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# Aryabhata's pioneer contribution in mathematics: The father of Indian mathematics

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## Abstract
Aryabhata was a prominent Indian mathematician and astronomer who is the Father of Indian Mathematics. He is well known all around the world for his elegant pieces of work, legacy and inventions in these fields. He was the first mathematician cum astronomer from the classical period of Indian mathematics and astronomy. His pieces of work include the Aryabhatiya (c. 499), the main surviving Scripture from Aryabhata's classical pieces of work which is scripted in 118 verses describing Hindu Mathematics up to that time, and the now lost Aryasiddhanta. The place Kusumapura is situated near Patalipurta (Patna), then the capital of the Gupta dynasty, where he wrote at least these two pieces of treatises. It studies subjects such as plane and spherical trigonometry, quadratic equations, arithmetic and algebra. Aryabhata is also considered a great pioneer physicist for his explicitly describing the relativity of motion. Aryabhata's work also includes his notions of cosine, sine, inverse sine, and versed sine, which forms the foundation of trigonometry. He was also the first mathematician to obtain the calculation based on sine and versine tables from 0 to 180 degrees with four decimal places of precision. Patliputra was the centre of communications for network channel which facilitated learning from other parts of the world to access it and also facilitated the mathematical, and astronomical contents by Aryabhata and his school to travel across India and also eventually into the Islamic world.

**Keywords:** God, Allah, religion, ganita, aryabhata, kuttaka, aryabhatiya, kala-kriya, algebra, indeterminate equations, solar system, zero (0), Pi (π), trigonometry, eclipse, heliocentrism, sidereal period

## Introduction
Parts of this survey work have been submitted a well as presented by the first author at the "29th International Research Conference 2024" held at J. D. Women's College, organized by Patliputra University, Patna, Bihar & Global Leaders Foundation, New Delhi, India during 15th-16th March, 2024.

It is befitting to start the introductory section of this survey work with the following quotation: "We owe a lot to the ancient Indians teaching us how to count. Without which most modern scientific discoveries would have been impossible". Albert Einstein

Indian civilization is one of the ancient civilizations in all over the world. This civilization was unknown to many people for their reluctance to give it proper recognition over many hundreds of years. It has a considerable and directly part playing character in the construction of mathematics of our current era. At a first glance, it is to point out that the very application of the term 'Indian Mathematics' is relative in context. This term may even mean mathematics originated in India that also developed in Pakistan, Nepal, Bangladesh and Sri Lanka.

A group of intellectuals including the Brahmans became designated priests and cult of learning. Another cult of soldiers and nobles became the designated Kshatriyas; the trading group and the agricultural one were called the Vaishyas. Furthermore, the artisans and labourers were embellished with the Shudra. This thus was the origin of the classification of the Hindus into four *varna*s (i.e., "classes"). The terms have nothing to do with caste system. It rather signifies possessor of respective kinds of knowledge required at the time for further enhancement. Religion was the source of motivation for all kinds of movements in ancient India.

It was a certain kind of absorbing all India Knowledge System (IKS) revealing inquisitive minds. It embraced not only worship and prayer but also philosophy, law, morality, and government organizations too. Religion got drenched and exemplified in educational endeavours as well, and the tuition of the Vedic literature became thus so crucial.

The child got opportunity for rudimentary education at home. The beginning of secondary education and formal schooling was performe by a solemn ceremony called the upanayana, i.e., thread ceremony, which was set for boys only and was more or less considered requisite for boys of the three higher classes. The Brahman boys had this formal opportunity at the age of 8, the Kshatriya boys at the age of 11, and the Vaishya boys at the age of 12. The boy used to leave his father's house and step into his preceptor's ashrama. An ashrama was a home situated in the middle of spaces surrounded with wooded regions. The acarya would consider him as his own baby, give him free education, and not charge anything for his lodging and boarding. The student had to tend the sacrificial fires (Agni), do the household work of his teacher (Professor) as well, and supervise his cattle too.

The study at this juncture consisted of the reading aloud the Vedic *mantra*s, i.e., "hymns" and the auxiliary scientific phonetics, the methodologies for the performance of the sacrifices, grammar, astronomy, etymology and prosody. The characteristics of education, however, were at variance according to the requirements of the prevailing class system. For a child of the clerical class, there used to be a specific syllabus for studies. The trayi-vidya or the accomplishment of learning of the three Vedas, the most primordial one of Hindu scriptures, was considered mandatory for him. During the whole course of time at school, as that at college, the student had to practice brahmacharya that is, wearing simple dress living on plain food, lying on the hard bed/chowki/cot, and leading a self-controlled life style.

The studentship period normally extended to 12 years. For those who desired to continue their endeavours, there was no age limit. After completing their education at an ashrama, they would join a higher centre of education or a university presided over by a kulapati *(*The Vice Chancellor, in modern jargon*)*, i.e., that is also used to be a founder of a school of thought. Higher strata pupils would also make improvement over their existing knowledge by taking participation in philosophical discussions at a parisad, or "academy." There were not any hindrances to women education and women empowerment, but normally girls were instructed at home.

The method of teaching varied according to the nature of the subject matter of study. The first duty of the student was to memorize the particular Veda of his school of thought, with special emphasis put on the pronunciation. In the study of such literary subject matter of study as logic, law, rituals, and prosody, it was the comprehension that played a very vital role. A third method was the application of parables, which were employed in the personal spiritual teaching relating to the Upanishads, or culmination of the Vedas. In higher education scenario, such as in the teaching of Dharma-Shastra, i.e., "Science of Righteousness", the most celebrated and utilitarian methodology was catechism in which the student was supposed to ask questions and the teacher to discourse incessantly on the related topics. The flashback, however, displayed so crucial its role.

It is pertaining here to take notice of the fact that neither Buddhism nor Jainism recognized the authority of the Vedas, and both challenged the exclusive claims of the Brahmans to become endowed with priesthood. They taught through the common language of the community, and gave education to all, irrespective of caste, creed, or sex. The establishment of the imperialistic Nanda dynasty about 413 BCE and then of the even stronger Maurya dynasty some 40 years later shook the very foundations of the Vedic structure of life, culture, and polity.

It is worth taking cognizance of the revolution that in the 3rd century BCE, Buddhism was able to get a great impetus under India's most celebrated ruler called Ashoka. Consequently; a Counter Reformation approach in Hinduism began to materialize in the whole country for Buddhism conjured up resistance. About the 1st century CE, there was also a global lay action among both Hindus and Buddhists. As a result of these phenomenal series of events, Buddhist monasteries began to undertake non-religious as well as religious education. There initiated a huge growth of popular fundamental education along with secondary and higher education.

Ancient India achieved success in providing mathematics with many engrossing ideas well ahead of their effective materialization elsewhere of the world. The earliest such mathematical pieces of work go back at least to the Vedas period (ca. 2500-1700 BCE). The numbers were the first mathematical notions which were discussed considerably in the Vedic texts. They open out through oral communication in combinations of numbers raised to a power of 10 from a hundred up to a trillion, which facilitated them to handle huge numbers so easily. Geometrical constructions and numerical computations were written in the Baudhayana Sulbasutra (around 800-600 BCE), out of religious ceremonies, especially for the requirement to construct and plan fire altars of specific shapes and sizes. The Indian numerals were discovered on pillars from the Ashokas authority. These numerals were, in fact, one of the most important mathematical fulfillments that made important contributions in the domain of mathematics.

The various arithmetical notions for computing the four operations: addition, subtraction, multiplication and division on positive and negative numbers, were mentioned by Brahmagupta (598-668 CE) in the comprehensive Indian pieces of work "the Brahmasphutasiddhanta". Mahavira, the ninth century mathematician, composed the first Sanskrit textbook committed completely to mathematics, rather than being an adjunct one to astronomy. Siddhantasiromani, composed by Bhaskara II (1114-1185 CE), was the pieces of work that discussed geometry, algebra, permutations and combinations, and even quadratic equations with many roots. It also studied linear system of equations in more than one variable. There is suggestive evidence that these equations have more than one solution. He solved Pell's equation by applying the Chakravakam method. The Indian mathematical school in Kerala too got success in giving higher contributions to mathematics, such as the developments of infinite series, especially, those associated with trigonometric functions.

The place Kusumapura is thought to be situated near Patliputra. It was again founded as Patna, Bihar in 1541 which is considered to be the birth place of Aryabhata. Kusumapura is on the Ganges and is more inclined towards the north side. There have been various other places too thought by some historians for places of his birth. Some affirm that he was born in South India, perhaps, Kodungallur which was the historical capital city of Thiruvanchikkulam of ancient Kerala. Some believe that he was born in Tamil Nadu; Asmaka or Andhra Pradesh; whereas still many others consider that he was born in the north-east of India, perhaps, in Bengal. The pieces of information that emanate from Bhaskara I, who describes Aryabhata as Asmakiya, is indicative of his hailing from the Asmaka country. During the Buddha's era, a group of the Asmaka community settled in the region between the Narmada and Godavari rivers in the Central India.

Kusumapura became one of the two major mathematical centres of excellence of learning in India, the other one was Ujjain. All of them are located in the north region except Kusumapura which is considered to be situated near Patalipurta, i.e., Patna, Bihar, then the capital of the Gupta dynasty, where he wrote at least these two pieces of work known as Aryabhatiya (c. 499), and the now lost Aryabhatasiddhanta. Pataliputra was the center of some communications for network medium which facilitated learning from other parts of the world to access it easily, and it also facilitated the mathematical as well as astronomical pieces of work by Aryabhata. His school facilitated to travel across India, and also eventually into the Islamic world.

It also seems that, at some point in time, he might have visited Kusumapura for higher studies and lived there for some time. As a matter of fact, Hindu and Buddhist, as well as Bhaskara I (CE 629), consider Kusumapura as Paṭliputra, modern day Patna. It is also described like that Aryabhata was the head of an institution called *Kulapa* at Kusumapura, and, since the University of Nalanda was located in Pataliputra at the time and had an astronomical observatory too, it is observed that Aryabhata might have been the head of the Nalanda University as well. Aryabhata is also known to have established an observatory at the Sun temple located in Taregana, Bihar.

Many of Aryabhata's pieces of work have been lost to time's unfortunate tidal wave of current, whereas some are so far available, and modern scholars hold them in high esteem for their tremendous authenticity. Thus, one ought to be aware of Aryabhata's notable discoveries, which have endowed India with so much pride and honour. Aryabhatasiddhanta disseminated mainly in the northwest of India and, through the Sasanian dynasty (224-651) of Iran, had an intellectual impact on the development of Islamic astronomy. Its contents are preserved up to some extent in the pieces of work by Varahamihira (c.550), Bhaskara I (629), Brahmagupta (598-c.665), and many more. It is one of the earliest astronomical pieces of work to compute the starting of each day to midnight.

The main continuing Scripture is Aryabhata's classical pieces of work named "Aryabhatiya*"*. It is a small astronomical book scripted in 118 verses in total. It describes a note of Hindu mathematics up to that time. Its mathematical section consists of 33 verses with 66 mathematical rules without proof. The Aryabhatiya consists of 10 verses for the introduction section, followed by a subsection on mathematical contents with, and as already mentioned, 33 verses. Thereafter, a section of 25 verses on the reckoning on time and planetary working models is composed. Its final chapter consists of 50 verses on the sphere and eclipses titling this section with 'Set of ten giti stanzas*'*. The mathematical section of the Aryabhatiya includes algebra, arithmetic, spherical trigonometry and plane trigonometry. It also includes continued fractions, quadratic equations, a table of sines and sums of power series.

**Aryabhata's Pioneer Pieces of Work**

It is said that Aryabhata studied at Nalanda University. He later on became the Head of the Department. His research performances at Nalanda include subjects in mathematics, astronomy, physics, medicine, biology, and other fields of study. He gained his crucial source of knowledge from Nalanda. His notable pieces of work were based on previous discoveries by Mesopotamians, Greeks, and Nalanda University itself. *Aryabhatiya*, a Compendium Commentary on Mathematics and Astronomy, was served as reference materials in the Indian mathematical literature that has sustained itself up to modern cadence. The mathematical section of the *Aryabhatiya* describes his comprehensive knowledge of plane trigonometry, arithmetic, algebra, and spherical trigonometry. It also contains continued fractions, quadratic equations, sums-of-power series, and a table on sines.

The empirical working attributes of Aryabhata are investigated only from the treatise named *Aryabhatiya*. His disciple Bhaskara I calls it *Ashmakatantra* or the treatise from the Ashmaka. It is also at times referred to as *Arya-shatas-aShTa* for there are 108 verses in the text materials. It is noted in the very concise manner typical of Sutra Literature. It is classified into four Padas or Chapters as follows.

1. *Gitikapada* with 13 compositions: Its various units of measurement of time: kalpa, Yuga and manvantra, which describe a cosmology other than what has been described in earlier texts such as Lagadha's Vedanga Jyotisha (c. first century BCE). There is also a table on sines, i.e., jya, scripted in single. The duration of the planetary revolutions during a mahayuga is recognized as 4.32 million years.
2. *Ganitapada* with 33 compositions: It investigates mensuration, i.e., kṣetra vyavahara, arithmetic and geometric progressions; gnomon, i.e., shadows (easy), indeterminate equations, quadratic, and simultaneous equations.
3. *Kalakriyapada* with 25 compositions: Its various units of measurement of time and a method for determining the positions of planets are for a given day, calculations relating to the intercalary month, and a seven-day week naming convention for the days of a week.
4. *Golapada* with 50 compositions: It investigates features of objects like the ecliptic, the node, the celestial equator, the geometric and the trigonometric fields of the celestial sphere, shape and size of the earth, factors for day and night, emergence of zodiacal signs on horizon, etc. Furthermore, some versions make citation of colophons as addendum affixed at the end, explaining the qualities of the pieces of work, etc.

He, in his treatise, Aryabhatiya, presented a number of innovations in mathematics and astronomy in the form of commentary versions, which have been impactful for centuries for the further development of the subject matter. He is familiar about his description of relativity of motion.

The "*Arya-siddhanta"*, the lost pieces of work on astronomical computations, is seen through making note of Aryabhata's contemporary, Varahamihira, and later mathematician commentators, including Brahmagupta and Bhaskara I. This classical work appears to be based on the earlier "Surya Siddhanta" which was a Sanskrit summary on Greek and Mesopotamian theories in astronomy and mathematics which applies exhibiting the mid-night day reckoning, as opposed to that of the sunrise in *Aryabhatiya*.

A third text, which may have sustained itself in the Arabic translation, is Al-nanf or Al ntf. It is said that it is a translation by Aryabhata, but the Sanskrit name of this work was unspecified so far by then. Perhaps from the 9th century, it is mentioned by Abu Rayhan al-Biruni who was the Persian Scholar and Analyst of India.

**Aryabhata's Pieces of Work on Algebra**

The honour of 'Father of Algebra' was also attributed to Aryabhata other than Al-Khwarizmi and Diophantus because of Aryabhata's notable understanding and explanation of planetary systems. These pieces of his work of Algebra were established out of studying the problem in astronomy for determining the periods of the planets. It describes integer solutions to equations of the form $by = ax + c$ and $by = ax - c$, where $a, b, c$ are integers. Aryabhata makes use of the algorithm called "Kuttaka" to solve problems of this type. It is well known that the word Kuttaka means "to pulverise" and the method of breaking the problem down into new problems where the coefficients become smaller and smaller with each step. The method here is essentially making use of the Euclidean algorithm to find the greatest common divisor of $a$ and $b$, and it is also related to continued fractions. In Aryabhatiya, Aryabhata provided elegant results for the summation of series of squares and cubes like those as follows:

$$1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6},$$

$$1^3 + 2^3 + \cdots + n^3 = (1 + 2 + 3 + \cdots + n)^2.$$

Aryabhata gives formulae for the areas of a triangle and that of a circle which are found to be correct. He introduced a mathematical algorithm for the sum of an arithmetic progression $S_n$. He also formulated another one for the number of terms $n$ given that the sum is clearly in the knowledge domain. His course of action for computing the sum of an arithmetic progression is based on the mathematical relationship between the initial term $a$ and the common difference $d$ of alternate terms.

**Aryabhata's Pieces of Work on Indeterminate Equations**

Indeterminate equations of the second or higher degree contain two or more unknowns to solve for. Diophantine system of equations is equations of polynomial expressions for which rational or integer solutions are much sought after. More often than not, the expression implies that one requires integer solutions, but in a sense these are analogous ones. In system of indeterminate equations, the number of unknowns exceeds the number of equations. Furthermore, these unknowns are subject to further constraints such as being integers, or positive integers, or rationals, etc. It is well known that indeterminate equations are not solvable uniquely. In fact, in some cases, it might even have infinitely many solutions. A problem of much gaining to Indian mathematicians since antiquity has been to find integer solutions to Diophantine equations that have the standard format of $ax + by = c$. This issue was also investigated in classical Chinese mathematics, and its solution is often referred to as the Chinese Remainder Theorem.

In the general run of things, Diophantine equations can be even notoriously difficult to work out. They were discussed extensively in ancient Vedic text in Sulba Sutras. Its rather classical parts might date to 800 BCE. Aryabhata's method of solving these types of mix-ups, as elaborated by Bhaskara in 621 CE, is called the kuṭṭaka which involves a recursive algorithm for writing the original factors in smaller ones. This algorithm gets converted into the standard method for solving first order diophantine equations in Indian mathematics. At the outset, the whole subject of algebra was interestingly called kuṭṭaka-gaṇita or kuṭṭaka. The Kuttaka method is even nowadays considered the standard method to find solutions to these types of equations.

**Aryabhata's Pieces of Work on Motions of the Solar System**

Aryabhata correctly affirmed for each day that the Earth rotates about its axis. The movement of the earth causes the movement of the star, contrary to the then prevailing viewpoint, that it is the sky that got rotated. This work can be glanced in the first chapter of the Aryabhatiya, where he discusses the number of rotations of the Earth in a Yuga, and more explicitly in his gola section.

He described a geocentric model of the Solar System, in which the Sun and the Moon are each brought by epicycles. They in turn revolve around the Earth. In this phenomenal work, which is also found out there in the Paitamahasiddhanta (c. 425 CE), the motions of the planets are stated to be each governed by two epicycles, larger sighra (i.e., fast) and a smaller manda (i.e., slow). The sequence of the planets in terms of distance from the earth is taken as: the Moon, the Mercury, the Venus, the Sun, the Mars, the Jupiter, the Saturn, and the asterisms.

The settings and periods of the planets were computed with respect to uniformly moving points. In case of Mercury and Venus, they move around the Earth at the same mean speed as the Sun. In the event of the Mars, the Jupiter, and the Saturn, they rotate the Earth at specific speeds, representing each planet's motion by the zodiac. Another theme in Aryabhata's facsimile, the sighrocca, the fundamental planetary period in relation to the Sun, is perceived by some as indicative of an underlying heliocentric replica.

He gives a systematic study of the setting of the planets in space. He gave the circumference of the earth as 4 967 yojanas and its diameter as $1581\frac{1}{24}$ yojanas. As 1 yojana = 5 miles, the circumference comes out to be 24 835 miles on the calculation, which is an excellent approximation to the currently accepted value at 24 902 miles. He believed that the rotation of the heavenly bodies was due to the axial moving around of the Earth. This is quite a notable view of the natural phenomena of the solar system which later commentators themselves could not bring to follow suit, and most of them got the text modified to save him from what they considered were stupidity of mere erroneous calculation!

He provides the radius of the planetary orbits in terms of the radius of the Earth/the Sun orbit as essentially their periods of moving around the Sun. He believes that the Moon and the planets shine by the reflected sunlight. Amazingly, he believes that the orbits of the planets are elliptical in shape. He correctly describes the causes of eclipses of the Sun and the Moon. His findings for the length of the year at 365 days, 6 hours, 12 minutes and 30 seconds are somehow overestimated for the exact value is less than 365 days and 6 hours.

**Aryabhata's Brief Work on Place Value System and Zero (0)**

The introduction of zero (0) brought a lot of drastic changes not only in mathematics but also in the day-to-day life of people. Zero(0) has so many different terms for it, for example, 'null', 'nil', void, empty, '0' as a digit, number, standing for absence of something, 'sunya' in Sanskrit, and so on. It is fascinating how the origin of zero brought out radical transformation that is nowadays illustrated in mathematics.

Aryabhatta utilized the concept of zero (0) in his mathematical pieces of work, but he did not ascribe a symbol for it. The actual symbol "0" and the origin of the word zero is said to come from the Persian al-Khwarizmi about 450 years later. If one really wants to give credit for the concept of zero, one needs to go back a hundred years before Aryabhatta to the Mayans or even 700 years back to the Babylonians. Although, it is stated to be fair to say that modern use of the concept of zero comes from Aryabhatta. He was acquainted with the notion of zero as well as the application of larger numbers up to 1018.

It is also stated that the knowledge of zero (0) was not explicitly mentioned in Aryabhata's place-value system as a place holder for the powers of ten having null coefficients. Moreover, he did not apply the Brahmi numerals. Keeping the Sanskritic tradition up from Vedic time onwards, he applied letters of the alphabet to depict numbers expressing quantities like the table of sines in the form of mnemonic. His calculation of Pi is a near approximation of the contemporary quantification that is considered the most accurate one amongst the ancient ones.

The trace of zero (0) introduced by Aryabhatta in India dates to the 5th century. Zero (0) was in use to be represented as a dot in mathematics. Furthermore, when it reached Arab, an oval shape was ascribed to the number that one today knows about as the digit or number '0'. This is why zero belongs to the Hindu-Arabic number system. Succeeding Aryabhatta, Bramhaputra is credited for further study of zero. In the 7th century, Bramhaputra started applying zero in mathematical operations. The first numeral zero originates from a Hindu astronomer and mathematician Brahmagupta in 628.

The modern day zero (0) was conceived when zero (0) reached the shore of China from India and then reached the Middle East. In approximately 773 AD, the mathematician Mohammad ibn-Musa al-Khowarizmi studied and synthesized Indian arithmetic and proved how zero functioned for the formulae he called 'al-jabr' that is today known as "Algebra". In AD 813, astronomical tables were prepared by him with Hindu numerals. About 825, he published a book synthesizing Greek and Hindu knowledge that also contained his own contribution to mathematics with an explanation of the application of zero (nought, naught, empty, nil). This book was later translated into Latin in the 12th century under the title "Algoritmi de numero Indorum". This title implies "al-Khwarizmi on the Numerals of the Indians". The word "Algoritmi" was the translator's Latinization of Al-Khwarizmi's own name and the term "Algorithm" or "Algorism" started to acquire a meaning of any arithmetic fixed on decimals.

Around 1200 AD, an Italian mathematician Fibonacci gave zero (0) in Europe. In the beginning, zero (0) was called 'Sunya' in India. It was called 'Sifr' in the Middle East when it reached Italy. It then was named 'Zefero' and thereafter, it was called 'Zero' in English.

## Aryabhata's Pieces of Work on the Value of $\pi$

The mathematical constant Pi is basically a story of the Egyptian mythology. The Egyptians thought that the pyramids of Giza were constructed on the principles of pi. The vertical measurements of the pyramids have the same correlation with the perimeter of their base as the relationship between a circle's radius and its circumference. The pyramids are architectural marvels and are considered one of the Seven Wonders of the World. The Physicist Larry Shaw initiated celebrating 14 March as Pi day at San Francisco's Exploratorium science museum. March 14 or 3/14 is considered pi day because 3.14 are the first digits of pi. The symbol for Pi has been in usage for over 250 years. The symbol was given by William Jones, a Welsh mathematician, in 1706. The symbol was made popular by the mathematician Leonhard Euler. As the accurate value of pi can never be computed, one can never find the exact area or circumference of a circle. Other dates when people celebrate pi include Pi Approximation Day on July 22. The value 22/7 in the form of day/month is an approximation of π, and June 28. The value 6.28 is an approximated value of $2\pi$ or tau.

The number $\pi$ is a constant in mathematics that is defined to be the ratio of a circle's circumference and its diameter. It is approximately equal to 3.14159. The number $\pi$ appears in many formulae across mathematics and physics. It is an irrational number, meaning that it cannot be expressed completely as a ratio of two integers, even though fractions such as $\frac{22}{7}$ are often employed to approximate it. Its decimal representation is of non-terminating and non-repeating pattern. It is a transcendental number, implying that it cannot be a solution of an equation involving only finite sums, products, powers, and integers. The transcendence of $\pi$ implies that it is impossible to solve the ancient challenge of squaring the circle with a straightedge and compass.

Aryabhata worked on the approximation for pi (π). It may have been observed that π is irrational number. In the second part of the Aryabhatiyam (gaṇitapada 10), he explains that for a circle whose diameter is 20000, the circumference will come out to be 62832.

That is, $\pi = \frac{62832}{20000}$.

It is speculated that Aryabhata used the word asanna (i.e., approaching), to mean that not only this is an approximation but that the value is incommensurable (irrational). If this is exact, it is quite a sophisticated and keen insight, since the irrationality of pi (π) was proved in Europe only in 1761 by Lambert.

This approximation was seen noted in Al-Khwarizmi's book on algebra, after Aryabhatiya was translated into Arabic (c. 820 CE).

## Aryabhata's Pieces of Pieces of Work on Trigonometry

Trigonometry forming from triangle (trigonon) and measure (metron) is a branch of mathematics related to relationships between angles and side lengths of triangles. Particularly, the trigonometric functions relate the angles of a right triangle with ratios of its side measurement. The field emerged in the Hellenistic World (The World of Alexander the Great) during the 3rd century BC from applications of geometry to astronomical investigations. The Greeks emphasized on the computation of chords, whereas mathematicians in India created the earliest known tables of values for trigonometric ratios which is also called trigonometric functions such as sine. Throughout the course of history, trigonometry has been applied in areas such as geodesy, surveying, celestial mechanics, and navigation. Trigonometry is known for its many identities.

These trigonometric identities are often applied for rewriting trigonometrical terms with the aim to simplify a trigonometric expression to find a more useful form of an expression or to solve an equation.

In Ganitapada 6, Aryabhata calculates the area of a triangle as that translates to: "for a triangle, the result of a perpendicular with the half-side is the area." He described the notion of sine in his work by the name of ardha-jya, which verbatim means "half-chord". For the sake of simplicity, people started calling it jya. When the Arabic writers translated his pieces of work from Sanskrit into Arabic, they expressed it as jiba. However, in Arabic writings, vowels are omitted, and it was abbreviated as jb. Later writers substituted it with jaib, meaning "pocket" or "fold" in a garment. It is to be noted that in Arabic, jiba is a meaningless word. Later in the 12th century, when Gherardo of Cremona translated these writings from Arabic into Latin, he replaced the Arabic jaib with its Latin counterpart, sinus, which means "cove" or "bay"; that way originates the English word sine.

The modern definition of the sine is first displayed in the **Surya Siddhanta**, and its characteristics were further prepared for documentation in the 5th century (AD) by Indian mathematician and astronomer Aryabhata.These Greek and Indian pieces of work were translated and expanded by medieval Islamic mathematicians too.

**Aryabhata's Pieces of Work on Motions of the Solar System**

Aryabhata correctly insisted that the Earth rotates about its axis daily. He also conceptualized the fact that the apparent movement of the stars is a relative motion caused by the rotation of the Earth, contrary to the then existing perception, that the sky got rotated. This is indicated in the first chapter of the Aryabhatiya, where he gives the number of rotations of the Earth in a Yuga, and made that more clearly in his gola chapter.

Aryabhata described a geocentric model of the Solar System, in which the Sun and the Moon are each carried by epicycles. They in turn revolve around the Earth. In this representation, which is also found in the Paitamahasiddhanta (c. 425 CE), the motions of the planets are each governed by two epicycles, a smaller manda (slow) and a larger sighra (fast). The sequence of the planets in terms of its distance from earth is considered as: the Moon, the Mercury, the Venus, the Sun, the Mars, the Jupiter, the Saturn, and the asterisms.

The positions and periods of the planets were found relative to uniformly moving points of objects. In case of the Mercury and the Venus, the movement around the Earth is considered at the same mean speed as the Sun. In case of the Mars, the Jupiter, and the Saturn, the movement around the Earth is at specific speeds, representing each planet's motion through the zodiac. Most historians of astronomy consider that these two epicycle representations reflect elements of pre-Ptolemaic Greek astronomy. Another point in Aryabhata's representation, the sighrocca, the basic planetary period in relation to the Sun, is seen by some historical observers as an indication of an underlying heliocentric representation.

**Aryabhata's Pieces of Work on Eclipses**

An eclipse is an astronomical occurrence that happens when an astronomical spacecraft or object is temporarily clouded by passing into the shadow of another heavenly body or by having another body go through it and the observer. This layout of three celestial bodies is understood as a syzygy. An eclipse is the result of either an occultation, completely hidden event, or a transit, partially hidden event. A "deep eclipse" or "deep occultation" is when a small astronomical body is behind a bigger one.

The eclipse is most often described as either a solar eclipse, when the Moon's shadow crosses the Earth's surface, or a lunar eclipse, when the Moon moves into the Earth's shadow. It can also refer to such events beyond the Earth: the Moon system: for instance, a planet moving into the shadow cast by one of its moons, a moon passing into the shadow cast by its host planet, or a moon passing into the shadow of another moon. A binary star system can also cause eclipses if the plane of the orbit of its constituent stars intersects the observer's stand at position.

Solar and lunar eclipses were scientifically explained by Aryabhata. He states that the Moon and the planets shine by reflected sunlight. Instead of the existing cosmogony in which eclipses were produced by Rahu and Ketu, identified as the pseudo-planetary lunar nodes, he explains eclipses in terms of shadows cast by and falling on the Earth. Thus, the lunar eclipse occurs when the Moon enters into the Earth's shadow. He discusses at length the size and extent of the Earth's shadow and then provides the computation and the size of the eclipsed part during an eclipse. Later Indian astronomers improved on the computations, but Aryabhata's methods provided the core concept in this respect. His computational paradigm was an accurate one that the 18th-century scientist Guillaume Le Gentil observed during a visit to Pondicherry, India.

**Aryabhata's Pieces of Work on Sidereal Periods**

**Sidereal period**, the time needed for a celestial body within the solar system to perform one complete revolution with respect to the fixed stars, i.e., as viewed from some fixed point outside the system. The sidereal period of a planet can be computed if its synodic period, i.e., the time for it to return to the same position with respect to the Sun and the Earth, is known. The sidereal period of the Moon or an artificial satellite of the Earth is the time required for it to return to the same position against the backdrop of stars.

In astronomy, Kepler's laws of planetary motion describe the orbits of planets around the Sun. The laws modified the heliocentric theory of Nicolaus Copernicus, replacing its circular orbits and epicycles with elliptical trajectories. It also explains how planetary velocities vary. The three laws state as follows

The orbit of a planet is an ellipse with the Sun at one of the two foci.

Equation of an ellipse: $r = \frac{p}{1+e\cos\theta}$, where where $p$ is the semi-latus rectum, $e$ is the eccentricity of the ellipse, $r$ is the distance from the Sun to the planet, and $\theta$ is the angle to the planet's current position from its nearest approach, as observed from the Sun. Thus $(r,\ \theta)$ are polar coordinates. When the eccentricity is zero, the orbit is a circle.

A line segment joining a planet and the Sun sweeps out equal areas during equal intervals of time.

In a small time, $dt,$ the planet sweeps out a small triangle having base line r and height $rd\theta$ and the area is given by $dA = \frac{1}{2}.r.rd\theta.$ Its constant areal velocity is $\frac{dA}{dt} = \frac{r^2}{2}\frac{d\theta}{dt}.$

The area enclosed by elliptical orbit is $\pi ab.$ Thus, its period T is satisfied by

$$T.\frac{r^2}{2}\frac{d\theta}{dt} = \pi ab.$$

Also, the mean motion of the planet around the sun is $n = \frac{2\pi}{T}$ satisfies $r^2.d\theta = abndt$.

Hence $\frac{dA}{dt} = \frac{abn}{2} = \frac{\pi ab}{T}$.

The square of a planet's orbital period is proportional to the cube of the length of the semi-major axis of its orbit.
This captures the relationship between the distance of planets from the Sun, and their orbital periods. In Newton's law of gravitation, this relation can be obtained in the case of a circular orbit by equating the centripetal force and the gravitational force:$mr\omega^2 = G\frac{mM}{r^2}$. Now representing the angular velocity $\omega$ in terms of the orbital period $T$, and then rearranging, Kepler's third law is as follows:

$$mr(\frac{2\pi}{T})^2 = G\frac{mM}{r^2} \to T^2 = \left(\frac{4\pi^2}{GM}\right)r^3 \to T^2 \propto r^3.$$

A more detailed version of derivation can be calculated with the general elliptical orbits, instead of circles, as well as orbiting the center of mass, in place of just the large mass. This results in replacing a circular radius, $r$, with the semi-major axis, $a$, of the elliptical relative motion of one mass relative to another, and also replacing the large mass $M$ with $M + m$. However, with planetary masses being so much smaller than the Sun, this correction is generally ignored. The complete analogous formula is as follows:
$\frac{a^3}{T^2} = G\frac{(m+M)}{4\pi^2} \approx \frac{GM}{4\pi^2} \approx 7.496 \times 10^{-6}\frac{AU^3}{days^2}$ is a constant, where $M$ is the mass of the Sun, m is the mass of the planet, $G$ is the gravitational constant, $T$ is the orbital period, and $a$ is the elliptical semi-major axis, and $AU$ is the astronomical unit, the average distance from the earth to the Sun.
The elliptical orbits of planets were indicated by calculations of the orbit of the Mars. From this, Kepler inferred that other bodies in the Solar System, including those farther away from the Sun, also have elliptical orbits. The second law helps to establish that when a planet is closer to the Sun, it travels faster. The third law expresses that the farther a planet is from the Sun, the slower its orbital speed, and vice versa. Isaac Newton exhibited in 1687 that relationships like Kepler's would apply in the Solar System as a result of his own laws of motion and law of universal gravitation.
Johannes Kepler's laws are improvements upon the model of Copernicus. As per Copernicus:

1. The planetary orbit is a circle with epicycles.
2. The Sun is approximately at the center of the orbit.
3. The speed of the planet in the main orbit is constant.

Despite being correct in stating that the planets revolved around the Sun, Copernicus was incorrect in defining their orbits. Introducing physical explanations for movement in space beyond just geometry, Kepler correctly defined the orbit of planets as follows:

1. The planetary orbit is not a circle with epicycles, it is an ellipse.
2. The Sun is not at the center but at a focal point of the elliptical orbit.
3. Neither the linear speed nor the angular speed of the planet in the orbit is constant, but the area speed is constant.

The eccentricity of the orbit of the Earth captures the time from the March equinox to the September equinox, around 186 days, unequal to the time from the September equinox to the March equinox, around 179 days. A diameter would intersect the orbit into equal parts, but the plane through the Sun parallel to the equator of the Earth cuts the orbit into two parts with areas in a 186 to 179 ratio. So the eccentricity of the orbit of the Earth is approximately

$$e \approx \frac{\pi}{4}\frac{186 - 179}{186 + 179} \approx 0.015.$$

This value is close to the correct value (0.016710218).
Aryabhata calculated the sidereal rotation, i.e., the rotation of the earth referencing the fixed stars, as 23 hours, 56 minutes, and 4.1 seconds. Its modern value stands at 23:56:4.091. Similarly, his calculated value for the duration of the sidereal year is 365 days, 6 hours, 12 minutes, and 30 seconds (i.e., 365.25858 days) with an error of 3 minutes and 20 seconds over the duration of a year, i.e., 365.25636 days.

**Aryabhata's Pieces of Work on Heliocentric**

Copernican heliocentrism is the astronomical model designed by Nicolaus Copernicus. This model got the position of the Sun at the center of the Universe, motionless, with the Earth and the other planets orbiting around it in circular paths, modified by epicycles, and at uniform speeds. The Copernican model displaced the geocentric model of Ptolemy that had existed for centuries, which had placed the Earth at the center of the Universe.
The concept of Indian heliocentrism has been advocated by B. L. van der Waerden. Aryabhata favored an astronomical model in which the Earth turns on its own axis. His model also gave corrections, i.e., the sigra anomaly for the speeds of the planets in the sky in terms of the mean speed of the Sun. Thus, it has been suggested that his computations were based on an underlying heliocentric model, in which the planets orbit the Sun, though this has been disproved. It has also been indicated that aspects of Aryabhata's system may have been obtained from an earlier, possibly pre-Ptolemaic Greek, heliocentric model of which Indian astronomers were unaware, though the evidence is scant. The general consensus is that a synodic anomaly, depending on the position of the Sun, does not infer a physically heliocentric orbit, such corrections being also present in late Babylonian astronomical texts, and that Aryabhata's system was not explicitly heliocentric.

**Aryabhata's Legacy and Invention**

Aryabhata had the most outstanding visionary approach. His pieces of work were of great applicability in the Indian astronomical tradition. It impacted several neighboring cultures and civilizations through translational pieces of work. The Arabic translational work during the Islamic Golden Age (c. 820 CE), was particularly noteworthy. Some of his results are got referenced by Al-Khwarizmi and in the 10th century. Al-Biruni stated that Aryabhata's followers believed that the Earth rotated on its axis.
His definitions of sine, cosine, versine, and inverse sine and that thus made way to the birth of trigonometry. He was also the first to specify sine and versine ($1-\cos x$) tables, in 3.75° intervals from 0° to 90°, to an accuracy of up to 4 decimal places.
In fact, modern names "sine" and "cosine" are mistranscriptions of the words jya and kojya as introduced by Aryabhata. As mentioned, they were translated as jiba and kojiba in Arabic and then misunderstood

by Gerard of Cremona while translating an Arabic geometry text to Latin. He assumed that jiba was the Arabic word jaib, which means "fold in a garment", L. sinus (c. 1150).

Aryabhata's astronomical calculation methods were also very impactful. Along with the trigonometric tables, they came to be widely applied in the Islamic world and computed many Arabic astronomical tables (zijes). Particularly, the astronomical tables in the work of the Arabic Spain scientist Al Zarqali of the 11th century were got translated into Latin as the Tables of Toledo in the 12th century and remained the most accurate ephemeris applied in Europe for centuries.

The Calendric calculations constructed by Aryabhata and his followers have been in continuous use in India for application oriented of fixing the Panchangam i.e., the Hindu calendar. In the Islamic world, they formed the basis of the Jalali calendar given in 1073 CE by a cult of astronomers including Omar Khayyam, compositions of which, modified in 1925, are the national calendars in use in Iran and Afghanistan today. The dates of the Jalali calendar are actually based on the solar transition, as in Aryabhata and previously Siddhanta calendars. This type of calendar requires an ephemeris for calculating dates. Although dates were difficult to compute, seasonal errors were less in the Jalali calendar than in the Gregorian calendar.

India's first satellite Aryabhata and the lunar crater Aryabhata are named in his respect. The Aryabhata satellite also presented on the reverse of the Indian 2-rupee currency. An Institute for conducting research in astrophysics, astronomy, and atmospheric sciences is the Aryabhata Research Institute of Observational Sciences (ARIES) near Nainital, India. The Bacillus aryabhata, a species of bacteria was discovered in the stratosphere by ISRO scientists in 2009 as is the inter-school Aryabhata Mathematics Competition that also got name after him.

Aryabhatta Knowledge University (AKU), Patna has been founded by the state Government of Bihar for the evolution and governance of educational infrastructure related to technical, medical, governance and associated professional education in his honour. The university is governed by Bihar State University Act 2008.

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