecasts of one series using the information about that series contained in all other time series. Villani (2001) [8] used the Swedish monetary data for seven variables to forecast the inflation of the country using Bayesian co-integrated VAR. Pesaran *et al.* (2004) [6], Dees *et al.* (2007) [2] and Gutierrez *et al.* (2014) [4] employed the global Vector Autoregressive (GVAR) methodology to analyze the global wheat market. Poskitt (2011) [7] developed a new methodology for identifying the structure of VARMA time series models. Deryugina and Ponomarenko (2015) [3] built Bayesian VAR model comprising 14 major domestic real price and monetary macroeconomic indicators as well as external sector variables for Russia. Xu and Lin (2015) [9] used a Vector Autoregressive model to examine the factors that influence changes in carbon dioxide emissions in China. They discovered that energy efficiency is essential in reducing carbon dioxide emissions. Iwok and Okoro (2016) [5] used probability multivariate time series models to forecast stocks of the Nigerian banking sector.

Azubuike and Kosemoni (2017) [1] used Average Monthly Exchange Rates (AMER) of Naira (Nigerian currency) to six other currencies of the World to evaluate and compared the performance of univariate and multivariate based time series models. Zadrozny and Chen (2020) [10] used weighted-covariance factor decomposition of VARMA models for forecasting quarterly U.S. Real GDP at monthly intervals.

Materials and methods

Multivariate time series models are used to explain the interdependencies and co-movements of multiple variables. The assumption in multivariate analysis is that dependency of variables not only depends on their past values but also depend on other variables. The VAR models are used to capture the linear interdependencies among multiple time series.

In general, VAR (p) model is given by

$$y_t = \delta_t + \sum_{i=1}^{p} A_i y_{t-i} + \varepsilon_t \ \dots \qquad (1)$$

Where, $y_t = (y_{1t}, y_{2t}, \dots\dots, y_{kt})'$, and the $A_i's\ (i = 1, 2, \dots\dots, p)$ are $(K \times K)$ coefficient matrices and $\varepsilon_t = (\varepsilon_{1t}, \varepsilon_{2t}, \dots\dots, \varepsilon_{kt})'$ and $\delta_t = \left(\delta_{t_{1t}}, \delta_{t_{2t}}, \dots\dots, \delta_{t_{kt}}\right)'$ is K-dimensional error term and constant term.A_{ij} is the (i, j)th element of A matrix and A_{ij} denotes the linear dependence of y_{it} on $y_{j,t-1}$ in the presence of $y_{i,t-1}$, hence A_{ij} is the conditional effect of y_{it} on $y_{j,t-1}$ given $y_{i,t-1}$. If $A_{12} = 0$, then y_{it} does not depend on $y_{j,t-1}$ and indicate that y_{it} only depend on its own lag period.
VAR model of order p may have some drawbacks. However, the A_i parameter matrices are unknown and can be replaced by estimators. VAR model of large order is required for an appropriate representation of time series data. Hence, a large number of parameters may be required to adequate description of the data. Given limited sample size, this usually lead to low estimation precision and forecasts based on VAR model with estimated coefficients may suffer from the uncertainty in the parameter estimations. As a result, it is useful to consider the larger model class of vector autoregressive moving average (VARMA) model which may be able to adequately describe the data in a more parsimonious manner. VARMA models for multivariate time series include the VAR model as well as moving average terms for each variable. VARMA (p, q) model is written as

$$y_t = \delta_t + \sum_{i=1}^{p} A_i y_{t-i} - \sum_{j}^{q} M_j \varepsilon_{t-j} + \varepsilon_t \ \dots \qquad (2)$$

Where $A_i\ (i = 1, 2, \dots, p)$ are $(K \times K)$ matrices of autoregressive parameters and $M_j\ (j = 1, 2, \dots, q)$ are $(K \times K)$ matrices of moving average parameters. Defining the VAR and MA operators:

$$A(B) = A_0 + A_1 B + A_2 B^2 + \cdots + A_p B^p \ \dots \qquad (3)$$

$$M(B) = M_0 - M_1 B - M_2 B^2 - \cdots - M_q B^q \ \dots \qquad (4)$$

The model can be written in more compact notation as

$$A(B) y_t = M(B) \varepsilon_t, t = 0, \pm 1, \pm 2, \dots\dots \ \dots \qquad (5)$$

Where, ε_t is a white noise process with mean zero, nonsingular, time- invariant covariance matrix $E(\varepsilon_t \varepsilon_t') = \Sigma$ and covariances zero, $E(\varepsilon_t \varepsilon_{t-h}') = 0$ for $h = 0, \pm 1, \pm 2, ..$ The A_0 and M_0 matrices are assumed to be nonsingular. They are often identical $A_0 = M_0$ and in many cases they will be equal to identity matrix $A_0 = M_0 = I_k$.

Comparison and Validation of the Developed Models

Model Selection

Information criteria such as Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) are used to select an appropriate model.

$$\text{AIC} = -2In(\text{L}) + 2k \ \dots \qquad (6)$$

$$\text{BIC} = -2In(\text{L}) + In(\text{N})k \qquad (7)$$

Where L is the value of the likelihood function evaluated at the parameter estimates, N is the number of observations and k is the number of estimated parameters. The models having minimum values of AIC and BIC will be selected as better model.

Model Evaluations

The model is evaluated quantitatively using the MAPE, SEP, RMSE and Relative Deviation (RD%). The SEP is used for the comparison of forecast from different models because of its dimension less.

$$\text{MAPE} = \frac{100}{\text{n}} \times \sum_{\text{i}=1}^{\text{n}} \left| \frac{\text{O}_\text{i} - \text{E}_\text{i}}{\text{O}_\text{i}} \right| \ \dots \qquad (8)$$

$$\text{SEP} = \frac{100}{\bar{\text{y}}} \text{RMSE where RMSE} = \left[\frac{1}{\text{n}} \sum_{\text{i}=1}^{\text{n}} (\text{O}_\text{i} - \text{E}_\text{i})^2 \right]^{\frac{1}{2}} \ \dots \qquad (9)$$

$$RD = \frac{O_i - E_i}{O_i} \times 100 \;\ldots \qquad (10)$$

Where, O_i, $\bar{y}$ and E_i are the observed, mean and predicted values and n is the number of observations in validation set.

Results

Vector Autoregression (VAR) model

In Table 1, the results of KPSS test show that the differenced series are stationary and hence, VAR and VARMA models can be fitted. The order of VAR and VARMA models is determined on the basis of lowest value of information Criteria such as Akaike information criterion (AIC) and Bayesian Information Coefficient (BIC).

Table 1: KPSS test for stationary checking

Markets	Level (actual series)		Differencing (d=1, D=1)	
	t-statistic	p-value	t-statistic	p-value
Gurugram	0.73	0.01	0.02	0.1
Yamunanagar	0.85	0.01	0.03	0.1
Panipat	0.74	0.01	0.04	0.1
Kurukshetra	0.81	0.01	0.03	0.1
Panchkula	1.36	0.01	0.05	0.1
Karnal	0.77	0.01	0.04	0.1

Table 2: Information criteria for VAR models of different orders

VAR (p)			VARMA (p,q)		
Order(p)	AIC	BIC	Order (p, q)	AIC	BIC
1	**67.92**	**68.95**	**1,1**	**67.01**	**70.82**
2	68.07	69.98	1,2	113.82	116.47
3	68.27	71.07	2,1	68.81	71.46
4	68.61	72.29	2,2	68.81	71.46
5	68.75	73.32	2,3	129.63	133.96

Table 2 shows that the results of information criteria for different order of p and q. Vector Autoregression of order 1 *i.e.* VAR (1) is selected on the basis of minimum values of AIC (67.92), BIC (68.95) and HQC (68.34) and VARMA (1,1) is chosen with the least value of AIC (67.01) and BIC (70.82) among the other models.

Parameter Estimation

The estimated parameters of VAR (1) models are given below:

$$\begin{bmatrix} \Delta\hat{Y}_{Gurugram,t} \\ \Delta\hat{Y}_{Yamunanagar,t} \\ \Delta\hat{Y}_{Panipat,t} \\ \Delta\hat{Y}_{Kurukshetra,t} \\ \Delta\hat{Y}_{Panchkula,t} \\ \Delta\hat{Y}_{Karnal,t} \end{bmatrix} = \begin{bmatrix} 13.67 \\ 16.61 \\ 17.15 \\ 17.06 \\ 15.69 \\ 9.41 \end{bmatrix} + \begin{bmatrix} 0.64^{**} & -0.22 & -0.44^{*} & -0.11 & -0.12^{**} & -0.02 \\ 0.67^{**} & -0.26 & -0.28 & -0.10 & -0.13 & -0.03 \\ 0.63^{**} & -0.23 & -0.50^{**} & -0.01 & -0.01 & -0.04 \\ 0.59^{**} & -0.06 & -0.31^{*} & -0.27 & -0.08 & -0.01 \\ 0.56^{*} & -0.27 & -0.22 & 0.39 & -0.44^{**} & -0.10 \\ 0.41^{*} & -0.33 & -0.27 & 0.18 & 0.01 & 0.05 \end{bmatrix} \begin{bmatrix} \Delta Y_{Gurugram,t-1} \\ \Delta Y_{Yamunanagar,t-1} \\ \Delta Y_{Panipat,t-1} \\ \Delta Y_{Kurukshetra,t-1} \\ \Delta Y_{Panchkula,t-1} \\ \Delta Y_{Karnal,t-1} \end{bmatrix}$$

'*' significant at 5% level of significance

'**' significant at 1% level of significance

Out of forty-two estimated parameters, only eleven parameters were found to be significant in this model. So, the non-significant parameters are restricted to zero because non-significant parameters reduce the accuracy of model and the following VAR (1) model is obtained.

$$\Delta\hat{Y}_{Gurugram,t} = 0.64\Delta\hat{Y}_{Gurugram,t-1} - 0.44\Delta\hat{Y}_{Panipat,t-1} - 0.12\Delta\hat{Y}_{Panchkula,t-1}$$

$$\Delta\hat{Y}_{Yamunanagar,t} = 0.67\Delta\hat{Y}_{Yamunanagar,t-1}$$

$$\Delta\hat{Y}_{Panipat,t} = 0.63\Delta\hat{Y}_{Gurugram,t-1} - 0.50\Delta\hat{Y}_{Panipat,t-1}$$

$$\Delta\hat{Y}_{Kurukshetra,t} = 0.59\Delta\hat{Y}_{Gurugram,t-1} - 0.31\Delta\hat{Y}_{Panipat,t-1}$$

$$\Delta\hat{Y}_{Panchkula,t} = 0.56\Delta\hat{Y}_{Gurugram,t-1} - 0.44\Delta\hat{Y}_{Panchkula,t-1}$$

$$\Delta\hat{Y}_{Karnal,t} = 0.41\Delta\hat{Y}_{Gurugram,t-1}$$

The obtained the estimated parameters of chosen VARMA (1,1) model given as below

$$\begin{bmatrix} \Delta\hat{Y}_{Gurugram,t} \\ \Delta\hat{Y}_{Yamunanagar,t} \\ \Delta\hat{Y}_{Panipat,t} \\ \Delta\hat{Y}_{Kurukshetra,t} \\ \Delta\hat{Y}_{Panchkula,t} \\ \Delta\hat{Y}_{Karnal,t} \end{bmatrix} = \begin{bmatrix} 11.57 \\ 13.09^{*} \\ 19.06^{*} \\ 15.03 \\ 14.81 \\ 6.64 \end{bmatrix} + \begin{bmatrix} 0.54 & -0.19 & -0.44^{**} & -0.53 & -0.21 & 0.13 \\ 0.38 & -0.08 & -0.03 & -0.53^{**} & -0.46 & 0.14 \\ 0.42 & 0.45 & -0.37 & -0.17 & -0.16 & 0.49^{**} \\ 0.78^{**} & 0.55^{**} & -0.33^{**} & -0.57 & -0.24 & 0.22 \\ 0.75 & 0.16 & -0.11 & -0.03 & -0.08 & -0.03 \\ 0.01^{**} & -0.33 & -0.21 & -0.08 & 0.23 & 0.16 \end{bmatrix} \begin{bmatrix} \Delta\hat{Y}_{Gurugram,t-1} \\ \Delta\hat{Y}_{Yamunanagar,t-1} \\ \Delta\hat{Y}_{Panipat,t-1} \\ \Delta\hat{Y}_{Kurukshetra,t-1} \\ \Delta\hat{Y}_{Panchkula,t-1} \\ \Delta\hat{Y}_{Karnal,t-1} \end{bmatrix}$$

$$- \begin{bmatrix} 0.74^{**} & 0.91 & -0.21^{*} & -0.49 & -0.17 & 0.59^{**} \\ -0.53^{**} & 0.57 & 0.41 & -0.70^{**} & -0.37^{**} & 0.36 \\ -0.71 & 0.27 & 0.36 & -0.47^{*} & -0.08 & 0.72^{**} \\ 0.01 & 0.07^{**} & -0.05^{*} & -0.64^{**} & -0.17 & 0.46^{**} \\ 0.21 & 0.45 & 0.19 & -0.06^{*} & 0.41^{**} & 0.48^{**} \\ 0.38 & 0.28 & 0.23 & -0.62 & 0.35^{**} & 0.37 \end{bmatrix} \begin{bmatrix} \hat{e}_{Gurugram,t-1} \\ \hat{e}_{Yamunanagar,t-1} \\ \hat{e}_{Panipat,t-1} \\ \hat{e}_{Kurukshetra,t-1} \\ \hat{e}_{Panchkula,t-1} \\ \hat{e}_{Karnal,t-1} \end{bmatrix}$$

'*' significant at 5% level of significance
'**' significant at 1% level of significance

Out of seventy-eight estimated parameters, only twenty-five parameters were found to be significant in this model. So, the non-significant parameters were restricted to zero because non-significant parameters reduce the accuracy of model and the following VARMA (1,1) model is obtained.

$$\Delta\hat{Y}_{Gurugram,t} = -0.44\Delta\hat{Y}_{Panipat,t-1} - 0.74\hat{e}_{Gurugram,t-1} + 0.21\hat{e}_{Panipat,t-1} - 0.59\hat{e}_{Karnal,t-1}$$

$$\Delta\hat{Y}_{Yamunanagar,t} = 13.09 - 0.53\Delta\hat{Y}_{Kurukshetra,t-1} + 0.53\hat{e}_{Gurugram,t-1} + 0.70\hat{e}_{Kurukshetra,t-1} + 0.37\hat{e}_{Panchkula,t-1}$$

$$\Delta\hat{Y}_{Panipat,t} = 19.06 + 0.49\Delta\hat{Y}_{Karnal,t-1} + 0.47\hat{e}_{Panipat,t-1} - 0.72\hat{e}_{Karnal,t-1}$$

$$\Delta\hat{Y}_{Kurukshetra,t} = 0.78\Delta\hat{Y}_{Gurugram,t-1} + 0.55\Delta\hat{Y}_{Yamunanagar,t-1} - 0.33\Delta\hat{Y}_{Panipat,t-1} - 0.07\hat{e}_{Yamunanagar,t-1}$$

$$+0.05\hat{e}_{Panipat,t-1} + 0.64\hat{e}_{Kurukshetra,t-1} - 0.46\hat{e}_{Karnal,t-1}$$

$$\Delta\hat{Y}_{Panchkula,t} = 0.06\hat{e}_{Kurukshetra,t-1} - 0.41\hat{e}_{Panchkula,t-1} + 0.48\hat{e}_{Karnal,t-1}$$

$$\Delta\hat{Y}_{Karnal,t} = 0.01\Delta\hat{Y}_{Gurugram,t-1} - 0.35\hat{e}_{Panchkula,t-1}$$

Table 3: Multivariate Ljung-Box Q statistic for VAR (1) and VARMA (1,1) models

Lag	VAR (1)			VARMA (1,1)		
	Q Statistic	**Df**	**p-value**	**Q Statistic**	**df**	**p-value**
1	16.88	36	0.97	22.6	36	0.96
2	69.25	72	0.56	56.6	72	0.91
3	109.88	108	0.43	89.3	108	0.95
4	137.68	144	0.63	117.6	144	0.81
5	175.29	180	0.58	163.3	180	0.84
6	204.93	216	0.69	195.2	216	0.82

The multivariate Ljung-Box test used to check the autocorrelation of residuals from selected VAR (1) and VARMA (1,1) models. Table 3 displays the results of multivariate Ljung-Box Q statistic is not significant at the 5% level of significance, then residuals series are not autocorrelated. As a result, the series may be categorized as white noise. As a result, the VAR (1) and VARMA (1,1) models are selected as an appropriate model for forecasting tomato prices. Tables 4, 5 and 6 show forecasting performance of selected as appropriate VAR and VARMA models in terms of relative deviation (RD (%)) for prices of tomato in Gurugram & Yamunanagar, Panipat & Kurukshetra and Panchkula & Karnal markets respectively.

Table 4: Forecasting performance of VAR (1) model for prices of tomato in Gurugram and Yamunanagar markets of Haryana

Months	Gurugram					Yamunanagar				
	Observed	VAR	RD (%)	VARMA	RD (%)	Observed	VAR	RD (%)	VARMA	RD(%)
Jan-20	1614.10	1292.00	19.955	1248.00	22.68	1992.36	1549.00	22.253	1235.00	38.01
Feb-20	1388.38	1085.00	21.851	1351.00	2.69	1527.39	1282.00	16.066	1340.00	12.27
March-20	1669.24	1062.00	36.378	1665.00	0.25	1543.43	1169.00	24.260	1322.00	14.35
April-20	1814.90	1146.00	36.856	2062.00	-13.62	1604.29	1169.00	27.133	1880.00	-17.19
May-20	937.56	1171.00	-24.899	1121.00	-19.57	857.97	1030.00	-20.051	959.00	-11.78
June-20	937.56	1092.00	-16.473	1137.00	-21.27	366.02	1011.00	-176.214	923.00	-152.17
July-20	2671.11	1488.00	44.293	1995.00	25.31	2806.55	1386.00	50.616	2074.00	26.10
Aug-20	2217.97	1551.00	30.071	1860.00	16.14	2635.12	1443.00	45.240	1913.00	27.40
Sept-20	3050.00	1587.00	47.967	1791.00	41.28	3430.55	1680.00	51.028	2825.00	17.65
Oct-20	3050.00	1602.00	47.475	1769.00	42.00	2929.84	1500.00	48.803	2268.00	22.59
Nov-20	2054.60	1605.00	21.883	1793.00	12.73	2928.72	1508.00	48.510	2066.00	29.46
Dec-20	1322.00	1601.00	-21.104	1821.00	-37.75	2126.59	1508.00	0.29	1788.00	15.92

Table 5: Forecasting performance of VAR (1) model for prices of tomato in Panipat and Kurukshetra markets of Haryana

Months	Panipat					Kurukshetra				
	Observed	VAR	RD (%)	VARMA	RD (%)	Observed	Predicted	RD (%)	VARMA	RD (%)
Jan-20	2274.09	1711.00	24.76	1075.00	52.73	2104.86	1465.00	30.40	1307.00	37.91
Feb-20	1334.74	1384.00	-3.69	1043.00	21.86	1176.79	1200.00	-1.97	1317.00	-11.91
March-20	1422.76	1225.00	13.90	1259.00	11.51	1616.69	1096.00	32.21	1339.00	17.18
April-20	1422.76	1202.00	15.52	1810.00	-27.22	1616.69	1106.00	31.59	1677.00	-3.73
May-20	691.46	1260.00	-82.22	942.00	-36.23	923.85	1175.00	-27.19	784.00	15.14
June-20	668.42	1349.00	-101.82	978.00	-46.32	491.27	1261.00	-156.68	848.00	-72.61
July-20	2006.78	1436.00	28.44	2004.00	0.14	2287.73	1339.00	41.47	1797.00	21.45
Aug-20	2001.09	1505.00	24.79	1878.00	6.15	1970.67	1398.00	29.06	1698.00	13.84
Sept-20	2712.35	1552.00	42.78	1790.00	34.01	3131.43	1435.00	54.17	1632.00	47.88
Oct-20	2740.49	1578.00	42.42	1732.00	36.80	2581.26	1455.00	43.63	1591.00	38.36
Nov-20	2658.78	1589.00	40.24	1734.00	34.78	2469.07	1463.00	40.75	1592.00	35.52
Dec-20	1598.77	1591.00	0.49	1754.00	-9.71	2208.14	1463.00	33.75	1609.00	27.13

Table 6: Forecasting performance of VAR (1) model for prices of tomato in Panchkula and Karnal markets of Haryana

Months	Panchkula					Karnal				
	Observed	VAR	RD (%)	VARMA	RD (%)	Observed	VAR	RD (%)	VARMA	RD (%)
Jan-20	2116.62	1644.00	22.33	2282.00	-7.81	1599.15	2006.00	-25.44	1118.00	30.09
Feb-20	1744.46	1325.00	24.05	2345.00	-34.43	1913.64	1523.00	20.41	1196.00	37.50
March-20	2328.11	1175.00	49.53	2115.00	9.15	1913.64	1262.00	34.05	1662.00	13.15
April-20	2313.11	1152.00	50.20	2445.00	-5.70	1699.05	1166.00	31.37	2070.00	-21.83
May-20	1425.64	1206.00	15.41	2066.00	-44.92	771.70	1171.00	-51.74	1108.00	-43.58
June-20	852.53	1291.00	-51.43	1162.00	-36.30	1586.34	1223.00	22.90	1328.00	16.29
July-20	3454.00	1976.00	42.79	2431.00	29.62	2795.52	1287.00	53.96	2184.00	21.88
Aug-20	3438.08	1846.00	46.31	2303.00	33.01	3613.59	1344.00	62.81	2036.00	43.66
Sept-20	2988.79	1995.00	33.25	2240.00	25.05	3569.22	1386.00	61.17	2915.00	18.33
Oct-20	3643.18	1924.00	47.19	2184.00	40.05	1834.54	1413.00	22.98	1864.00	-1.61
Nov-20	4295.95	1537.00	64.22	2165.00	49.60	1976.39	1426.00	27.85	1863.00	5.74
Dec-20	3059.26	1541.00	49.63	2162.00	29.33	1498.78	1431.00	4.52	1882.00	-25.57

Table 7: Forecasting accuracy measures of selected models for price of tomato in all selected markets

Models	1 month		3 months		6 months		9 months		12 months	
	MAPE	SEP	MAPE	SEP	MAPE	SEP	MAPE	SEP	MAPE	SEP
Gurugram										
VAR	19.96	17.01	26.06	22.90	26.07	22.51	30.97	39.65	30.77	41.61
VARMA	22.68	19.33	8.54	11.22	13.35	11.20	18.09	27.49	21.27	31.97
Yamunanagar										
VAR	22.25	21.50	20.86	17.64	47.66	20.11	48.10	44.37	46.61	48.45
VARMA	38.01	36.72	21.54	22.70	40.96	20.32	35.21	25.45	32.07	27.20
Panipat										
VAR	24.76	31.38	14.12	19.27	40.32	34.86	37.55	32.77	35.09	38.10
VARMA	52.73	66.82	28.70	40.05	32.64	31.01	26.24	30.66	26.45	34.58
Kurukshetra										
VAR	30.40	34.01	21.53	25.32	46.67	27.42	44.97	42.31	43.57	44.83
VARMA	37.91	42.40	22.33	26.28	26.41	20.40	26.85	32.89	28.56	36.16
Panchkula										
VAR	22.33	17.91	31.97	28.77	35.49	28.18	37.25	37.95	41.36	51.20

VARMA	7.81	6.27	17.13	14.41	23.05	15.13	25.11	24.80	28.75	36.83
Karnal										
VAR	25.44	19.71	26.64	24.10	30.99	22.71	40.43	59.36	34.93	52.32
VARMA	30.09	23.31	26.91	25.17	27.07	21.00	27.37	33.94	23.27	29.92

Discussions

Forecasting accuracy measures *i.e.*, MAPE and SEP of VAR and VARMA models for forecasting horizons of 1, 3, 6, 9 and 12 months was presented in Table 7 for prices of tomato in all selected markets. Table 7 shows the combined results of VAR and VARMA models at various forecasting horizons, indicate that VARMA model provides better forecast accuracy measures in terms of lower value of MAPE and SEP as compared to VAR model for various forecasting horizons except 1 month ahead. In Yamunanagar market, VAR model performed better than VARMA model for 1, 3 and 6 months ahead forecasting horizon but VARMA model better performed for 9 and 12 months ahead. VAR model performed better than VARMA model for 1 and 3 months ahead forecasting horizon but VARMA model better performed for 6, 9 and 12 months ahead in Panipat, Kurukshetra and Karnal market. In Panchkula market, VARMA model performed better than VAR for each forecasting horizon.

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