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Three-dimensional analysis of incompressible couette flow in a visco-elastic medium

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Abstract
An investigation is carried out to study the three-dimensional unsteady incompressible Couette flow of a visco-elastic fluid through a porous channel bounded by two horizontal infinite parallel plates in presence of heat and mass transfer. The second-order Rivlin-Ericksen fluid model is taken throughout the study. Approximate solutions for velocity, temperature, concentration and other flow characteristics of the problem have been obtained by using regular perturbation technique. The influence of different pertinent parameters on the flow region has been discussed through graphical illustrations. Newtonian results are found to come out as limiting cases of the present analysis.

Keywords: Visco-elastic, couette flow, porous channel, heat and mass transfer

1. Introduction

Couette flow is crucial in numerous mechanisms regarding the relative motion of two surfaces. It is widely used in transpiration cooling system. In this phenomenon, several engines can be secured from the influence of hot gases. The application of this process is noticed in rocket and turbojet engines *viz.* gas turbine blades, exhaust nozzles and combustion chamber walls. The visco-elastic flows through porous medium in presence of heat and mass transfer have large number of applications in engineering and geophysics, for example, drying of porous solids, enhanced oil recovery, underground energy transport, cooling of nuclear reactors, to study the underground water resources in the field of agriculture engineering, for filtration and purification processes in chemical engineering etc.

In view of these applications, various researchers have worked on this field. Aboeldahab and Azzam [1] have investigated the unsteady three-dimensional combined heat and mass transfer for convective flow over a stretching surface with time dependent chemical reaction. The study of heat and mass transfer on free convective three-dimensional unsteady flow over a porous vertical plate has been presented by Ahmed [2]. Chaudhary and Sharma [3] have analysed the three-dimensional unsteady convection and mass transfer flow through porous medium. Influence of viscous dissipation and radiation on unsteady MHD free convection flow past an infinite heated vertical plate in a porous medium with time dependent suction has been investigated by Cookey *et al.* [4]. Sahin [5] has analysed the transient three-dimensional flows through a porous medium with transverse permeability oscillating with time. Flow and heat transfer in a second-grade fluid over a stretching sheet has been studied by Vajravelu and Roper [6]. Rajagopal and Gupta [7] have investigated an exact solution for the flow of a non-Newtonian fluid past an infinite plate on boundary conditions for fluids of differential type.

In the present study, we have analysed the three-dimensional incompressible couette flow of a visco-elastic fluid through a porous channel in presence of heat and mass transfer.

2. Mathematical Formulation

We consider the unsteady incompressible flow of a visco-elastic fluid between two infinite horizontal flat plate separated by a distance d. Let U_w be the uniform velocity of the upper plate in the direction of the flow. We assume a cartesian coordinate system with origin on the lower stable plate. $\bar{X}$-axis is taken along the flow, $\bar{Y}$-axis normal to the plate and $\bar{Z}$-axis is taken perpendicular to the $\bar{X}\bar{Y}$-plane.

The upper plate is subjected to a constant injection $-v_0$ and stable plate a normal sinusoidal suction velocity disposition of the form $\bar{V} = -v_0\left[1 + \varepsilon\cos\left(\frac{\pi\bar{Z} - U_w\bar{t}}{d}\right)\right]$. Due to periodic suction, the flow emerges as three-dimensional. Now employing Boussinesq approximation, the governing equations of the flow are as follow:

$$\frac{\partial\bar{V}}{\partial\bar{Y}} + \frac{\partial\bar{W}}{\partial\bar{Z}} = 0 \tag{1}$$

$$\rho\left(\frac{\partial\bar{U}}{\partial\bar{t}} + \bar{V}\frac{\partial\bar{U}}{\partial\bar{Y}} + \bar{W}\frac{\partial\bar{U}}{\partial\bar{Z}}\right) = \mu_{[1]}\left(\frac{\partial^2\bar{U}}{\partial\bar{Y}^2} + \frac{\partial^2\bar{U}}{\partial\bar{Z}^2}\right) + \mu_{[2]}\left(\frac{\partial^3\bar{U}}{\partial\bar{Y}^2\partial\bar{t}} + \frac{\partial^3\bar{U}}{\partial\bar{Z}^2\partial\bar{t}} + 2\frac{\partial\bar{V}}{\partial\bar{Y}}\frac{\partial^2\bar{U}}{\partial\bar{Y}^2} + \bar{V}\frac{\partial^3\bar{U}}{\partial\bar{Y}^3} + \frac{\partial\bar{U}}{\partial\bar{Y}}\frac{\partial^2\bar{V}}{\partial\bar{Y}^2} + \frac{\partial\bar{U}}{\partial\bar{Z}}\frac{\partial^2\bar{W}}{\partial\bar{Y}^2} + 2\frac{\partial\bar{W}}{\partial\bar{Y}}\frac{\partial^2\bar{U}}{\partial\bar{Y}\partial\bar{Z}} + \bar{W}\frac{\partial^3\bar{U}}{\partial\bar{Y}^2\partial\bar{Z}} + \frac{\partial^2\bar{V}}{\partial\bar{Z}^2}\frac{\partial\bar{U}}{\partial\bar{Y}} + 2\frac{\partial\bar{V}}{\partial\bar{Z}}\frac{\partial^2\bar{U}}{\partial\bar{Y}\partial\bar{Z}} + \bar{V}\frac{\partial^3\bar{U}}{\partial\bar{Z}^2\partial\bar{Y}} + \frac{\partial^2\bar{W}}{\partial\bar{Z}^2}\frac{\partial\bar{U}}{\partial\bar{Z}} + 2\frac{\partial\bar{W}}{\partial\bar{Z}}\frac{\partial^2\bar{U}}{\partial\bar{Z}^2} + \bar{W}\frac{\partial^3\bar{U}}{\partial\bar{Z}^3}\right) + \mu_{[3]}\left(\frac{\partial^2\bar{V}}{\partial\bar{Y}^2}\frac{\partial\bar{U}}{\partial\bar{Y}} + 2\frac{\partial^2\bar{U}}{\partial\bar{Y}^2}\frac{\partial\bar{V}}{\partial\bar{Y}} + 2\frac{\partial\bar{W}}{\partial\bar{Y}}\frac{\partial^2\bar{U}}{\partial\bar{Y}\partial\bar{Z}} + \frac{\partial\bar{U}}{\partial\bar{Z}}\frac{\partial^2\bar{W}}{\partial\bar{Y}^2} + 2\frac{\partial\bar{V}}{\partial\bar{Z}}\frac{\partial^2\bar{U}}{\partial\bar{Y}\partial\bar{Z}} + \frac{\partial\bar{U}}{\partial\bar{Z}}\frac{\partial^2\bar{W}}{\partial\bar{Z}^2} + \frac{\partial\bar{U}}{\partial\bar{Y}}\frac{\partial^2\bar{V}}{\partial\bar{Z}^2} + 2\frac{\partial\bar{W}}{\partial\bar{Z}}\frac{\partial^2\bar{U}}{\partial\bar{Z}^2}\right) + g\beta(\bar{T} - T_0) + g\beta^*(\bar{C} - C_0) \tag{2}$$

$$\rho\left(\frac{\partial\bar{V}}{\partial\bar{t}} + \bar{V}\frac{\partial\bar{V}}{\partial\bar{Y}} + \bar{W}\frac{\partial\bar{V}}{\partial\bar{Z}}\right) = -\frac{\partial\bar{P}}{\partial\bar{Y}} + \mu_{[1]}\left(\frac{\partial^2\bar{V}}{\partial\bar{Z}^2} + \frac{\partial^2\bar{V}}{\partial\bar{Y}^2}\right) + \mu_{[2]}\left(\frac{\partial^3\bar{V}}{\partial\bar{Y}^2\partial\bar{t}} + \frac{\partial^3\bar{V}}{\partial\bar{Z}^2\partial\bar{t}} + 3\frac{\partial\bar{V}}{\partial\bar{Z}}\frac{\partial^2\bar{W}}{\partial\bar{Y}^2} + \bar{V}\frac{\partial^3\bar{V}}{\partial\bar{Y}^3} + \bar{W}\frac{\partial^3\bar{V}}{\partial\bar{Y}^2\partial\bar{Z}} + \bar{V}\frac{\partial^3\bar{V}}{\partial\bar{Z}^2\partial\bar{Y}} + \bar{W}\frac{\partial^3\bar{V}}{\partial\bar{Z}^3} + 3\frac{\partial\bar{V}}{\partial\bar{Z}}\frac{\partial^2\bar{V}}{\partial\bar{Y}\partial\bar{Z}} + 2\frac{\partial\bar{U}}{\partial\bar{Z}}\frac{\partial^2\bar{U}}{\partial\bar{Y}\partial\bar{Z}} + 4\frac{\partial\bar{U}}{\partial\bar{Y}}\frac{\partial^2\bar{U}}{\partial\bar{Y}^2} + 2\frac{\partial\bar{U}}{\partial\bar{Y}}\frac{\partial^2\bar{U}}{\partial\bar{Z}^2} + 2\frac{\partial\bar{W}}{\partial\bar{Y}}\frac{\partial^2\bar{V}}{\partial\bar{Y}\partial\bar{Z}} + 13\frac{\partial^2\bar{V}}{\partial\bar{Y}^2}\frac{\partial\bar{V}}{\partial\bar{Y}} + 4\frac{\partial^2\bar{W}}{\partial\bar{Y}^2}\frac{\partial\bar{W}}{\partial\bar{Y}} + \frac{\partial\bar{V}}{\partial\bar{Y}}\frac{\partial^2\bar{V}}{\partial\bar{Z}^2}\right) + \mu_{[3]}\left(8\frac{\partial^2\bar{V}}{\partial\bar{Y}^2}\frac{\partial\bar{V}}{\partial\bar{Y}} + 2\frac{\partial^2\bar{U}}{\partial\bar{Y}^2}\frac{\partial\bar{U}}{\partial\bar{Y}} + \frac{\partial\bar{U}}{\partial\bar{Z}}\frac{\partial^2\bar{U}}{\partial\bar{Y}\partial\bar{Z}} + \frac{\partial\bar{U}}{\partial\bar{Y}}\frac{\partial^2\bar{U}}{\partial\bar{Z}^2} + 2\frac{\partial\bar{V}}{\partial\bar{Z}}\frac{\partial^2\bar{V}}{\partial\bar{Y}\partial\bar{Z}} + 2\frac{\partial\bar{W}}{\partial\bar{Y}}\frac{\partial^2\bar{W}}{\partial\bar{Y}^2} + 2\frac{\partial\bar{W}}{\partial\bar{Y}}\frac{\partial^2\bar{V}}{\partial\bar{Y}\partial\bar{Z}} + 2\frac{\partial\bar{V}}{\partial\bar{Z}}\frac{\partial^2\bar{W}}{\partial\bar{Y}^2}\right) \tag{3}$$

$$\rho\left(\frac{\partial\bar{W}}{\partial\bar{t}} + \bar{V}\frac{\partial\bar{W}}{\partial\bar{Y}} + \bar{W}\frac{\partial\bar{W}}{\partial\bar{Z}}\right) = -\frac{\partial\bar{P}}{\partial\bar{Z}} + \mu_{[1]}\left(\frac{\partial^2\bar{W}}{\partial\bar{Y}^2} + \frac{\partial^2\bar{W}}{\partial\bar{Z}^2}\right) + \mu_{[2]}\left(\frac{\partial^3\bar{W}}{\partial\bar{Y}^2\partial\bar{t}} + \frac{\partial^3\bar{W}}{\partial\bar{Z}^2\partial\bar{t}} + 3\frac{\partial\bar{W}}{\partial\bar{Y}}\frac{\partial^2\bar{V}}{\partial\bar{Z}^2} + \bar{W}\frac{\partial^3\bar{W}}{\partial\bar{Z}^3} + \bar{W}\frac{\partial^3\bar{W}}{\partial\bar{Y}^2\partial\bar{Z}} + \bar{V}\frac{\partial^3\bar{W}}{\partial\bar{Z}^2\partial\bar{Y}} + \bar{V}\frac{\partial^3\bar{W}}{\partial\bar{Y}^3} + 2\frac{\partial\bar{U}}{\partial\bar{Y}}\frac{\partial^2\bar{U}}{\partial\bar{Y}\partial\bar{Z}} + 2\frac{\partial\bar{U}}{\partial\bar{Z}}\frac{\partial^2\bar{U}}{\partial\bar{Y}^2} + 4\frac{\partial\bar{U}}{\partial\bar{Z}}\frac{\partial^2\bar{U}}{\partial\bar{Z}^2} + 2\frac{\partial\bar{V}}{\partial\bar{Z}}\frac{\partial^2\bar{W}}{\partial\bar{Y}\partial\bar{Z}} + 3\frac{\partial\bar{W}}{\partial\bar{Y}}\frac{\partial^2\bar{W}}{\partial\bar{Y}\partial\bar{Z}} + 13\frac{\partial^2\bar{W}}{\partial\bar{Z}^2}\frac{\partial\bar{W}}{\partial\bar{Z}} + 4\frac{\partial^2\bar{V}}{\partial\bar{Z}^2}\frac{\partial\bar{V}}{\partial\bar{Z}} + \frac{\partial\bar{W}}{\partial\bar{Z}}\frac{\partial^2\bar{W}}{\partial\bar{Y}^2}\right) + \mu_{[3]}\left(8\frac{\partial^2\bar{W}}{\partial\bar{Z}^2}\frac{\partial\bar{W}}{\partial\bar{Z}} + 2\frac{\partial^2\bar{U}}{\partial\bar{Z}^2}\frac{\partial\bar{U}}{\partial\bar{Z}} + \frac{\partial\bar{U}}{\partial\bar{Y}}\frac{\partial^2\bar{U}}{\partial\bar{Y}\partial\bar{Z}} + \frac{\partial\bar{U}}{\partial\bar{Z}}\frac{\partial^2\bar{U}}{\partial\bar{Y}^2} + 2\frac{\partial\bar{W}}{\partial\bar{Y}}\frac{\partial^2\bar{W}}{\partial\bar{Y}\partial\bar{Z}} + 2\frac{\partial\bar{W}}{\partial\bar{Y}}\frac{\partial^2\bar{V}}{\partial\bar{Z}^2} + 2\frac{\partial\bar{V}}{\partial\bar{Z}}\frac{\partial^2\bar{W}}{\partial\bar{Y}\partial\bar{Z}} + 2\frac{\partial\bar{V}}{\partial\bar{Z}}\frac{\partial^2\bar{V}}{\partial\bar{Z}^2}\right) \tag{4}$$

$$\frac{\partial\bar{T}}{\partial\bar{t}} + \bar{V}\frac{\partial\bar{T}}{\partial\bar{Y}} + \bar{W}\frac{\partial\bar{T}}{\partial\bar{Z}} = \frac{k}{\rho C_p}\left(\frac{\partial^2\bar{T}}{\partial\bar{Y}^2} + \frac{\partial^2\bar{T}}{\partial\bar{Z}^2}\right) \tag{5}$$

$$\frac{\partial\bar{C}}{\partial\bar{t}} + \bar{V}\frac{\partial\bar{C}}{\partial\bar{Y}} + \bar{W}\frac{\partial\bar{C}}{\partial\bar{Z}} = D\left(\frac{\partial^2\bar{C}}{\partial\bar{Y}^2} + \frac{\partial^2\bar{C}}{\partial\bar{Z}^2}\right) \tag{6}$$

The relevant boundary conditions are:

$$\bar{U} = \bar{L}\frac{\partial\bar{U}}{\partial\bar{Y}}, \bar{V} = -v_0\left[1 + \varepsilon\cos\left(\frac{\pi\bar{Z} - U_w\bar{t}}{d}\right)\right], \bar{W} = 0, \bar{T} = T_0, \bar{C} = C_0 \text{ at } \bar{Y} = 0 \tag{7}$$

$$\bar{U} = U_w, \bar{V} = -v_0, \bar{W} = 0, \bar{P} = \overline{P_w}, \bar{T} = T_1, \bar{C} = C_1 \text{ at } \bar{Y} = d \tag{8}$$

where $\bar{U}, \bar{V}, \bar{W}$ are the velocity components in the $\bar{X}, \bar{Y}, \bar{Z}$ directions respectively, $\bar{T}$ is the temperature, $\bar{C}$ is the species concentration, $\bar{P}$ is the fluid pressure, g is the acceleration due to gravity, β is the coefficient of the thermal expansion, β^* is the coefficient of volume expansion for mass transfer and $\varepsilon, \rho, k, D, C_p$ are the amplitude of the suction velocity, density, thermal conductivity, diffusion coefficient and specific heat at constant pressure respectively.
We introduce the dimensionless quantities:

$$Y = \frac{\bar{Y}}{d}, Z = \frac{\bar{Z}}{d}, t = \frac{U_w\bar{t}}{d}, h = \frac{\bar{L}}{d}, P = \frac{\bar{P}}{\rho U_w^2}, U = \frac{\bar{U}}{U_w}, V = \frac{\bar{V}}{U_w}, W = \frac{\bar{W}}{U_w}, \theta = \frac{\bar{T} - T_0}{T_1 - T_0}, \phi = \frac{\bar{C} - C_0}{C_1 - C_0}, Gr = \frac{g\beta(T_1 - T_0)}{U_w^3}, Gm = \frac{g\beta^*(C_1 - C_0)}{U_w^3}, Re = \frac{dU_w}{v_1}, Pr = \frac{\rho v_1 C_p}{k}, Sc = \frac{v_1}{D}, S = \frac{v_0}{U_w}, \alpha_1 = \frac{v_2}{d^2}, \alpha_2 = \frac{v_3}{d^2} \tag{9}$$

Substitution of (10) into the Eqs. (2) to (7) yields the following dimensionless equations:

$$\frac{\partial V}{\partial Y} + \frac{\partial W}{\partial Z} = 0 \tag{10}$$

$$\frac{\partial U}{\partial t} + V\frac{\partial U}{\partial Y} + W\frac{\partial U}{\partial Z} = \frac{1}{Re}\left(\frac{\partial^2 U}{\partial Y^2} + \frac{\partial^2 U}{\partial Z^2}\right) + \alpha_1\left(\frac{\partial^3 U}{\partial Y^2\partial t} + \frac{\partial^3 U}{\partial Z^2\partial t} + 2\frac{\partial V}{\partial Y}\frac{\partial^2 U}{\partial Y^2} + V\frac{\partial^3 U}{\partial Y^3} + \frac{\partial U}{\partial Y}\frac{\partial^2 V}{\partial Y^2} + \frac{\partial U}{\partial Z}\frac{\partial^2 W}{\partial Y^2} + 2\frac{\partial W}{\partial Y}\frac{\partial^2 U}{\partial Y\partial Z} + W\frac{\partial^3 U}{\partial Y^2\partial Z} + \frac{\partial^2 V}{\partial Z^2}\frac{\partial U}{\partial Y} + 2\frac{\partial V}{\partial Z}\frac{\partial^2 U}{\partial Y\partial Z} + V\frac{\partial^3 U}{\partial Z^2\partial Y} + \frac{\partial^2 W}{\partial Z^2}\frac{\partial U}{\partial Z} + 2\frac{\partial W}{\partial Z}\frac{\partial^2 U}{\partial Z^2} + W\frac{\partial^3 U}{\partial Z^3}\right) + \alpha_2\left(\frac{\partial^2 V}{\partial Y^2}\frac{\partial U}{\partial Y} + 2\frac{\partial^2 U}{\partial Y^2}\frac{\partial V}{\partial Y} + 2\frac{\partial W}{\partial Y}\frac{\partial^2 U}{\partial Y\partial Z} + \frac{\partial U}{\partial Z}\frac{\partial^2 W}{\partial Y^2} + 2\frac{\partial V}{\partial Z}\frac{\partial^2 U}{\partial Y\partial Z} + \frac{\partial U}{\partial Z}\frac{\partial^2 W}{\partial Z^2} + \frac{\partial U}{\partial Y}\frac{\partial^2 V}{\partial Z^2} + 2\frac{\partial W}{\partial Z}\frac{\partial^2 U}{\partial Z^2}\right) + Gr\theta + Gm\phi \tag{11}$$

$$\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial Y} + W\frac{\partial V}{\partial Z} = -\frac{\partial P}{\partial Y} + \frac{1}{Re}\left(\frac{\partial^2 V}{\partial Z^2} + \frac{\partial^2 V}{\partial Y^2}\right) + \alpha_1\left(\frac{\partial^3 V}{\partial Y^2\partial t} + \frac{\partial^3 V}{\partial Z^2\partial t} + 3\frac{\partial V}{\partial Z}\frac{\partial^2 W}{\partial Y^2} + V\frac{\partial^3 V}{\partial Y^3} + W\frac{\partial^3 V}{\partial Y^2\partial Z} + V\frac{\partial^3 V}{\partial Z^2\partial Y} + W\frac{\partial^3 V}{\partial Z^3} + 3\frac{\partial V}{\partial Z}\frac{\partial^2 V}{\partial Y\partial Z} + 2\frac{\partial U}{\partial Z}\frac{\partial^2 U}{\partial Y\partial Z} + 4\frac{\partial U}{\partial Y}\frac{\partial^2 U}{\partial Y^2} + 2\frac{\partial U}{\partial Y}\frac{\partial^2 U}{\partial Z^2} + 2\frac{\partial W}{\partial Y}\frac{\partial^2 V}{\partial Y\partial Z} + 13\frac{\partial^2 V}{\partial Y^2}\frac{\partial V}{\partial Y} + 4\frac{\partial^2 W}{\partial Y^2}\frac{\partial W}{\partial Y} + \frac{\partial V}{\partial Y}\frac{\partial^2 V}{\partial Z^2}\right) + \alpha_2\left(8\frac{\partial^2 V}{\partial Y^2}\frac{\partial V}{\partial Y} + 2\frac{\partial^2 U}{\partial Y^2}\frac{\partial U}{\partial Y} + \frac{\partial U}{\partial Z}\frac{\partial^2 U}{\partial Y\partial Z} + \frac{\partial U}{\partial Y}\frac{\partial^2 U}{\partial Z^2} + 2\frac{\partial V}{\partial Z}\frac{\partial^2 V}{\partial Y\partial Z} + 2\frac{\partial W}{\partial Y}\frac{\partial^2 W}{\partial Y^2} + 2\frac{\partial W}{\partial Y}\frac{\partial^2 V}{\partial Y\partial Z} + 2\frac{\partial V}{\partial Z}\frac{\partial^2 W}{\partial Y^2}\right) \tag{12}$$

$$\frac{\partial W}{\partial t}+V\frac{\partial W}{\partial Y}+W\frac{\partial W}{\partial Z}=-\frac{\partial P}{\partial Z}+\frac{1}{Re}\left(\frac{\partial^2 W}{\partial Y^2}+\frac{\partial^2 W}{\partial Z^2}\right)+\alpha_1\left(\frac{\partial^3 W}{\partial Y^2\partial t}+\frac{\partial^3 W}{\partial Z^2\partial t}+3\frac{\partial W}{\partial Y}\frac{\partial^2 V}{\partial Z^2}+W\frac{\partial^3 W}{\partial Z^3}+W\frac{\partial^3 W}{\partial Y^2\partial Z}+V\frac{\partial^3 W}{\partial Z^2\partial Y}+V\frac{\partial^3 W}{\partial Y^3}+2\frac{\partial U}{\partial Y}\frac{\partial^2 U}{\partial Y\partial Z}+2\frac{\partial U}{\partial Z}\frac{\partial^2 U}{\partial Y^2}+4\frac{\partial U}{\partial Z}\frac{\partial^2 U}{\partial Z^2}+2\frac{\partial V}{\partial Z}\frac{\partial^2 W}{\partial Y\partial Z}+3\frac{\partial W}{\partial Y}\frac{\partial^2 W}{\partial Y\partial Z}+13\frac{\partial^2 W}{\partial Z^2}\frac{\partial W}{\partial Z}+4\frac{\partial^2 V}{\partial Z^2}\frac{\partial V}{\partial V}+\frac{\partial W}{\partial Z}\frac{\partial^2 W}{\partial Y^2}\right)+\alpha_2\left(8\frac{\partial^2 W}{\partial Z^2}\frac{\partial W}{\partial Z}+2\frac{\partial^2 U}{\partial Z^2}\frac{\partial U}{\partial Z}+\frac{\partial U}{\partial Y}\frac{\partial^2 U}{\partial Y\partial Z}+\frac{\partial U}{\partial Z}\frac{\partial^2 U}{\partial Y^2}+2\frac{\partial W}{\partial Y}\frac{\partial^2 W}{\partial Y\partial Z}+2\frac{\partial W}{\partial Y}\frac{\partial^2 V}{\partial Z^2}+2\frac{\partial V}{\partial Z}\frac{\partial^2 W}{\partial Y\partial Z}+2\frac{\partial V}{\partial Z}\frac{\partial^2 V}{\partial Z^2}\right) \tag{13}$$

$$\frac{\partial \theta}{\partial t}+V\frac{\partial \theta}{\partial Y}+W\frac{\partial \theta}{\partial Z}=\frac{1}{RePr}\left(\frac{\partial^2 \theta}{\partial Y^2}+\frac{\partial^2 \theta}{\partial Z^2}\right) \tag{14}$$

$$\frac{\partial \phi}{\partial t}+V\frac{\partial \phi}{\partial Y}+W\frac{\partial \phi}{\partial Z}=\frac{1}{ScRe}\left(\frac{\partial^2 \phi}{\partial Y^2}+\frac{\partial^2 \phi}{\partial Z^2}\right) \tag{15}$$

The relevant boundary conditions are:

$$U=h\frac{\partial U}{\partial Y}, V=-S[1+\varepsilon\cos(\pi Z-t)], W=0, \theta=0, \phi=0 \text{ at } Y=0 \tag{16}$$

$$U=1, V=-S, W=0, P=P_w, \theta=1, \phi=1 \text{ at } Y=1 \tag{17}$$

3. Method of Solution

Solving the system of equations (11) to (15) using perturbation technique for $\varepsilon \ll 1$ and $\alpha_1 \ll 1$ [Nowinski and Ismail [8]] we obtain the expression for velocity, temperature and concentration profiles of the problem.

4. Results and Discussion

Once the velocity, temperature and concentration profiles are known, we can now calculate some important dimensionless flow parameters *viz*. shearing stress in the main flow, Nusselt number and Sherwood number.

To get the physical significance of the problem, the numerical computations have been implemented for velocity, temperature and concentration fields. During the computations, we use different values of the visco-elastic parameter α_1, thermal Grashof number Gr, solutal Grashof number Gm, Reynolds number Re, Prandtl number Pr and Schmidt number Sc with fixed values of $S=1$, $\varepsilon=0.3$, $h=0.5$, $t=0.2$ and $Z=0.2$. Only the real part of the problem has been taken into account throughout the computation. The prospect of this study is to highlight the influence of visco-elastic parameter α_1 on the flow region. Newtonian results are found to emerge as limiting cases of the present analysis by setting $\alpha_1=0$.

Figures 1 to 6 illustrate the variation of main flow velocity U against Y with different values of other flow parameters. The graphs reveal that the main flow velocity increases near the fixed plate but gradually decreases away from it. Also, with the growth of the absolute values of the visco-elastic parameter α_1 (α_1=0, -0.04, -0.08) the main flow velocity enhances in comparison with Newtonian flow phenomenon. The rising values of thermal Grashof number Gr (Figs: 1 and 2) and solutal Grashof number Gm (Figs: 1 and 3) depict an accelerating trend of fluid velocity. This implies that both the thermal and concentration buoyancy forces have tendency to enhance the fluid velocity. The similar result is observed in case of Reynolds number Re (Figs 1 and 4), Prandtl number Pr (Figs 1 and 5) and Schmidt number Sc (Figs 1 and 6). The cross flow velocity component W against Y has been depicted in figure 7. It is observed that the cross velocity first diminishes near the fixed plate then boost up to noticeable amount. Also, the cross velocity decelerates with the rising values of $|\alpha_1|$.

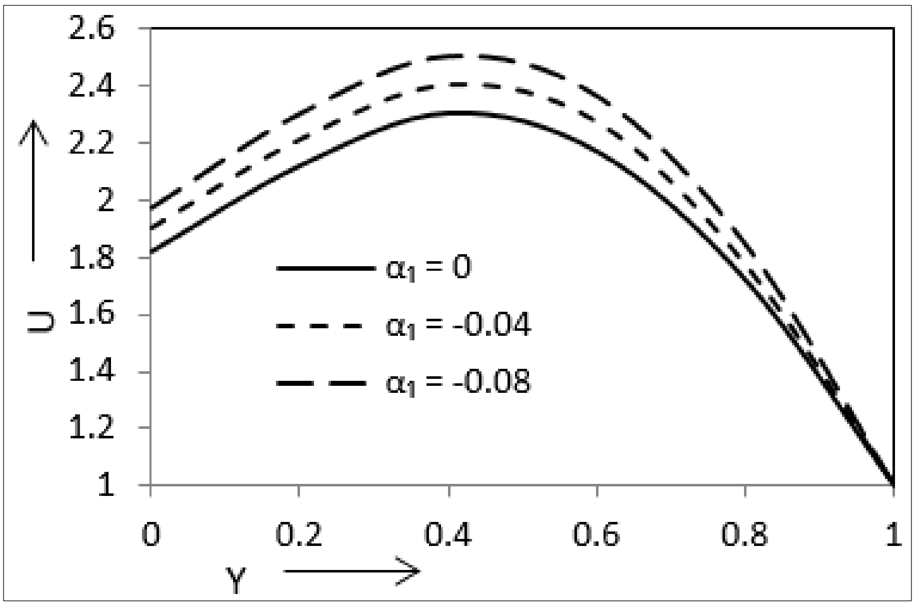

Fig 1: Variation of U against Y for $Gr=3, Gm=4, Re=2, Pr=1.5$ and $Sc=0.6$.

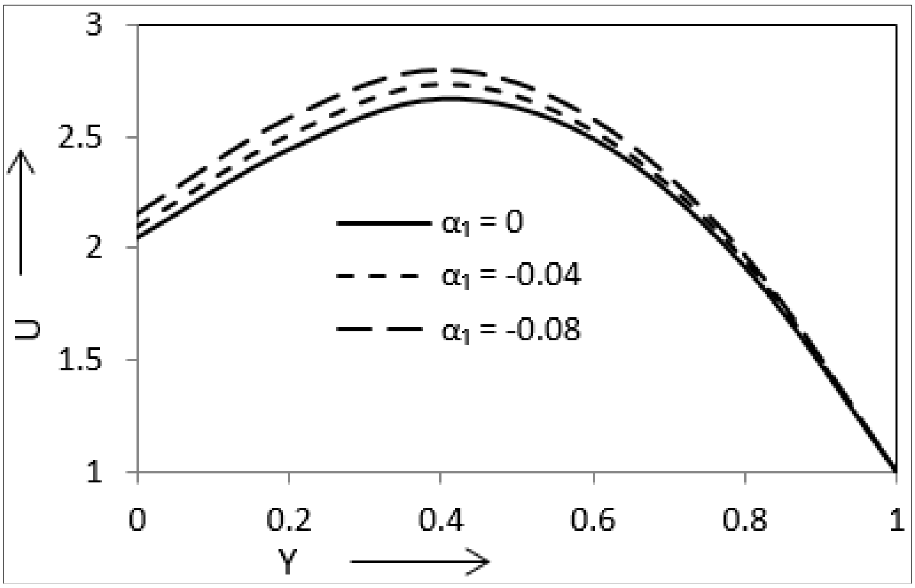

Fig 2: Variation of U against Y for $Gr = 5, Gm = 4, Re = 2, Pr = 1.5$ and $Sc = 0.6$

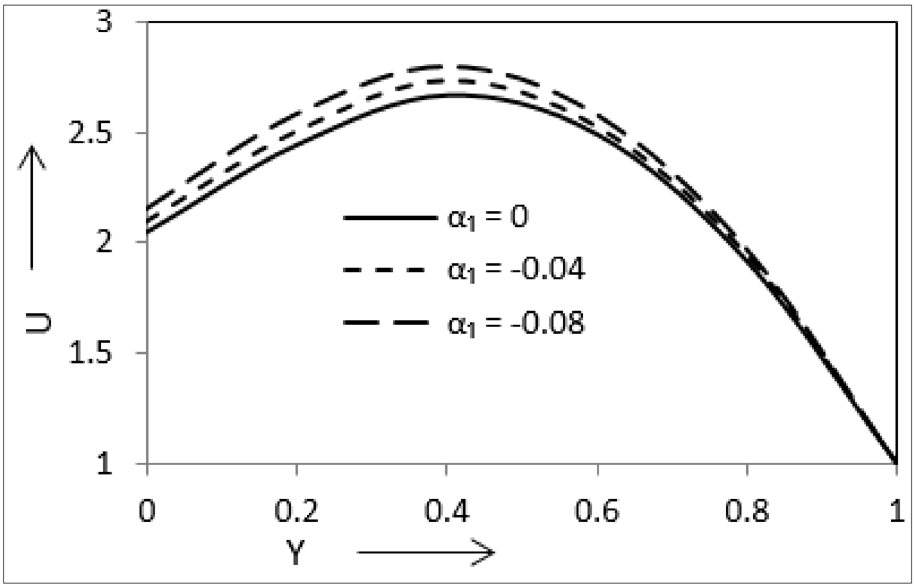

Fig 3: Variation of U against Y for $Gr = 3, Gm = 6, Re = 2, Pr = 1.5$ and $Sc = 0.6$

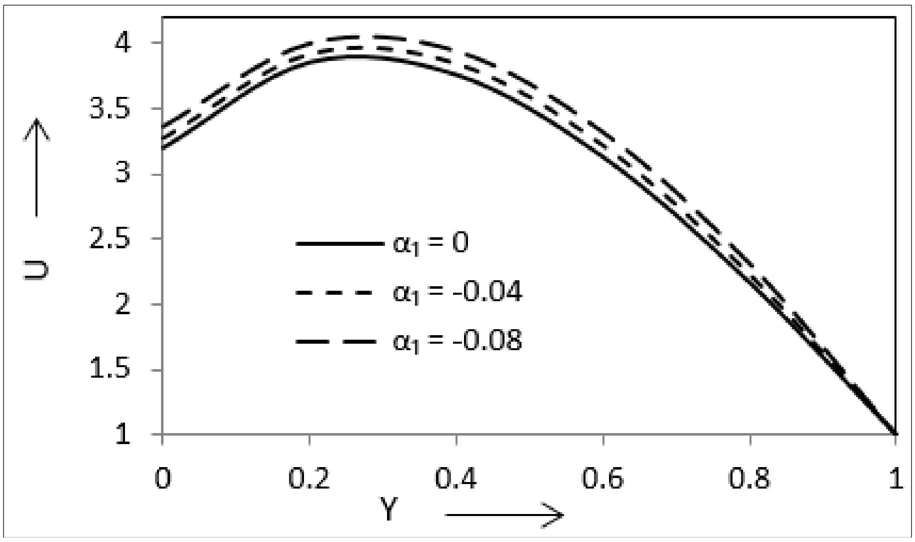

Fig 4: Variation of U against Y for $Gr = 3, Gm = 4, Re = 4, Pr = 1.5$ and $Sc = 0.6$.

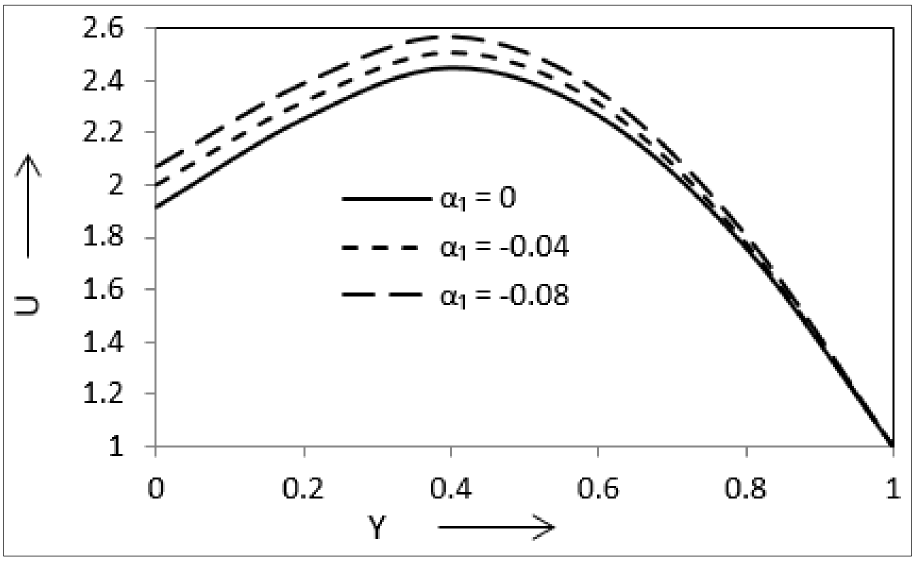

Fig 5: Variation of U against Y for $Gr = 3, Gm = 4, Re = 2, Pr = 3.5$ and $Sc = 0.6$.

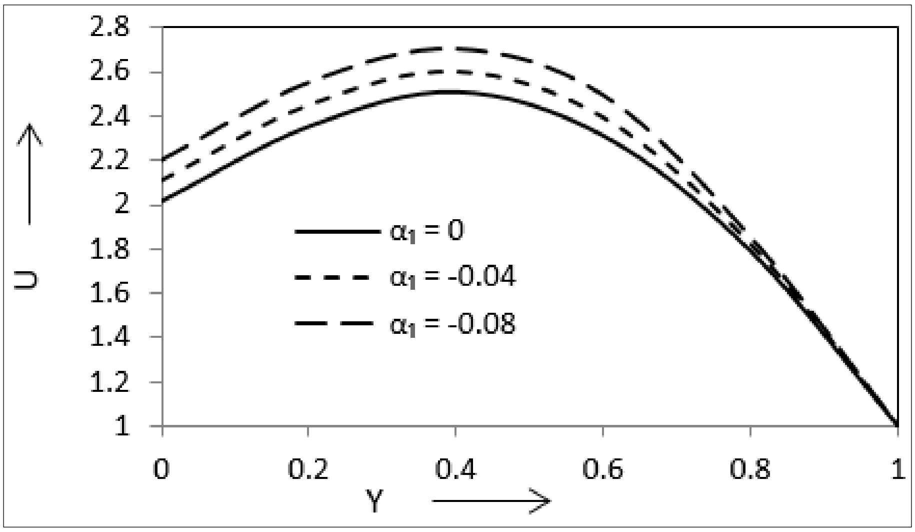

Fig 6: Variation of U against Y for $Gr = 3, Gm = 4, Re = 2, Pr = 1.5$ and $Sc = 1.4$

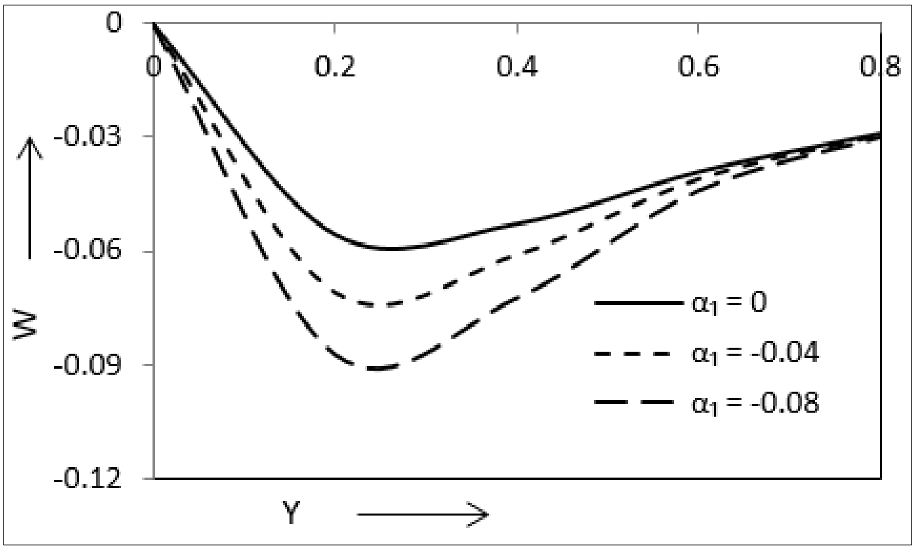

Fig 7: Variation of W against Y for $Gr = 3, Gm = 4, Re = 4, Pr = 1.5$ and $Sc = 0.6$

5. Conclusion

The study leads to the following conclusions:

- The main flow velocity and the cross flow velocity components are significantly affected at each point of the fluid flow region by the visco-elastic parameter α_1.

- The enhancement of the absolute value of the visco-elastic parameter α_1 depicts the accelerating trend of the main flow velocity in comparison with Newtonian fluid flow phenomenon but opposite result is noticed in case of cross flow velocity.
- With the enhancement of different significant flow parameters the main flow velocity affects considerably by both Newtonian and non-Newtonian cases.
- The temperature and concentration profiles are not considerably influenced by the visco-elastic parameter α_1. This is due to the restraining effect played by the elasticity of the fluid.

6. References

1. Aboeldahab EM, Azzam GED. Unsteady three-dimensional combined heat and mass transfer for convective flow over a stretching surface with time dependent chemical reaction. Acta Mech. 2006;184:121-136.
2. Ahmed S. The study of heat and mass transfer on free convective three-dimensional unsteady flow over a porous vertical plate. J Energy Heat Mass Trans. 2009;31:89-110.
3. Chaudhary RC, Sharma PK. Three-dimensional unsteady convection and mass transfer flow through porous medium. Heat Mass Trans. 2003;39:765-779.
4. Cookey CI, Ogulu A, Omubo-Pepple VM. Influence of viscous dissipation and radiation on unsteady MHD free convection flow past an infinite heated vertical plate in a porous medium with time dependent suction. Int J Heat Mass Trans. 2003;46:2305-2311.
5. Sahin A. Transient three-dimensional flows through a porous medium with transverse permeability oscillating with time. Emirates J Engg Res. 2008;13:11-17.
6. Vajravelu K, Roper T. Flow and heat transfer in a second-grade fluid over a stretching sheet. Int J Non-linear Mech. 1999;34:1031-1036.
7. Rajagopal KR, Gupta AS. An exact solution for the flow of a non-Newtonian fluid past an infinite plate on boundary conditions for fluids of differential type. Meccanica. 1984;19:158-160.
8. Nowinski SK, Ismail IA. Application of multi-parameter perturbation methods to elasto-statics in development of theoretical and applied mechanics. N A Shaw, 1965, 11.