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Analysis of an M/M/1/∞ model with Bernoulli schedule during busy period, additional task, customers fluctuating priorities behavior during working vacation

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Abstract
This paper analyzes the customers (Clients) fluctuating priorities behavior during working vacation (WV) with Bernoulli scheduled busy period in an M/M/1/∞ model along with the assumption that some additional task apart from regular job is assigned to the server during working vacation. Customers arrive at the system with Poisson's distributed rate λ and get served with exponentially distributed rate μ. During busy period (State) clients might enter the queue with probability r and could leave with probability q. After completing busy period, the server moves to a single working vacation with exponentially distributed rate γ where service is provided at slower pace than busy state as some additional task is assigned to server apart from regular job. In this paper we have introduced a new behavior of customers named as "Customers Fluctuating Behavior" during working vacation. According to this behavior customers standing in a queue wait for their turn might leave the queue with exponentially distributed rate ξ during working vacation not due to impatience or long queue or they have any issue with the system but it is due to their (Clients) fluctuating or can say confusing behavior on their priorities of doing work. If some clients are present at an epoch of WV completion, then the server return back to busy state with probability p otherwise do some secondary tasks with rate ϕ and probability m. During performing some secondary task server will not serve any client. If client arrives during this duration, then server will resume busy state with rate ψ. In this paper we have formulated various system probabilities in steady state condition by using PGF method. Various system measures such as average queue length in busy period, working vacation and in the system respectively, Mean sojourn time in the system, abandonment rate of clients in WV period is obtained explicitly. The impact of some parameters on some system measures has been illustrated numerically and graphically.

Keywords: Busy period, Bernoulli schedule, fluctuating priorities, working vacation, impatience of customers, rate of abandonment of customers

Introduction
Many authors in past have discussed application of different queueing models in various real-life fields. Recently, Rathore [13, 14] discussed vastly the application of different queueing models in Hospital sector and use of stochastic modelling in queueing system for Crowding control and congestion respectively. In real-life scenarios, the application of queueing system with server vacation finds relevance in various fields where entities or tasks wait in line for service from a server that may occasionally take breaks or go on vacation. Some real-life applications are customer service centers, manufacturing systems, computer systems, healthcare services, call centers, transportation systems, retail checkout lines, data processing centers. The application of queueing models with server vacation is widespread and helps in optimizing various processes by considering the intermittent unavailability of servers, ensuring a balance between resource utilization and service efficiency. In queueing theory, a "vacation" is when a server requires to take a moment or break from serving clients continuously, being tired, or for other reasons, thus they are not available in the system. There are many circumstances where the server continuously serves the customers during vacation rather than ceasing all operations. In queueing theory, a working vacation is a concept that allows a service provider to temporarily reduce their service rate without completely stopping service. This is typically employed when the demand for service is lower than usual, and the service

provider wants to optimize resource utilization. In queueing theory, a Bernoulli schedule refers to a scheduling policy where a server makes a decision about whether to take a vacation or serve a customer based on a Bernoulli process. The vacation models in which server takes vacation with some probability p or stay in system with some probability 1-p are called vacation model with Bernoulli schedule which helps in providing control on system congestion.

Brief Literature Review

Servi *et al.* [15] introduced the notion of one and many working vacation in a markovian queuing model having single server. Later, Liu *et al.* [7] and Tian *et al.* [16] derived the stochastic decompositions of a one server queue with one and more than one working vacation respectively. Lin *et al.* [6] extended the research of Tian *et al.* [16] to the many servers system. Selvaraju *et al.* [8] analyzed impatient behavior of clients in M/M/1 queue with one and multiple (More than one) WV. Yue *et al.* [1] mentioned client's impatience in working vacations in a one server queue following markovian distribution. Veena [17] analyzed customer's impatience, vacation interruption during Bernoulli schedule in one server markovian queue with multiple (more than one) WV. Laxmi *et al.* [12] represented a comprehensive analysis of a single server queueing system with a variant of working vacations with customer balking. Rama devi *et al.* [18] analyzed a markovian one server queueing model with working vacation, deterioration of server and intolerance in customer. Faud *et al.* [2] discussed retention of reneged customer and state dependent reneging in an M/M/1/∞ queueing system with one working vacation. Keilson *et al.* [5] introduced Bernoulli scheduled service for first time in GI/G/1 queue, then Kella [9] presented a generalized form of bernoulli scheme in an M/G/1 Queue in which a server takes q vacations in succession with some probability P_r. Choudhury *et al.* [3] introduced bernoulli vacation in a two-phase queueing system with multiple vacation. Li *et al.* [19] investigated reliability analysis of M/G/1 queueing system with bernoulli vacation. Zhang *et al.* [4] discussed bernoulli scheduled-controlled vacation with vacation interruption in a M/M/1 queue. Manoharan *et al.* [10] used probability generating function method to study impatience in customers with bernoulli scheduled working vacation, vacation interruption, setup time in a M/M/1 queue. Poonam *et al.* [11] investigated bernoulli schedule in a one server retrial markovian queue with interruption in working vacation for study of balking of customers.

Model Description

In this paper we are considering a M/M/1 queueing model with customers(clients) fluctuating priorities during working vacation, Bernoulli scheduled busy state along with the assumption that some additional task apart from regular job is assigned to the server during working vacation. Client comes to the system follows Poisson's distribution with rate λ and service time follows exponential distribution with rate μ. When server is occupied in busy period clients might enter and exit the system with probability' r' and probability 1-r=q respectively. The server goes for only one working vacation with exponentially distributed rate γ when system is free from customers. During WV state the server gives service at a lower rate η than busy state as server is busy in performing some additional work apart from regular job. The customers during working vacation might leave the queue with exponentially distributed rate ξ. The abandonment of clients during working vacation is not due to client's intolerance or long queue while it is considered that they may have some other priorities or important work to do. That's why after waiting for some time in queue the clients may leave the queue for finishing other prioritize work. The server will move to busy state with probability p if at least one client is present at epoch of working vacation completion otherwise moves to do some secondary work with rate ϕ and probability m, where server will not provide any service to client. During performing some secondary task if client arrives then server will resume busy state with exponentially distributed rate ψ. The clients are served on basis of FCFS queue discipline. The inter arrival times, vacation time, service time, client's abandon time during working vacation all are mutually independent. The rate Transition diagram of the model is shown below in Figure.1

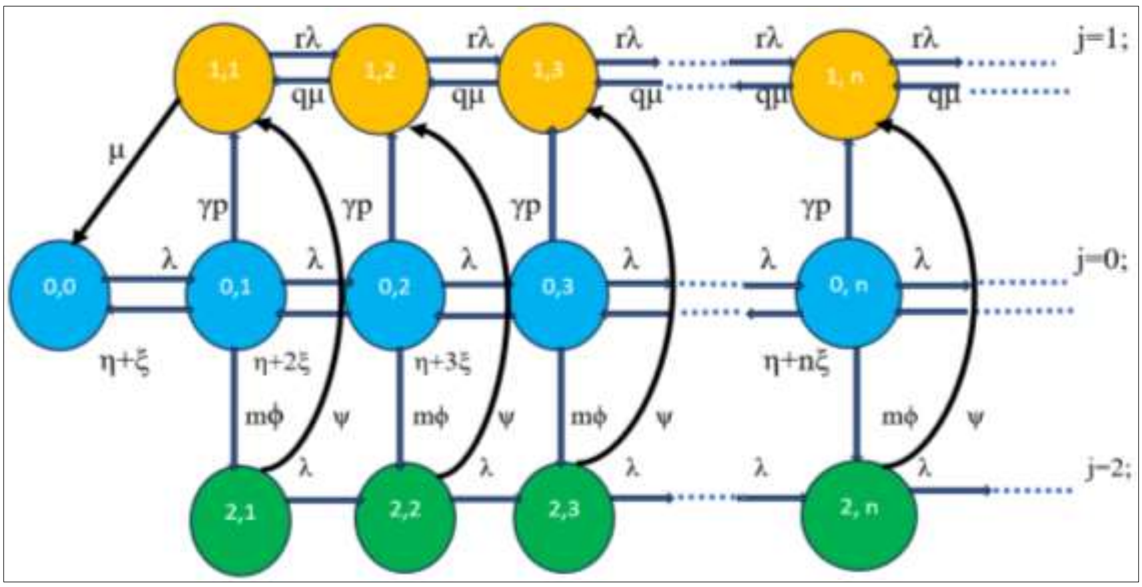

Fig 1: Transition rate diagram of different states of server in the model.

Mathematical Formulation

Let N (t)=Number of customers in the system at time t, J(t)=State of server at time t with
j=0 denotes server is in working vacation state.

j=1 denotes server is in busy state.
j=2 denotes server is busy in performing some secondary task.
Consider a continuous time Markov process (J(t), N(t)) with transition rate diagram as shown in figure 1.
Let P_{jn} =P$\{J(t) = j, N(t) = n; j = 0,1,2; n = 0,1,2 \dots\dots\dots\}$ denotes the steady state probabilities of the system.
P_{on}=Probability of system having n customers in the system when server is in working vacation state, n≥0.
P_{1n}= Probability of system having n customers when server is in busy state, n≥1.
P_{2n}= Probability of system having n customers when server is busy in performing some secondary task, n≥1.

Balance equations for each server state is given as follows
For j=0:

$$\lambda P_{00}= (\xi+\eta)P_{01}+\mu P_{11},\ n=0 \tag{1}$$

$$(\lambda+ n\xi +\eta+ \gamma p+ m\phi)P_{on}=\lambda P_{0,n-1}+ [\eta+(n+1)\ \xi]P_{o,n+1},\ n\geq 1 \tag{2}$$

For j=1:

$$(r\lambda+\mu)P_{11}=\psi P_{21}+q\mu P_{12}+\gamma p P_{01},\ n=1 \tag{3}$$

$$(r\lambda+q\mu)P_{1n}=\psi P_{2n} + q\mu P_{1,n+1} + \gamma p P_{0n} + r\lambda P_{1,n-1},\ n\geq 2 \tag{4}$$

Where q=1-r.
For j=2:

$$P_{21}(\lambda + \psi) = m\phi P_{01},\ n=1 \tag{5}$$

$$P_{2n}(\lambda + \psi) = m\phi P_{0n} + \lambda P_{2\ n-1},\ n\geq 2 \tag{6}$$

Steady State Probabilities and Some System Performances
Now we use Probability generation function technique to derive various system performances and steady state probabilities,

$$\text{Let } D_0(z)=\sum_{n=0}^{\infty} P_{0n}\, z^n,\ D_0'(z)= \sum_{n=0}^{\infty} nP_{on}\, z^{n-1} \tag{7}$$

$$D_1(z)=\sum_{n=1}^{\infty} P_{1n}z^n,\ D_1'(z)= \sum_{n=1}^{\infty} nP_{1n}z^{n-1} \tag{8}$$

$$D_2(z)=\sum_{n=1}^{\infty} P_{2n}z^n,\ D_2'(z)= \sum_{n=1}^{\infty} nP_{2n}z^{n-1} \tag{9}$$

Such that $D_0(1) + D_1(1) + D_2(1)$ =1.
Multiply equation (6) by $z^n$and taking summation over n, we get,

$$\sum_{n=2}^{\infty}(\lambda + \psi)P_{2n}\, z^n=m\phi\sum_{n=2}^{\infty} P_{0n}\, z^n+\lambda \sum_{n=2}^{\infty} P_{2\ n-1}\, z^n$$

On further simplifying above equation, we get

$$D_0(z) = \frac{1}{m\phi}[D_2(z)\{\lambda + \psi - \lambda z\} - (\lambda + \psi)zP_{21} + m\phi P_{00} + m\phi zP_{01}] \tag{10}$$

Taking $\lim_{z\to 1} D_0\,(z)$, we get

$$D_0(1) = \frac{1}{m\phi}[\psi D_2(1) - (\lambda + \psi)P_{21} + m\phi P_{00} + m\phi P_{01}] \tag{11}$$

where $D_0(1)$ =P(WV)=Probability of server being in state of working vacation,
Multiply equation (2) by $z^n$and taking summation over n, we get,

$$\sum_{n=1}^{\infty}(\lambda + \eta + n\xi + m\phi + \gamma p)P_{0n}z^n=\lambda\sum_{n=1}^{\infty} P_{0\ n-1}\, z^n+\sum_{n=1}^{\infty}(\eta + (n + 1)\xi)P_{0\ n+1}z^n$$

On further simplifying above equation, we get

$$\frac{1}{m\phi}[\lambda z + \eta z + \gamma pz + \phi mz - \lambda z^2 - \eta][D_2(z)(\lambda + \psi - \lambda z) - (\lambda + \psi)zP_{21} + m\phi P_{00} + m\phi zP_{01}] = (\lambda z + \eta z + \gamma pz + m\phi z - \eta)P_{00} - \eta zP_{01} - \xi zP_{01} + \xi z(1 - z)D_0'(z) \tag{12}$$

Taking limit z tends to 1 in (12), we get

$$D_2(1) = \frac{1}{\psi}\left[(\lambda+\psi)P_{21} + \frac{m\phi}{(m\phi+\gamma p)}\{\lambda P_{00} - (m\phi+\gamma p+\eta+\xi)P_{01}\}\right] \quad (13)$$

where $D_2(1)$ =P(ST)= Probability that server is busy in performing some secondary task.
Multiply equation (4) by $z^n$and taking summation over n,we get,

$$\sum_{n=2}^{\infty}(r\lambda+q\mu)P_{1n}z^n = r\lambda\sum_{n=2}^{\infty}P_{1\,n-1}z^n + \gamma p\sum_{n=2}^{\infty}P_{0\,n}z^n + \psi\sum_{n=2}^{\infty}P_{2\,n}z^n + q\mu\psi\sum_{n=2}^{\infty}P_{1\,n+1}z^n$$

On further simplifying above equation, we get

$$D_1(z)\{r\lambda z + q\mu z - r\lambda z^2 - \mu q\} = \gamma pz\,D_0(z) + \psi z\,D_2(z) - \gamma pzP_{00} - \gamma pz^2P_{01} - \psi z^2P_{21} + P_{11}\{(r\lambda+q\mu)z^2 - q\mu z\} - q\mu z^2P_{12} \quad (14)$$

Taking limit z tends to 1 in (14), we get

$$D_1(1) = \frac{1}{(r\lambda-q\mu)}[\psi D_2(1) + \gamma pD_0(1) - \gamma pP_{00} - \mu(1+q)P_{11}] \quad (15)$$

Where,$D_1(1)$ =P(B)= Probability of server being in busy state.
Now differentiate equation (10) with respect to z taking limit z tends to 1, we get

$$D_0'(1) = \frac{1}{(\gamma p+m\phi)}[D_1(1)(q\mu - r\lambda) - \gamma pD_0(1) - (\lambda+\psi)D_2(1) - (\lambda+\psi)P_{21} + (m\phi+2\gamma p)P_{01} + \gamma pP_{00} + 2\psi P_{21} - (2r\lambda + q\mu)P_{11} + 2q\mu P_{12}] \quad (16)$$

where $D_0'(1)$ =E $[L_0]$ =Expected queue length during working vacation.
Now differentiate equation (14) w.r.t. z and taking limit z tends to 1in it, we get

$$D_1'(1) = \frac{1}{(r\lambda-q\mu)}[\psi D_2'(1) + \gamma pD_0'(1) - \mu P_{11} - q\mu\, D_1(1)] \quad (17)$$

Where $D_1'(1)$ =$E[L_1]$=Expected queue length during busy state.
Now differentiate equation (10) w.r.t. z and taking limit z tends to 1 in it, we get

$$D_2'(1) = \frac{1}{\psi}[\lambda D_2(1) + m\phi D_0'(1) + (\lambda+\psi)P_{21} - m\phi P_{01}] \quad (18)$$

Where $D_2'(1)$ =$E[L_2]$=Expected queue length when server is performing some secondary task.
Now, Expected queue length of system is E[L]= E $[L_0]$ + $E[L_1]$ + E$[L_2]$.
and Abandonment rate of a customer during working vacation is R=ξ E $[L_0]$.
Rearrange terms of recurrence relation (1), (2), (3), (4), (5), (6) we get,

$$P_{21} = \left[\frac{m\phi}{(\lambda+\psi)}\right]P_{01} = K_1P_{01}, \text{ where } K_1 = \left[\frac{m\phi}{(\lambda+\psi)}\right]$$

$$P_{00} = \left(\frac{\eta+\xi}{\lambda}\right)P_{01} = K_2P_{01},\ K_2 == \left(\frac{\eta+\xi}{\lambda}\right)$$

$$P_{11} = \frac{1}{\mu}\left(\frac{m\phi\psi}{(\lambda+\psi)} + \gamma p\right)P_{01} = K_3P_{01},\ K_3 = \frac{1}{\mu}\left(\frac{m\phi\psi}{(\lambda+\psi)} + \gamma p\right)$$

$$P_{12} = \left(\frac{r\lambda}{q\mu}\right)P_{11} = \left(\frac{r\lambda}{q\mu}\right)K_3P_{01} = K_4P_{01},\ K_4 = \left(\frac{r\lambda}{q\mu}\right)K_3$$

By using above$P_{nj}{}'$s, we can rewrite $D_0(1)$, $D_1(1)$, $D_2(1)$ in terms of P_{01} as follows,

$$D_2(1) = F_2P_{01}, \text{ where } F_2 = \frac{1}{\psi}\left[(\lambda+\psi)K_1 + \frac{m\phi}{(m\phi+\gamma p)}\{\lambda K_2 - (m\phi+\gamma p+\eta+\xi)\}\right]$$

$$D_0(1) = F_0P_{01}, \text{ where } F_0 = \frac{1}{m\phi}[\psi F_2 - (\lambda+\psi)K_1 + m\phi(K_2+1)]$$

$$D_1(1) = F_1P_{01}, \text{ where } F_1 = \frac{1}{(r\lambda-q\mu)}[\psi F_2 + \gamma pF_0 - \gamma pK_2 - \mu(1+q)K_3]$$

Since, $D_0(1)$, $D_1(1)$, $D_2(1)$, $D_0'(1)$, $D_1'(1)$, $D_2'(1)$, P_{21},P_{00}, P_{11}, P_{12} all are expressed in terms of P_{01}.Therefore, by normalization condition, we can easily obtain the value of P_{01} as

$D_0(1)$ + $D_1(1)$ + $D_2(1)$ =1,

$\therefore P_{01} = \frac{1}{[F_0 + F_1 + F_2]} = [F_0 + F_1 + F_2]^{-1}$

Numerical Analysis

Table 1: Impact of Joining probability r on E (L_0), E (L_1), E (L_2), E(L), W and R.

r	E (L_0)	E (L_1)	E (L_2)	E(L)	W	R
0.2	1.458316	0.041117	0.364580	1.864014	0.6213380	1.458316
0.4	0.937488	0.106322	0.234373	1.278183	0.4260611	0.937488
0.5	0.93749	0.273433	0.234374	1.445297	0.4817658	0.93749
0.6	0.937488	0.749352	0.234373	1.921213	0.640404	0.937488
0.8	1.367906	53.315955	0.341978	55.025839	18.341946	1.367906
0.9	4.218694	136.7170	1.054675	141.99036	47.330123	4.218694

Table 1 have shown effect of Joining probability r on E (L_0), E (L_1), E (L_2), E(L), W and R. It is clear from Table 1,if we take the parameters λ=3,μ=5,η=4,ξ=1,ψ=1,ϕ=0.5,γ=0.5,p=0.5,m=0.5, and increase the value of Joining probability r, value of E(L_0), E(L_2) and R follows same pattern i.e. as r increases E(L_0), E(L_2) and R decreases for r=0.2,0.4 and remains constant for r=0.5,0.6 and then increases abruptly for r=0.8,0.9 while E(L_1) E(L) and W increases as r increases.

Table 2: Impact of working vacation rate γ on various steady state probabilities and abandonment rate R of clients during working vacation

γ	P(WV)	P(B)	P(ST)	R
0.05	0.8333356	0.065623	0.0000009	0.3105858
0.1	0.8333352	0.084373	0.0000008	0.5624880
0.25	0.833345	0.140619	0.0000033	0.7499448
0.5	0.833342	0.234369	0.0000025	0.937464
1	0.8333386	0.421869	0.0000016	1.1249794

Table 2 have shown the impact of working vacation rate γ on P(WV), P(B), P(ST) and R. It is clear from Table 2, if we take the parameters λ=3, μ=5, η=4, ξ=1, ψ=1, ϕ=0.5, r=0.5, p=0.5, m=0.5, as the value of working vacation rate γ increases, P(B) and R increases while P(WV) and P(ST) remains constant.

Graphical Illustration

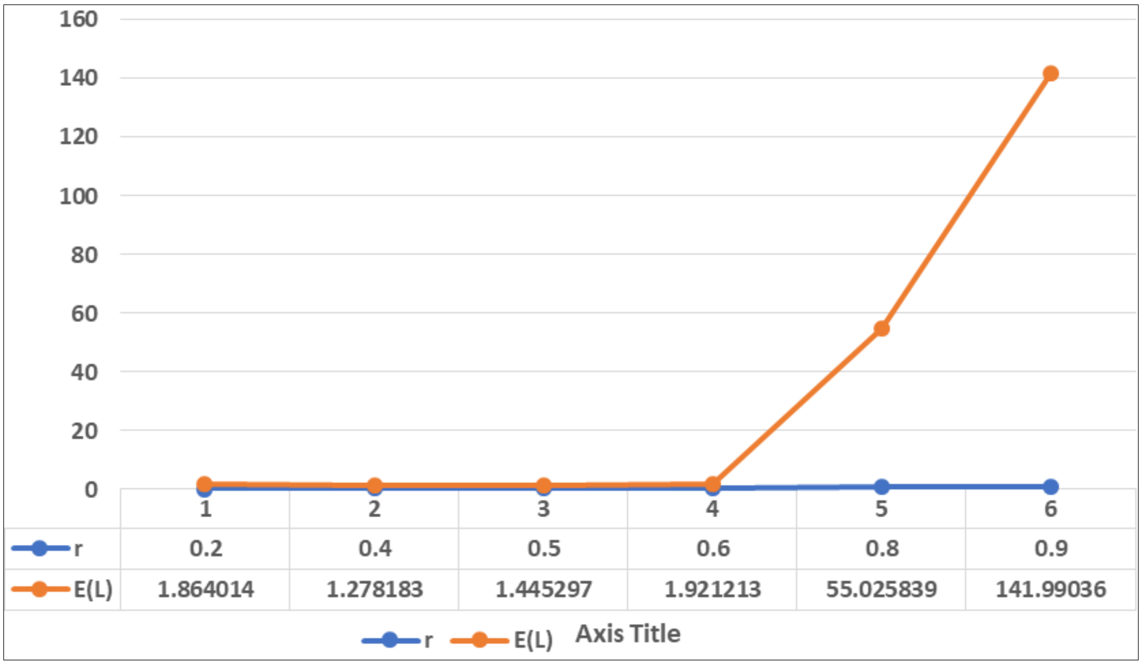

Fig 2: Impact of joining probability r on expected queue length of system E(L)

Figure 2 shows graphical illustration of impact of joining probability r on expected queue length E(L).It has been observed from Figure 2, that when we take λ=3,μ=5,η=4,ξ=1,ψ=1,ϕ=0.5,γ=0.5,p=0.5,m=0.5, as the value of joining probability r increases,value of E(L) increases gradually for starting values of r=0.2,0.4,0.5 and then a rapid increment have been noticed for r=0.6,0.8,0.9.

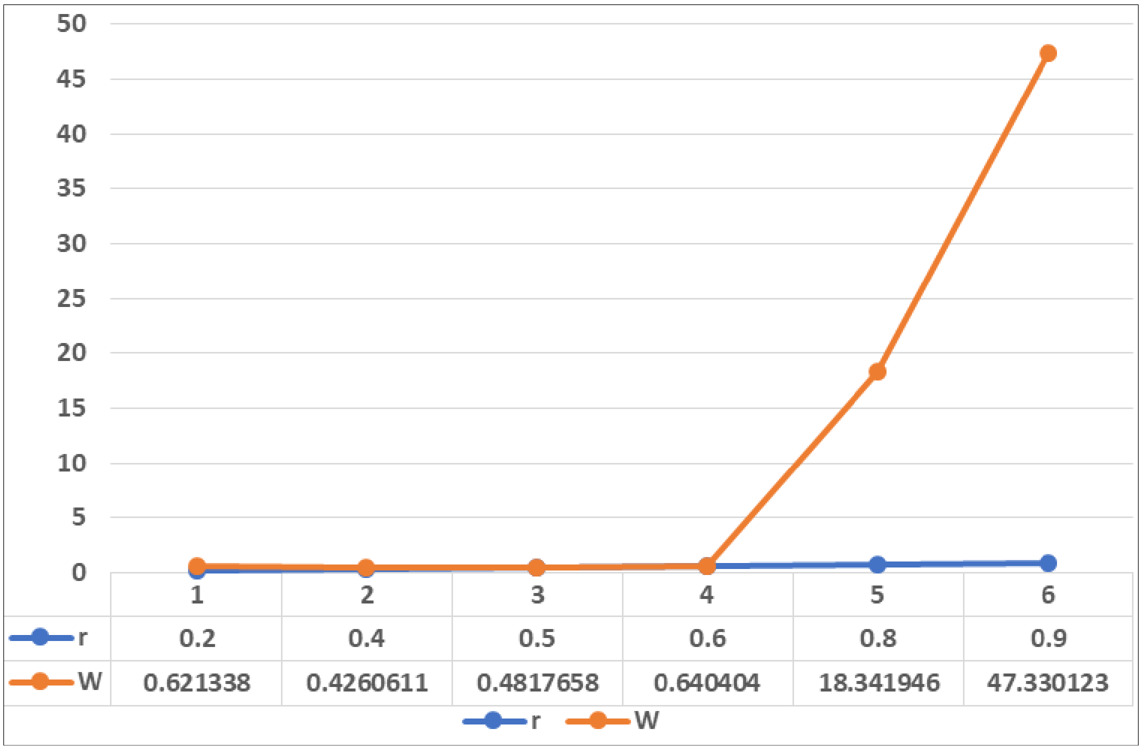

Fig 3: Impact of joining probability r on mean wait time of system W

Figure 3 shows graphical illustration of impact of joining probability r on mean wait time of system W. It has been observed from Figure 3 when we take λ=3, μ=5, η=4, ξ=1, ψ=1, ϕ=0.5, γ=0.5, p=0.5, m=0.5, as the value of joining probability r increases, value of W follows same Pattern of E(L) i.e. W increases gradually for starting values of r=0.2,0.4,0.5 and then a rapid increment has been noticed for r=0.6,0.8,0.9.

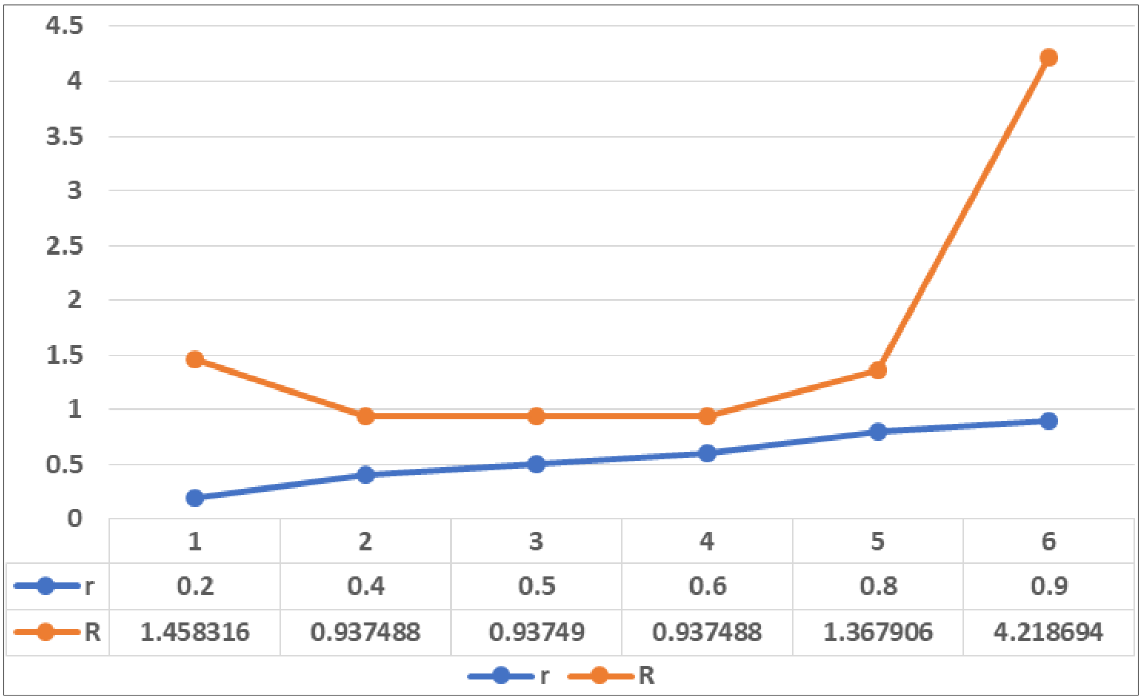

Fig 4: Impact of joining probability r on abandonment rate R of clients during working vacation

Figure 4 shows graphical illustration of impact of joining probability r on abandonment rate R of clients during working vacation. It has been observed from Figure 4, when we take λ=3, μ=5, η=4, ξ=1, ψ=1, ϕ=0.5, γ=0.5, p=0.5, m=0.5, as the value of joining probability r increases, value of R decreases for starting values of r=0.2, 0. 4, remains constant for r= 0.5, 0.6 and then an abrupt increment has been noticed for r =0.8,0.9.

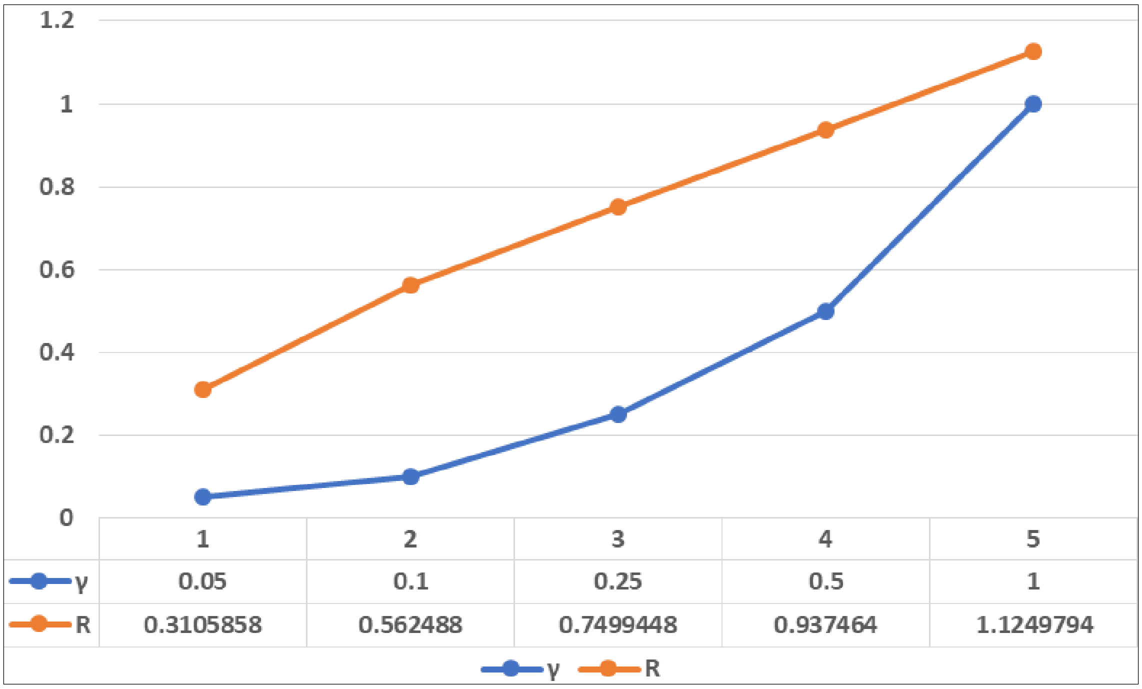

Fig 5: Impact of working vacation rate γ on abandonment rate R of clients during Working Vacation

Figure 5 shows graphical illustration of impact of working vacation rate γ on abandonment rate R of customers. It has been observed from Figure 5, when we take parameters $\lambda=3$, $\mu=5$, $\eta=4$, $\xi=1$, $\psi=1$, $\phi=0.5$, $r=0.5$, $p=0.5$, $m=0.5$, as the value of working vacation rate γ increases value of R increases.

Application of the Model

Our model could be applied in several real-life situations. Here we are discussing the real-life application of our model in banking sector. In case of banking sector customers often visit a bank branch with different priorities based on their preferences, financial needs, urgency of transaction etc. For example, some customers may have requirement of cash withdrawal or deposit, FD renewal, pass book printing, financial advice, loan application etc. Some customers have limited time due to their personal or professional activities so they may often visit the bank during WV period for managing their financial needs. In many real-life scenarios like in service industries, Bank, customers often change their priorities based on their needs or circumstances. Customers standing for service might leave the queue for performing some other prioritize work. Since no organization wants to leave their clients at any cost. This model can be applied in above scenario to analyze or evaluate various system performances like average queue length of the system, mean wait time of system, rate of abandonment of clients due to intolerance which may help in staff scheduling during busy and working vacation state, to minimize wait time and improve service efficiency. It also helps banks in implementing efficient queueing system which ultimately lead to the cultivation of customer loyalty and retention.

Conclusion

In this paper customers fluctuating priorities behavior during working vacation state with Bernoulli schedule during busy state in a $M/M/1/\infty$ model have been studied. Various system performances such as expected queue length during busy state, working vacation state and during performing secondary task in the system, average sojourn time customers spent in the system, rate of abandonment of customers during WV State is obtained explicitly by using PGF method. The impact of few parameters on some system performances have been shown numerically and graphically. If someone apply this model practically then as per the selection of values of model parameters the result will be obtained accordingly. Finally, it has been concluded that in many real-life scenarios like service industries, Banks etc. customers often change their priorities based on their needs or circumstances. The service industries need to work to prevent losing customers. The service staff needs to check or keep eye on this type of customers and prioritize or provide service according to the convenience of customers based on the severity of their conditions. It has been clear WV in queuing theory, by reducing service rate can potentially lead to increase customer's impatience, especially when customer experiences longer waiting time. Effective management, clear and transparent communication, implementing appropriate queue management policies and understanding customer behavior are some essential elements in mitigating potential negative effects on customers patience during such period or situation.

Future Scope: This work could be extended further by using heterogeneous servers with retrial and feedback queue etc.

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