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The optimal replacement strategy for a system failure even with longer repair time frames

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Abstract
The goal of this research is to determine the most efficient plan of action for replacing and maintaining a functional system. This is broken down and suffers unpredictable impacts. The failure mechanism is represented as a generalized delta shock process, while the repair time is represented as a geometric process. We also express the system's long-term average cost per unit time. This is a component of the threshold replacement plan. Our average cost function assists us in determining the optimal replacement policy (N*) and reduces the average cost. With additional numerical examples, we further demonstrate the uniqueness of the optimal policy N*. This study develops a model that is applicable to numerous systems in the real world.

Keywords: Delta shock model, geometric process, long run average cost, poisson process, reliability, replacement, stochastic process

1. Introduction

To measure the system's failure, time distribution and its associated factors are required. Researchers extensively studied the system's time-to-failure under normal operating conditions. However, collecting this kind of information has been challenging due to the significant level of reliability of today's products, the short period of time between design and delivery. Investigators have been motivated by this issue to create innovative techniques, gather up-to-date data on the reliability of components of products and materials. A catastrophic piece of equipment or facilities break down, stop production in the manufacturing sector. A disruption of this kinds negatively impact a corporation's revenue and customer service. To save costs, experts have searched for machine reliability and the best replacement plans. The issue of system maintenance during the phases of operation and repair are extensively researched. "A.O. Charles Elegbede and Chengbin Chu (2003) [1]" state that parallel-series systems should be allocated redundancy and reliability while keeping system costs to a minimum. This work establishes that, subject to a nonrestrictive condition on the reliability cost functions of the individual components, every stage of a parallel-series system's components must have the same reliability. According to "Cheng Guoqing, Li Ling, Liu Bingxiang, *et al*. (2013) [4]," a replacement plan is suggested for a malfunctioning system that is susceptible to delta-shock. The foundation of this approach is the notion that, as maintenance times increase, failure thresholds will geometrically shrink. This method lowers the average life-cycle cost and identifies the optimal replacement policy. "Lam Y (1991) [5]" investigates a geometric process maintenance model" for a failing system in a chaotic setting that assumes a counting process is formed by the quantity of random shocks that the random environment produces over time. The operational time of the system is shortened each time a random shock occurs. The sequential decreases in the system's operating time are random variables that are statistically independent and have an identical distribution. "M. M. Chen *et al*. (2010) [6]" discussed age and a random replacement policy in which the system is replaced at time T, Y, or failure, whichever occurs first. "Y. L. Zhang, G.J. Wang (2007) [7]" discussed a replacement strategy M = (N1, N2,..., Nk) depending on the quantity of component 1, component 2,..., and component k failures, respectively, and finding the best replacement strategy to reduce the average cost rate. "Karol Andrzejczak (2015) [9]" argues for the necessity of stochastic modeling, and concepts used in maintenance engineering are mentioned.

Additionally, the theoretical underpinnings of a stochastic process were described, and its different attributes were determined as objective models gets more complicated and extensive; reliability strategies must be created and enhanced. "Y.L. Zhang, G.J. Wang (2009) [12]" study a degrading cold standby repairable system made up of two distinct parts and a single repairman. Assume that component 1 has precedence in use and repair, and that the subsequent working times for each component form a diminishing geometric process while the subsequent repair times represent an increasing geometric process. "Babu D *et al*. (2020) [13]" introduced the partial sum process and its application to models for deteriorating system maintenance under imprecise delayed repair is examined. Explicitly deduced is an equation for the mean cost over the long term for an N-policy. "Safaei, F., Ahmadi, J. and Balakrishnan, N. (2018) [14]" discussed a maintenance model for system based on probability and average profit. "Waziri, T. A. and Yusuf, I. (2020) [15]" studied age based replacement model involving minimal repair. "M. Brown and F. Proschan (1983)"suggests an imperfect repair model, in which a repair is perfect with probability, and a minimal repair with probability.

In this study, we provide a delta shock model that practitioners can use to choose the best maintenance and replacement strategies for their equipment. A renewal process with exponentially dispersed shock interval and a geometrical increase in the threshold of a deadly shock is what we predict will cause the shocks to occur. The threshold distribution function also has an exponential distribution. This study's goal is to lower overall costs or long-term maintenance expenses.

2. Definitions

2.1 Stochastic Process

"The study of stochastic processes involves the analysis of a collection of random variables, their interdependence, and their change over time, and limiting behavior, among others (Ross, 2000) [2]".

Let S be a subset of [0, ∞).

A family of random variables $\{X_t\}_{t\in S}$, indexed by S, is called a Stochastic (or random) process.

2.2 Geometric Process

A stochastic process $\{S_n$, n = 1, 2, 3, ----------$\}$ is called a geometrically increasing or decreasing process if there exists a real number a $(0 < a \leq 1 \; or \; a > 1)$ then $\{a^{n-1}, S_n = 1, 2, 3 - - - - - -\}$ from a new renewal process. The real value a is called the ratio of the geometric process. Consider $E(S_1) = \varsigma$ and var$(S_1) = \sigma^2$, we have $E(S_n) = \frac{\varsigma}{a^{n-1}}$ and var $(S_n) = \frac{\sigma^2}{a^{2n-2}}$

In this way geometric process has independent on three parameters a, ς and σ^2.

3. Assumption

1. At time t = 0, system is 'good as new'. At after t_n the system fails, then based on system condition, it is either repair or replaced with a new one.
2. Failure follows Poisson process with rate λ_1 or $E(Y_j) = \frac{1}{\lambda_1}$, when Y_j are two consecutive failure of j[th] interval of time.
3. Let H_j (j= 1, 2 ------) be the time interval between (j–1)[th] and j[th] shocks. Consider H_j be exponentially distributed with distribution function H(x). Consider θ_j is another exponentially distributed random variable according with y_j. Here we consider the sequence $\{\theta_j$, j = 1, 2, --------$\}$ be a increasing Geometric process with $0 < a \leq 1$. Then $\theta_j$has cumulative distribution function $a(a^{j-1}x)$ and taking J = 1, then cumulative distribution function of θ_1 is $\alpha(x)$
4. If $Y_j \leq \theta_j$ and system breakdown j[th] shock, then $\{Y_j, \theta_j\}$ follow delta shock model. If one satisfying the above condition the life time or the operating time is the sum of all Y_j unit.
5. After (n – 1)[th] repair, let T_n be the operating time and through delta shock model it is Stochastically decreasing random variable sequence.
6. After n[th] breakdown, let B_n be the repair time and follows an increasing Geometric process with $0 < b \leq 1$ and c.d.f of B_n is $(\beta^{n-1}Y)$ if n = 1 then $\beta(Y)$ is c.d.f of B_1 with E (B_1) =$\mu > 0$.
7. T_n and B_n are two independent variable where n = 1, 2, ----------.
8. Consider R_c is the replacement cost which is fixed, r is the operating rate and r_c be the repair cost.
9. Let V = $r_p R_t$
10. Where r_p= during replacement, rate of cost per unit time.
11. R_t= Replacement time.
12. In this method, system fails N times then it will be replaced and N replacement method adopted.

4. Long period average cost per unit time:

Under supposed cost structure, the average cost function of the N – replacement policy, consider Q (N) be the long period average cost per unit time is given by

$$Q(N) = \frac{E(T_c)}{E(L)} \tag{1}$$

Where E (T_c) denotes the expected cost over a renewal cycle and E(L) denotes the expected length of renewal cycle.

According to assumption (3) H(x) and $\alpha(x)$ are exponentially distributed then

$$H(x) = 1 - e^{-\lambda_1 x} \; x \geq 0 \tag{2}$$

$$\alpha(a^{n-1}x) = 1 - e^{-a^{n-1}\lambda_2 x} \; x \geq 0 \tag{3}$$

Between (n – 1)th and nth repair, let expected operating time of the system be E (T_n). Again (n – 1)th repair Let l_{nj} be the be the length of time interval between (J – 1)th and jth shocks. The next failure is given by

$$F_n = \min \{f \mid ln1 > a^{n-1}\delta_1, \ldots \ldots \ldots l_{n(m-1)} > a^{n-1}\delta_1, l_{nj} < a^{n-1}\delta_1\} \quad (4)$$

Then $T_n = \sum_{J=0}^{F_n} l_{nj}$ (5)

Thus F_n denotes the number of shocks under the first deadly shocks, and it has geometric distribution.

Therefore P (Fn = s) $= q_n^{s-1} p_n$, s = 1, 2, 3,……………. (6)

Where p_n is the probability of a failure with (n – 1)th repair and $q_n = 1 - p_n$
Therefore from (5)

$$E(T_n) = E\left(\sum_{J=0}^{F_n} l_{nj}\right) = \frac{E(l_{n1})}{p_n} \quad (7)$$

Now since $E(l_{n1}) = \int_0^\infty x dH(x)$

$$= \int_0^\infty x d\left(1 - e^{-\lambda_1 x}\right) = \frac{1}{\lambda_1} \quad (8)$$

Again since l_{nj} and δ_n are exponentially distributed and are independent with mean $\frac{1}{\lambda_1}$ and $\frac{1}{a^{n-1}\lambda_1}$
Therefore $p_n = P(l_{nj} < \delta_n)$

$$= \int_0^\infty H(x) d\alpha(x)$$
$$= \int_0^\infty (1 - e^{-\lambda_1 x} d(1 - e^{-a^{n-1}\lambda_2 x})$$

$$= \frac{\lambda_1}{\lambda_1 + a^{n-1}\lambda_2} \quad (9)$$

Then $E(T_n) = \frac{E(l_{n1})}{p_n} = \frac{\lambda_1 + a^{n-1}\lambda_2}{\lambda_1^2}$ (10)

and $E\left(\sum_{n=1}^N T_n\right) = \sum_{n=1}^N E(T_n)$

$$= \sum_{n=1}^N \frac{\lambda_1 + a^{n-1}\lambda_2}{\lambda_1^2} \quad (11)$$

Operating cost of the system is denoted by O_c then

$$O_c = \sum_{n=1}^N r \sum(T_n)$$

$$= r \sum_{n=1}^N \frac{\lambda_1 + a^{n-1}\lambda_2}{\lambda_1^2} \quad (12)$$

Since B_n is an increasing G. P with ratio $0 < b \leq 1$ and n = 1, 2, …..…..
then

$$E(B_n) = \frac{\mu}{b^{n-1}}$$

Then the repair cost function will be

$$R_{c1} = \sum_{n=1}^{N-1} E(r_c B_n)$$

$$= \sum_{n=1}^{N-1} \frac{r_c \mu}{b^{n-1}} \quad (13)$$

Based on assumption 8, replacement cost can be given by
$R_{c2} = E(R_c + v)$

$$= R_c + r_p t \quad (14)$$

Under replacement policy N, if T_c is the total expected cost of renewal of the system then

$$T_c = (R_{c1} - O_c + R_{c2}) \tag{15}$$

and L be the length of renewal cycle, then expected length of a renewal cycle of the system be

$$E(L) = E(\sum_{n=1}^{N} T_n) + E(\sum_{n=1}^{N-1} B_n) + E(R_t) \tag{16}$$

Hence under replacement policy, the long period average cost Q(N) is given by

$$Q(N) = \frac{r_c \sum_{n=1}^{N-1} \frac{\mu}{b^{n-1}} - r \sum_{n=1}^{N} \frac{\lambda_1 + a^{n-1}\lambda_2}{\lambda_1^2} + R_c + r_p t}{\sum_{n=1}^{N} \frac{\lambda_1 + a^{n-1}\lambda_2}{\lambda_1^2} + \sum_{n=1}^{N-1} \frac{\mu}{b^{n-1}} + t} \tag{17}$$

For optimal N*, (17) can be written as

$$Q(N) = \frac{(r_c + r) \sum_{n=1}^{N-1} \frac{\mu}{b^{n-1}} - r \sum_{n=1}^{N} \frac{\lambda_1 + a^{n-1}\lambda_2}{\lambda_1^2} + R_c + (r + r_p)t}{\sum_{n=1}^{N} \frac{\lambda_1 + a^{n-1}\lambda_2}{\lambda_1^2} + \sum_{n=1}^{N-1} \frac{\mu}{b^{n-1}} + t} - r \tag{18}$$

To optimize Q (N) is similar to optimize the first term of (18) and it is denoted by O (N)

$$O(N) = \frac{(r_c + r) \sum_{n=1}^{N-1} \frac{\mu}{b^{n-1}} + R_c + (r_p + r)t}{\sum_{n=1}^{N} \frac{\lambda_1 + a^{n-1}\lambda_2}{\lambda_1^2} + \sum_{n=1}^{N-1} \frac{\mu}{b^{n-1}} + t} \tag{19}$$

Now

$$O(N+1) - O(N) = \frac{(r_c + r) \sum_{n=1}^{N} \frac{\mu}{b^{n-1}} + R_c + (r_p + r)t}{\sum_{n=1}^{N+1} \frac{\lambda_1 + a^{n-1}\lambda_2}{\lambda_1^2} + \sum_{n=1}^{N} \frac{\mu}{b^{n-1}} + t} - \frac{(r_c + r) \sum_{n=1}^{N-1} \frac{\mu}{b^{n-1}} + R_c + (r_p + r)t}{\sum_{n=1}^{N} \frac{\lambda_1 + a^{n-1}\lambda_2}{\lambda_1^2} + \sum_{n=1}^{N-1} \frac{\mu}{b^{n-1}} + t}$$

$$= (r_c + r) \left\{ \frac{\sum_{n=1}^{N} \frac{\mu}{b^{n-1}}}{\sum_{n=1}^{N+1} \frac{\lambda_1 + a^{n-1}\lambda_2}{\lambda_1^2} + \sum_{n=1}^{N} \frac{\mu}{b^{n-1}} + t} - \frac{\sum_{n=1}^{N-1} \frac{\mu}{b^{n-1}}}{\sum_{n=1}^{N} \frac{\lambda_1 + a^{n-1}\lambda_2}{\lambda_1^2} + \sum_{n=1}^{N-1} \frac{\mu}{b^{n-1}} + t} \right\}$$

$$+ (R_c + (r_p + r)t) \left\{ \frac{\sum_{n=1}^{N} \frac{\lambda_1 + a^{n-1}\lambda_2}{\lambda_1^2} + \sum_{n=1}^{N-1} \frac{\mu}{b^{n-1}} + t - \sum_{n=1}^{N+1} \frac{\lambda_1 + a^{n-1}\lambda_2}{\lambda_1^2} - \sum_{n=1}^{N} \frac{\mu}{b^{n-1}} - t}{\left(\sum_{n=1}^{N+1} \frac{\lambda_1 + a^{n-1}\lambda_2}{\lambda_1^2} + \sum_{n=1}^{N} \frac{\mu}{b^{n-1}} + t\right)\left(\sum_{n=1}^{N} \frac{\lambda_1 + a^{n-1}\lambda_2}{\lambda_1^2} + \sum_{n=1}^{N-1} \frac{\mu}{b^{n-1}} + t\right)} \right\}$$

$$= \left(\frac{1}{(\sum_{n=1}^{N+1} \frac{\lambda_1 + a^{n-1}\lambda_2}{\lambda_1^2} + \sum_{n=1}^{N} \frac{\mu}{b^{n-1}} + t)(\sum_{n=1}^{N} \frac{\lambda_1 + a^{n-1}\lambda_2}{\lambda_1^2} + \sum_{n=1}^{N-1} \frac{\mu}{b^{n-1}} + t)} \right) [(r_c + r)\{\sum_{n=1}^{N} \frac{\mu}{b^{n-1}} \left(\sum_{n=1}^{N} \frac{\lambda_1 + a^{n-1}\lambda_2}{\lambda_1^2} + \sum_{n=1}^{N-1} \frac{\mu}{b^{n-1}} + t\right) - \sum_{n=1}^{N-1} \frac{\mu}{b^{n-1}} \left(\sum_{n=1}^{N+1} \frac{\lambda_1 + a^{n-1}\lambda_2}{\lambda_1^2} + \sum_{n=1}^{N} \frac{\mu}{b^{n-1}} + t\right)\} + ((R_c + (r_p + r)t) \{-\frac{\mu}{b^{N-1}} - \frac{\lambda_1 + a^{N}\lambda_2}{\lambda_1^2}\}]$$

$$= \left(\frac{1}{(\sum_{n=1}^{N+1} \frac{\lambda_1 + a^{n-1}\lambda_2}{\lambda_1^2} + \sum_{n=1}^{N} \frac{\mu}{b^{n-1}} + t)(\sum_{n=1}^{N} \frac{\lambda_1 + a^{n-1}\lambda_2}{\lambda_1^2} + \sum_{n=1}^{N-1} \frac{\mu}{b^{n-1}} + t)} \right) \left[(r_c + r) \left\{ \left((\sum_{n=1}^{N-1} \frac{\mu}{b^{n-1}} + \frac{\mu}{b^{n-1}}) \right) \left(\sum_{n=1}^{N} \frac{\lambda_1 + a^{n-1}\lambda_2}{\lambda_1^2} + \sum_{n=1}^{N-1} \frac{\mu}{b^{n-1}} + t\right) \right\} - \sum_{n=1}^{N-1} \frac{\mu}{b^{n-1}} \left(\sum_{n=1}^{N} \frac{\lambda_1 + a^{n-1}\lambda_2}{\lambda_1^2} + \frac{\lambda_1 + a^{N}\lambda_2}{\lambda_1^2} + \sum_{n=1}^{N-1} \frac{\mu}{b^{n-1}} + \frac{\mu}{b^{N-1}} + t\right) + ((R_c + (r_p + r)t) \left\{-\frac{\mu}{b^{N-1}} - \frac{\lambda_1 + a^{N}\lambda_2}{\lambda_1^2}\right\}\right]$$

$$= \left(\frac{1}{(\sum_{n=1}^{N+1} \frac{\lambda_1 + a^{n-1}\lambda_2}{\lambda_1^2} + \sum_{n=1}^{N} \frac{\mu}{b^{n-1}} + t)(\sum_{n=1}^{N} \frac{\lambda_1 + a^{n-1}\lambda_2}{\lambda_1^2} + \sum_{n=1}^{N-1} \frac{\mu}{b^{n-1}} + t)} \right) \left[(r_c + r) \left\{ \sum_{n=1}^{N-1} \frac{\mu}{b^{n-1}} \cdot \sum_{n=1}^{N} \frac{\lambda_1 + a^{n-1}\lambda_2}{\lambda_1^2} + \left(\sum_{n=1}^{N-1} \frac{\mu}{b^{n-1}}\right)^2 + t \sum_{n=1}^{N-1} \frac{\mu}{b^{n-1}} + \frac{\mu}{b^{n-1}} \sum_{n=1}^{N} \frac{\lambda_1 + a^{n-1}\lambda_2}{\lambda_1^2} + \frac{\mu}{b^{N-1}} \sum_{n=1}^{N-1} \frac{\mu}{b^{n-1}} + t \frac{\mu}{b^{N-1}} - \sum_{n=1}^{N-1} \frac{\mu}{b^{n-1}} \sum_{n=1}^{N} \frac{\lambda_1 + a^{n-1}\lambda_2}{\lambda_1^2} - \frac{\lambda_1 + a^{N}\lambda_2}{\lambda_1^2} \sum_{n=1}^{N-1} \frac{\mu}{b^{n-1}} - \left(\sum_{n=1}^{N-1} \frac{\mu}{b^{n-1}}\right)^2 - \frac{\mu}{b^{N-1}} \sum_{n=1}^{N-1} \frac{\mu}{b^{n-1}} - t \sum_{n=1}^{N-1} \frac{\mu}{b^{n-1}} \right\} + \left(R_c + (r_p + r)t\right) \left\{-\frac{\mu}{b^{N-1}} - \frac{\lambda_1 + a^{N}\lambda_2}{\lambda_1^2}\right\}\right]$$

$$= \left(\frac{1}{(\sum_{n=1}^{N+1} \frac{\lambda_1 + a^{n-1}\lambda_2}{\lambda_1^2} + \sum_{n=1}^{N} \frac{\mu}{b^{n-1}} + t)(\sum_{n=1}^{N} \frac{\lambda_1 + a^{n-1}\lambda_2}{\lambda_1^2} + \sum_{n=1}^{N-1} \frac{\mu}{b^{n-1}} + t)} \right) \left[(r_c + r) \left\{ \frac{\mu}{b^{N-1}} \sum_{n=1}^{N} \frac{\lambda_1 + a^{n-1}\lambda_2}{\lambda_1^2} + t \frac{\mu}{b^{N-1}} - \sum_{n=1}^{N-1} \frac{\mu}{b^{n-1}} \left(\frac{\lambda_1 + a^{N}\lambda_2}{\lambda_1^2}\right) \right\} + \left(R_c + (r_p + r)t\right) \left\{-\frac{\mu}{b^{N-1}} - \frac{\lambda_1 + a^{N}\lambda_2}{\lambda_1^2}\right\}\right]$$

$$= \left(\frac{1}{(\sum_{n=1}^{N+1} \frac{\lambda_1 + a^{n-1}\lambda_2}{\lambda_1^2} + \sum_{n=1}^{N} \frac{\mu}{b^{n-1}} + t)(\sum_{n=1}^{N} \frac{\lambda_1 + a^{n-1}\lambda_2}{\lambda_1^2} + \sum_{n=1}^{N-1} \frac{\mu}{b^{n-1}} + t)} \sum_{n=1}^{N-1} \frac{\mu}{b^{n-1}} \left(\frac{\lambda_1 + a^{N}\lambda_2}{\lambda_1^2}\right) \right) \left[(r_c + r)\mu \left\{ \frac{1}{b^{N-1}} \sum_{n=1}^{N} \frac{\lambda_1 + a^{n-1}\lambda_2}{\lambda_1^2} + t \frac{1}{b^{N-1}} - \sum_{n=1}^{N-1} \frac{1}{b^{n-1}} \left(\frac{\lambda_1 + a^{N}\lambda_2}{\lambda_1^2}\right) \right\} + \left(R_c + (r_p + r)t\right) \left\{-\frac{\mu}{b^{N-1}} - \frac{\lambda_1 + a^{N}\lambda_2}{\lambda_1^2}\right\}\right]$$

$$=\left(\frac{1}{(\sum_{n=1}^{N+1}\frac{\lambda_1+a^{n-1}\lambda_2}{\lambda_1^2}+\sum_{n=1}^{N}\frac{\mu}{b^{n-1}}+t)(\sum_{n=1}^{N}\frac{\lambda_1+a^{n-1}\lambda_2}{\lambda_1^2}+\sum_{n=1}^{N-1}\frac{\mu}{b^{n-1}}+t)}\right)\frac{1}{b^{N-1}}\left[(r_c+r)\mu\left\{\sum_{n=1}^{N}\frac{\lambda_1+a^{n-1}\lambda_2}{\lambda_1^2}+t-\sum_{n=1}^{N-1}b^{N-n}\frac{\lambda_1+a^{N}\lambda_2}{\lambda_1^2}\right\}-\left(R_c+(r_p+r)t\right)\left\{\mu+\left(\frac{\lambda_1+a^{N}\lambda_2}{\lambda_1^2}\right)b^{N-1}\right\}\right] \tag{20}$$

The avability function is given by

$$A_v(N)=\frac{(r_c+r)\mu\left\{\sum_{n=1}^{N}\frac{\lambda_1+a^{n-1}\lambda_2}{\lambda_1^2}-\left(\frac{\lambda_1+a^{N}\lambda_2}{\lambda_1^2}\right)\sum_{n=1}^{N-1}b^{N-n}+t\right\}}{\left((R_c+(r_p+r)t\right)\left\{\mu+\left(\frac{\lambda_1+a^{N}\lambda_2}{\lambda_1^2}\right)b^{N-1}\right\}} \tag{21}$$

Again;

$A_v(N+1)-A_v(N)$

$$=\frac{(r_c+r)\mu\left\{\sum_{n=1}^{N+1}\frac{\lambda_1+a^{n-1}\lambda_2}{\lambda_1^2}-\left(\frac{\lambda_1+a^{N+1}\lambda_2}{\lambda_1^2}\right)\sum_{n=1}^{N}b^{N+1-n}+t\right\}}{\left((R_c+(r_p+r)t\right)\left\{\mu+\left(\frac{\lambda_1+a^{N+1}\lambda_2}{\lambda_1^2}\right)b^{N}\right\}}-\frac{(r_c+r)\mu\left\{\sum_{n=1}^{N}\frac{\lambda_1+a^{n-1}\lambda_2}{\lambda_1^2}-\left(\frac{\lambda_1+a^{N}\lambda_2}{\lambda_1^2}\right)\sum_{n=1}^{N-1}b^{N-n}+t\right\}}{\left((R_c+(r_p+r)t\right)\left\{\mu+\left(\frac{\lambda_1+a^{N}\lambda_2}{\lambda_1^2}\right)b^{N-1}\right\}}$$

$$=\frac{(r_c+r)\mu}{\left((R_c+(r_p+r)t\right)}\left[\frac{\sum_{n=1}^{N+1}\frac{\lambda_1+a^{n-1}\lambda_2}{\lambda_1^2}-\left(\frac{\lambda_1+a^{N+1}\lambda_2}{\lambda_1^2}\right)\sum_{n=1}^{N}b^{N+1-n}+t}{\left\{\mu+\left(\frac{\lambda_1+a^{N+1}\lambda_2}{\lambda_1^2}\right)b^{N}\right\}}-\frac{\sum_{n=1}^{N}\frac{\lambda_1+a^{n-1}\lambda_2}{\lambda_1^2}-\left(\frac{\lambda_1+a^{N}\lambda_2}{\lambda_1^2}\right)\sum_{n=1}^{N-1}b^{N-n}+t}{\left\{\mu+\left(\frac{\lambda_1+a^{N}\lambda_2}{\lambda_1^2}\right)b^{N-1}\right\}}\right]$$

$$=\left(\frac{(r_c+r)\mu}{\left((R_c+(r_p+r)t\right)}\right)\frac{1}{\left\{\mu+\left(\frac{\lambda_1+a^{N+1}\lambda_2}{\lambda_1^2}\right)b^{N}\right\}\left\{\mu+\left(\frac{\lambda_1+a^{N}\lambda_2}{\lambda_1^2}\right)b^{N-1}\right\}}\left[\mu\sum_{n=1}^{N+1}\frac{\lambda_1+a^{n-1}\lambda_2}{\lambda_1^2}-\mu\left(\frac{\lambda_1+a^{N+1}\lambda_2}{\lambda_1^2}\right)\sum_{n=1}^{N}b^{N+1-n}+\mu t+\left(\frac{\lambda_1+a^{N}\lambda_2}{\lambda_1^2}\right)b^{N-1}\sum_{n=1}^{N+1}\frac{\lambda_1+a^{n-1}\lambda_2}{\lambda_1^2}-\left(\frac{\lambda_1+a^{N}\lambda_2}{\lambda_1^2}\right)b^{N-1}\left(\frac{\lambda_1+a^{N+1}\lambda_2}{\lambda_1^2}\right)\sum_{n=1}^{N}b^{N+1-n}+\left(\frac{\lambda_1+a^{N}\lambda_2}{\lambda_1^2}\right)b^{N-1}t-\mu\sum_{n=1}^{N}\frac{\lambda_1+a^{n-1}\lambda_2}{\lambda_1^2}+\mu\left(\frac{\lambda_1+a^{N}\lambda_2}{\lambda_1^2}\right)\sum_{n=1}^{N-1}b^{N-n}-\mu t-\left(\frac{\lambda_1+a^{N+1}\lambda_2}{\lambda_1^2}\right)b^{N}\sum_{n=1}^{N}\frac{\lambda_1+a^{n-1}\lambda_2}{\lambda_1^2}+\left(\frac{\lambda_1+a^{N+1}\lambda_2}{\lambda_1^2}\right)b^{N}\left(\frac{\lambda_1+a^{N}\lambda_2}{\lambda_1^2}\right)\sum_{n=1}^{N-1}b^{N-n}-\left(\frac{\lambda_1+a^{N+1}\lambda_2}{\lambda_1^2}\right)b^{N}t\right]$$

$$=\left(\frac{(r_c+r)\mu}{\left((R_c+(r_p+r)t\right)}\right)\frac{1}{\left\{\mu+\left(\frac{\lambda_1+a^{N+1}\lambda_2}{\lambda_1^2}\right)b^{N}\right\}\left\{\mu+\left(\frac{\lambda_1+a^{N}\lambda_2}{\lambda_1^2}\right)b^{N-1}\right\}}\left[\mu\frac{\lambda_1+a^{N}\lambda_2}{\lambda_1^2}-\mu\left(\frac{\lambda_1+a^{N+1}\lambda_2}{\lambda_1^2}\right)\sum_{n=1}^{N}b^{N+1-n}+\mu\left(\frac{\lambda_1+a^{N}\lambda_2}{\lambda_1^2}\right)\sum_{n=1}^{N-1}b^{N-n}+\frac{\lambda_1+a^{N}\lambda_2}{\lambda_1^2}b^{N-1}\sum_{n=1}^{N+1}\frac{\lambda_1+a^{n-1}\lambda_2}{\lambda_1^2}-\frac{\lambda_1+a^{N+1}\lambda_2}{\lambda_1^2}b^{N}\sum_{n=1}^{N}\frac{\lambda_1+a^{n-1}\lambda_2}{\lambda_1^2}+t\frac{\lambda_1+a^{N}\lambda_2}{\lambda_1^2}b^{N-1}-t\frac{\lambda_1+a^{N+1}\lambda_2}{\lambda_1^2}b^{N}-\left(\frac{\lambda_1+a^{N+1}\lambda_2}{\lambda_1^2}\right)\left(\frac{\lambda_1+a^{N}\lambda_2}{\lambda_1^2}\right)\sum_{n=1}^{N-1}b^{2N-n}-\frac{\lambda_1+a^{N+1}\lambda_2}{\lambda_1^2}\frac{\lambda_1+a^{N}\lambda_2}{\lambda_1^2}b^{N}+\left(\frac{\lambda_1+a^{N+1}\lambda_2}{\lambda_1^2}\right)\left(\frac{\lambda_1+a^{N}\lambda_2}{\lambda_1^2}\right)\sum_{n=1}^{N-1}b^{2N-n}\right]$$

$$=\left(\frac{(r_c+r)\mu}{\left((R_c+(r_p+r)t\right)}\right)\frac{1}{\left\{\mu+\left(\frac{\lambda_1+a^{N+1}\lambda_2}{\lambda_1^2}\right)b^{N}\right\}\left\{\mu+\left(\frac{\lambda_1+a^{N}\lambda_2}{\lambda_1^2}\right)b^{N-1}\right\}}\left[\mu\frac{\lambda_1+a^{N}\lambda_2}{\lambda_1^2}-\mu\left(\frac{\lambda_1+a^{N+1}\lambda_2}{\lambda_1^2}\right)\sum_{n=1}^{N}b^{N+1-n}+\mu\frac{\lambda_1+a^{N}\lambda_2}{\lambda_1^2}\sum_{n=1}^{N}b^{N-n}-\mu\frac{\lambda_1+a^{N}\lambda_2}{\lambda_1^2}+\frac{\lambda_1+a^{N}\lambda_2}{\lambda_1^2}b^{N-1}\sum_{n=1}^{N+1}\frac{\lambda_1+a^{n-1}\lambda_2}{\lambda_1^2}-\frac{\lambda_1+a^{N+1}\lambda_2}{\lambda_1^2}b^{N}\sum_{n=1}^{N+1}\frac{\lambda_1+a^{n-1}\lambda_2}{\lambda_1^2}+\left(\frac{\lambda_1+a^{N+1}\lambda_2}{\lambda_1^2}\right)\left(\frac{\lambda_1+a^{N}\lambda_2}{\lambda_1^2}\right)b^{N}+t\frac{\lambda_1+a^{N}\lambda_2}{\lambda_1^2}b^{N-1}-t\frac{\lambda_1+a^{N+1}\lambda_2}{\lambda_1^2}b^{N}-\left(\frac{\lambda_1+a^{N+1}\lambda_2}{\lambda_1^2}\right)\left(\frac{\lambda_1+a^{N}\lambda_2}{\lambda_1^2}\right)b^{N}\right]$$

$$=\frac{(r_c+r)\mu}{\left((R_c+(r_p+r)t\right)}\Bigg)\frac{1}{\left\{\mu+\left(\frac{\lambda_1+a^{N+1}\lambda_2}{\lambda_1^2}\right)b^{N}\right\}\left\{\mu+\left(\frac{\lambda_1+a^{N}\lambda_2}{\lambda_1^2}\right)b^{N-1}\right\}}\left[\mu\left(\frac{\lambda_1+a^{N}\lambda_2}{\lambda_1^2}b^{-1}-\frac{\lambda_1+a^{N+1}\lambda_2}{\lambda_1^2}\right)\sum_{n=1}^{N}b^{N+1-n}+\left(\frac{\lambda_1+a^{N}\lambda_2}{\lambda_1^2}b^{-1}-\frac{\lambda_1+a^{N+1}\lambda_2}{\lambda_1^2}\right)b^{N}\sum_{n=1}^{N+1}\frac{\lambda_1+a^{n-1}\lambda_2}{\lambda_1^2}+t\left(\frac{\lambda_1+a^{N}\lambda_2}{\lambda_1^2}b^{-1}-\frac{\lambda_1+a^{N+1}\lambda_2}{\lambda_1^2}\right)b^{N}\right] \tag{22}$$

From (20) we see that the sign of denominator is always +v so that the sign of O (N + 1) – O (N) is dependent of numerator of (21).Therefore O (N + 1) ≥ O (N) or O (N + 1) ≤ O (N).

$\because 0 < a \le 1 \Longrightarrow \frac{\lambda_1+a^{n-1}\lambda_2}{\lambda_1^2}$ is a decreasing or increasing function for n.

While $0 < b \le 1 \Longrightarrow \frac{\lambda_1+a^{N}\lambda_2}{\lambda_1^2} \ge \frac{\lambda_1+a^{N+1}\lambda_2}{\lambda_1^2} \ge b\frac{\lambda_1+a^{N+1}\lambda_2}{\lambda_1^2}$

Therefore $A_v(N+1) \ge A_v(N)$

Hence for obtaining optimal N* by

N* = min$\{N|\ A_v(N) \ge 1\}$

If A_v (N*) greater then 1for certain value for N* then N* is unique. Since $A_v(N)$ is non decreasing in N, therefore there exist integer N* such that $A_v(N) \geq 1 \Leftrightarrow N \geq N^*$ and $A_v(N) < 1 \Leftrightarrow N < N^*$

Numerical Example

Consider the parametric values are:

$\lambda_1 = 0.02, \lambda_2 = 0.04, \mu = 6,\ r_p = 5, t = 25,\ R_c = 6000, r = 20, r_c = 5, a = 0.88, b = 0.87$

Table 1: Q (N) for different values of N.

N	Q(N)	N	Q(N)	N	Q(N)
1	17.85714	16	-12.51728	31	-5.69861
2	1.23824	17	-12.32308	32	-5.06243
3	-4.67493	18	-12.08381	33	-4.42780
4	-7.66749	19	-11.80097	34	-3.79968
5	-9.44386	20	-11.47586	35	-3.18277
6	-10.59400	21	-11.10978	36	-2.58137
7	-11.37577	22	-10.70418	37	-1.99930
8	-11.91930	23	-10.26078	38	-1.43985
9	-12.29697	24	-9.78166	39	-0.90568
10	-12.55191	25	-9.26934	40	-0.39889
11	-12.71110	26	-8.72682	41	0.07906
12	-12.79207	27	-8.15758	42	0.52727
13	-12.80646	28	-7.56559	43	0.94536
14	-12.76214	29	-6.95523	44	1.33343
15	-12.66450	30	-6.95523	45	1.69198

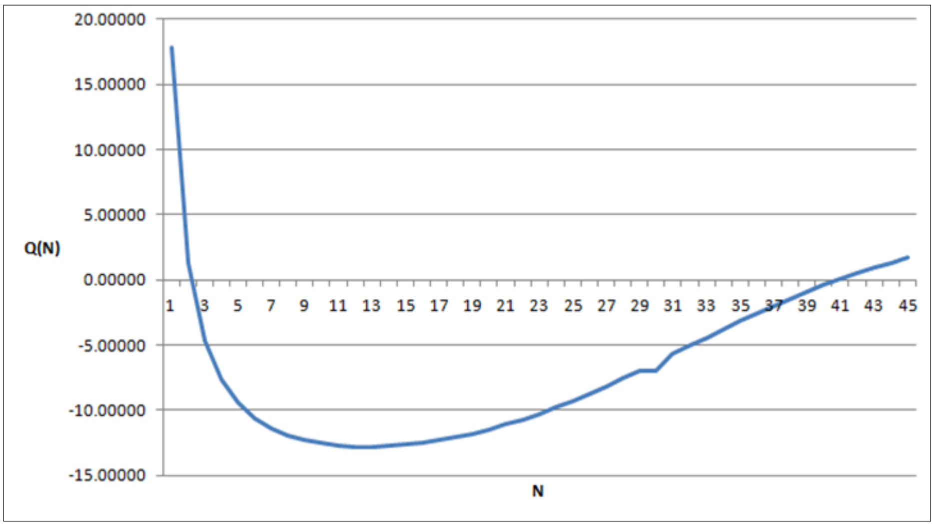

Fig 1: Graph of Q (N) for different values of N.

Table 2: A_v (N) for different values of N

N	Av(N)	N	Av(N)	N	Av(N)
1	0.02752	16	1.93959	31	6.83166
2	0.03916	17	2.23854	32	7.11884
3	0.05890	18	2.55320	33	7.39788
4	0.08872	19	2.88041	34	7.66909
5	0.13080	20	3.21693	35	7.93290
6	0.18744	21	3.55957	36	8.18974
7	0.26099	22	3.90534	37	8.44008
8	0.35372	23	4.25152	38	8.68440
9	0.46772	24	4.59577	39	8.92318
10	0.60476	25	4.93609	40	9.15687
11	0.76613	26	5.27092	41	9.38593
12	0.95256	27	5.59905	42	9.61078
13	1.16410	28	5.91961	43	9.83180
14	1.40012	29	6.23206	44	10.04937
15	1.65928	30	6.53610	45	10.26382

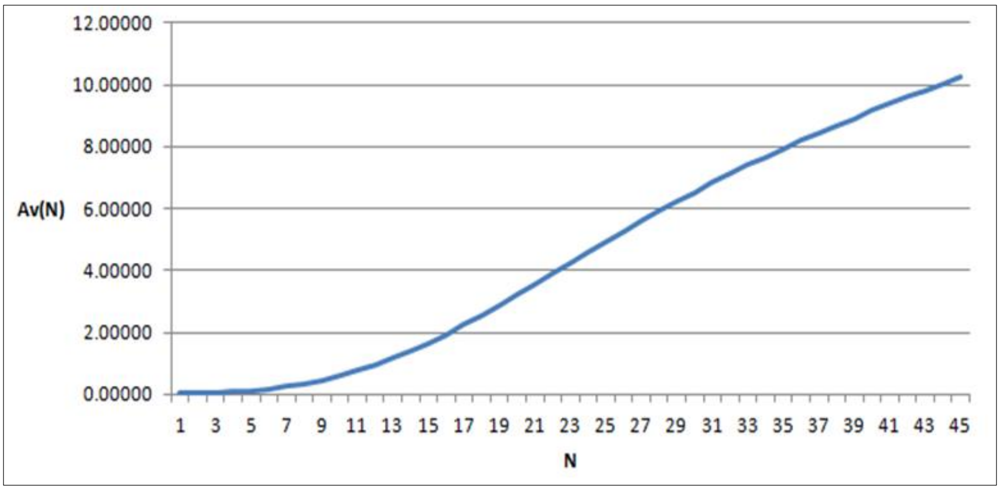

Fig 2: Graph of A_v (N) against N.

From table1 we see that at N = 13and Q (N) is minimum. Also at N = 6 is the first value where A_v (N) ≥ 1 and it is A_v (N) = 1.16410 > 1. Therefore 13th failure the system should be replaced. From the table1show that Q (13) = -12.80646 is the minimum average cost rate of the system.

5. Conclusion

In this study, we looked into the ideal replacement approach for broken-down systems that can still be fixed. We determine the minimum average cost at which the system should be replaced using the delta shock model. I also validated the proposed model with a numerical illustration. It is important to look at the system-deteriorating maintenance policy from a theoretical and practical standpoint. The method is efficient on a large scale because it can be used on a variety of reliable systems, including computer systems and electronic equipment. Our model can be used by managers to evaluate system performance and develop the optimum maintenance plan. Further research in this field can be enhanced by considering non-homogeneous Poisson and Renewal processes.

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