International Journal of Statistics and Applied Mathematics 2025; 10(10): 38-45

International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452
NAAS Rating (2025): 4.49
Maths 2025; 10(10): 38-45

https://www.mathsjournal.com
Received: 20-08-2025
Accepted: 22-09-2025

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Estimation of the common location parameter μ of several distributions when their common scale parameter is proportional to μ with known coefficient of variation by ranked set sampling

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DOI: https://www.doi.org/10.22271/maths.2025.v10.i10a.2178

Abstract
This article describes a technique that uses ranked set sampling (RSS) of independent but not identically distributed (inid) random variables to estimate the common location parameter μ of several distributions with a common scale parameter that is proportional to μ. For example, we use RSS to estimate the common mean of double exponential and normal distributions with known coefficients of variation. A comparison of the suggested estimator's efficiency utilizing RSS and order statistics of inid random variables is also conducted.

Keywords: Normal distribution, Double Exponential distribution, Known coefficient of variation, Order statistics of inid random variables, Ranked set sampling of inid random variables.

Introduction
Order statistics is concerned with the characteristics and uses of ordered random variables as well as functions of these variables. If the random variables $X_1, X_2, \dots, X_m$ are arranged in ascending order of magnitude as $X_{1:m} \leq X_{2:m} \leq \cdots \leq X_{m:m}$, then $X_{j:m}$ is called the j^{th} order statistic, j=1,2,…,m. Additional information regarding order statistics and its uses may be found in Balakrishnan and Cohen (1991) [2] and David and Nagaraja (2003) [4].

The literature reports on certain biological and physical science problems where the scale parameter is proportional to the location parameter (see Glesser and Healy, 1976) [5]. The issue of estimating the mean has been thoroughly covered in the literature that is currently available if the parent distribution is univariate Normal, more details, see, Arnholt and Hebert (1995) [1], Searls (1964) [19], Khan (1968) [8], Kunte (2000) [9], and Guo and Pal (2003) [6].

The best linear unbiased estimator of the mean of the Normal distribution with known coefficient of variation using order statistics is discussed by Thomas and Sajeevkumar(2003) [21]. Estimating the mean of logistic distribution with known coefficient of variation using order statistics is discussed by Sajeevkumar and Thomas (2005) [13, 16-17]. Estimating a parameter of the exponential distribution with known coefficient of variation by order statistics are discussed in Sajeevkumar and Irshad (2011) [7].

In order to increase the accuracy of the sample mean as an estimator of the population mean, McIntyre (1952) [11] originally proposed the idea of ranked set sampling (RSS). When ranking a group of sampling units can be accomplished with ease using the judgment approach, the RSS outlined in McIntyre (1952) [11] is appropriate. The process of ranked set sampling involves selecting m sets of units of size m at random from the population of interest. The units in each set are then ranked using a judgment method or a low-cost technique without the units being measured. After measuring the unit ranked as one from the first set, the unit ranked as two from the second set is also measured.

This method is repeated until the unit from the mth set that is ranked as m is measured. The observations that come from real measurements on units selected in the manner outlined above are then referred to as ranked set sampling. Ranked set sampling (RSS) offers several advantages over using order statistics (like median, quantiles etc.) from simple random sampling. The following are the advantages of RSS over order statistics.

i). Higher Efficiency

RSS provide more precise estimators (lower variance) for population parameters than simple random sampling (SRS), especially for the mean or median. This provide efficiency comes without increasing sample size.

ii). Cost- effectiveness

In RSS, ranking is done without actual measurement (eg. Visual inspection), and only a subset is measured. This reduces cost and effort when measurement is expensive or destructive.

The theory and applications of ranked set sampling (RSS) and the estimation of parameters of certain distributions using RSS (efficiency better than order statistics) are covered in detail in Chen *et al*. (2004), Lam *et al*. (1994) [10], Stokes (1995) [20], and Zheng and Moderres (2006) [23]. Irshad and Sajeevkumar (2011) [7] discuss the use of ranked set sampling to estimate the location parameter μ of the exponential distribution for which the coefficient of variation is known.

The problem of estimating the common location and common scale parameter of several distributions using order statistics of independent but not identically distributed (inid) random variables are discussed by Sajeevkumar and Thomas (2005) [13, 16-17]. The ranked set sampling (RSS) of independent non-identically distributed (inid) random variables was suggested by Priya and Thomas (2016) [12].

RSS offers several advantages in the multiple population context - such as comparing two or more groups, populations, or treatments. Hence are the main benefits.

a). Increased Efficiency Across Population

RSS improve estimation precision (eg. Means, variances, quintiles) for each population individually, when comparing multiple populations, it lead to more powerful statistical tests (eg. ANOVA, t-tests).

b). Better Resource Allocation

In multiple population studies, RSS allows you to minimize measurement costs while still obtaining reliable comparisons. Especially useful when measurements are expensive but ranking is cheap.

Sajeevkumar and Thomas (2006) [18] describe the use of order statistics of inid random variables to estimate the common location parameter μ of multiple distributions where their common scale parameter is proportional to μ. Sajeevkumar (2025) [14] discusses the use of ranked set sampling to estimate the common mean of logistic and normal distributions with known coefficients of variation. With the use of ranked set sampling of inid random variables, we hope to estimate the common location parameter μ of multiple distributions where their common scale parameter is proportional to μ in this article by applying the notion of Sajeevkumar (2025) [14].

2. Estimation of the Common Location Parameter μ of Several Distributions When the Scale Parameter of Each of the Distributions is $c\mu$ for Known c Using Order Statistics

The estimation of a common mean across multiple population is statistically significant in many applied fields because it allows for a pooled understanding of a shared parameter. Here's why it matters.

i) **i). Efficient Data Utilization**
When populations are believed to have similar or identical means, estimating a common mean allows for combining data from all groups, increasing statistical power and precision.

ii) **ii). Simplified Modeling**
A common mean reduces model complexity. Instead of modeling each group separately, one parameter represents the central tendency of all, making interpretation simpler.

iii) **iii). Cost Reduction**
Infields like agriculture, medicine or industry, assuming a common mean means fewer measurements may be needed from each population, reducing costs and resources.

The distribution theory of order Statistics of independent but not identically distributed (inid) random variables are discussed in Vaughan and Venables (1972) [22]. Sajeevkumar and Thomas (2005) [13, 16-17] used order statistics of inid random variables to determine the best linear unbiased estimator of common location and common scale parameters of multiple distributions.

Let $X_{j1}, X_{j2}, \ldots, X_{jm_t}$ be random sample of size m_i drawn from a distribution with pdf of the form

$$f_j(x_j;\mu, c\mu) = \frac{1}{c\mu} f_{jo}\left(\frac{x_i-\mu}{c\mu}\right),\ \mu > 0,\ c > 0, \tag{1}$$

where the form of $f_{jo}(.)$ is known and j=1,2,…,t. Let $m = m_1 + m_2 + \ldots + m_t$ .For convenience we assume that all the t distributions defined by the pdf defined in (1) have same support.

For the pooled sample of all m observations, let $\underline{X}^{(p)} = (X_{1:m(p)}, X_{2:m(p)}, ..., X_{m:m(p)})'$ be the vector of order statistics. Thus, the vector of order statistics above can be seen as a vector of order statistics of m independent random variables that are not identically distributed (inid). Vaughan and Venables (1972) [22] review the distribution theory of order statistics of inid random variables. Let $Z_{j1}, Z_{j2}, ..., Z_{jm_j}$ are the observations of a random sample of size m_j drawn from a population with pdf $f_{jo}(z_j), j = 1,2,...,t$ and let $Z_{1:m(p)}, Z_{2:m(p)}, ..., Z_{m:m(p)}$ are the order statistics of the pooled sample of the random samples, then $(Z_{1:m(p)}, Z_{2:m(p)}, ..., Z_{m:m(p)})' \overset{d}{=} (\frac{X_{1:m(p)} - \mu}{c\mu}, \frac{X_{2:m(p)} - \mu}{c\mu}, ..., \frac{X_{m:m(p)} - \mu}{c\mu})'$. Let $E(Z_{r:m(p)}) = \alpha_{r:m(p)}, r = 1,2,...,m$, $V(Y_{r:m(p)}) = \beta_{r,r,:m}$, for r =1,2,…,m. and $Cov(Y_{r:m(p)}, Y_{s:m(p)}) = \beta_{r,s:m(p)}$ for $1 \le r < s \le m$.Then the BLUE of the common location parameter μ is given in the following theorem.

Theorem: 2.1 : Let $\underline{X}^{(p)} = (X_{1:m(p)}, X_{2:m(p)}, ..., X_{m:m(p)})'$ be the vector of order statistics of the pooled sample of t samples, where the j^{th} sample contain m_j observations drawn from the distribution with pdf defined in (1), for j=1,2,…,t and $m = m_1 + m_2 + ... + m_t$ .Let $\underline{\alpha}^{(p)} = (\alpha_{1:m(p)}, \alpha_{2:m(p)}, ..., \alpha_{m:m(p)})'$ and $V^{(p)} = ((\beta_{r,s:m(p)}))$ be the vector of means and dispersion matrix of vector of order statistics of the pooled sample arising from $f_{jo}(z_j), j = 1,2,...,t$ and $m = m_1 + m_2 + ... + m_t$. Then the BLUE of μ namely $\hat{\mu}$ and its variance is given by

$$\hat{\mu} = \frac{(c\underline{\alpha}^{(p)} + \underline{1})'(V^{(p)})^{-1}}{(c\underline{\alpha}^{(p)} + \underline{1})'(V^{(p)})^{-1}(c\underline{\alpha}^{(p)} + \underline{1})} \underline{X}^{(p)} \tag{2}$$

and

$$V(\hat{\mu}) = \frac{c^2\mu^2}{(c\underline{\alpha}^{(p)} + \underline{1})'(V^{(p)})^{-1}(c\underline{\alpha}^{(p)} + \underline{1})}. \tag{3}$$

More details, see, Sajeevkumar and Thomas (2006) [16-17].

3. Estimation of Common Location Parameter μ of Several Distributions When the Scale Parameter of Each of the Distributions is Equals to c μ for known c using RSS

The ranked set sampling of independent non-identically distributed (inid) random variables was suggested by Priya and Thomas (2016) [12] and it is described below.

Suppose there are t populations and the j^{th} population has a distribution function $F_j(x)$ with pdf $f_j(x)$ and suppose we fix up the ultimate number of observations to be drawn from the j^{th} population of size m_j, j=1,2,..,t, with $\sum_{j=1}^{t} m_j = m.$ Then we define RSS from t populations in the following manner. Choose randomly m sets of units where each set contain m units comprising of further t subsets of units such that the j^{th} subset consists of m_j units drawn from the j^{th} population for j=1,2,…,t. Now by an inexpensive method of rank the units of each set.From the r^{th} set choose the unit with rank r and measure the characteristic of interest, which is denoted by $X_{(r:m(p))r}$ for r = 1,2,…,m. Then the observations $X_{(1;m(p))1}$, $X_{(2:m(p))2}$, …, $X_{(m:m(p))m}$, taken together is said to form the ranked set sampling drawn from t populations with m_j observations from the j^{th} population, j=1, 2,…,t. Using this RSS of inid random variables Priya and Thomas (2016) [12] estimate the common location and common scale parameter of several distributions using RSS of inid random variables. In this article we have used the RSS of inid random variable suggested by Priya and Thomas (2016) [12] to estimate the common location Parameter μ of several distributions,when the Scale Parameter of each of the distributions is equals to c μ for known c using RSS of inid random variables.

Let $X_{j1}, X_{j2}, ..., X_{jm_j}$ be a random sample of size m_j drawn from a distribution with pdf of the form

$$f_j(x_j;\mu,c\mu)=\frac{1}{c\mu}f_{j0}(\frac{x_j-\mu}{c\mu})\ ,\mu>0\ ,c>0 \quad ; \tag{4}$$

where the form of $f_{j0}(.)$ is known and j=1,2,…,t. Let m= $m_1+m_2+...+m_t$.

Let $X_{(1;m(p))1}, X_{(2:m(p))2},\ldots, X_{(m:m(p))m}$ be the RSS of inid random variables from (4).

Define $Z_{(r:m(p))r}=(\frac{X_{(r:m(p))r}-\mu}{c\mu}), r=1,2,...,m.$ Then clearly $Z_{(1:m(p))1}, Z_{(2:m(p))2},\ldots, Z_{(m:m(p))m}$ be the RSS of inid random variables from $f_{j0}(z_j)$, $j=1,2,...,t$. Now clearly

$$(Z_{(1:m(p))1}, Z_{(2:m(p))2},..., Z_{(m:m(p))m})' \stackrel{d}{=} (\frac{X_{(1:m(p))1}-\mu}{c\mu},\frac{X_{(2:m(p))2}-\mu}{c\mu},...,\frac{X_{(m:m(p))m}-\mu}{c\mu})'$$

Let $E(Z_{(r:m(p))r})=\alpha_{r:m(p)}$ and $V(Z_{(r:m(p))r})=\beta_{r,r:m(p)}$, r =1,2,…,m.

Also clearly $Cov(Z_{(r:m(p))r}, Z_{(s:m(p))s})=0$ for $1<r<s<m$.

Let $\underline{\alpha}^{(p)}=(\alpha_{1:m(p)},\alpha_{2:m(p)},...,\alpha_{m:m(p)})'$ and $B^{(p)}=((\beta_{r,s:m(p)}))$. Then the BLUE of μ using RSS is given in the following theorem.

Theorem: 3.1: Let $\underline{X}^{(p)}=(X_{(1:m(p))1}, X_{(2:m(p))2},..., X_{(m:m(p))m})'$ be the vector of RSS arising from (4). Also let $\underline{\alpha}^{(p)}=(\alpha_{1:m(p)},\alpha_{2:m(p)},...,\alpha_{m:m(p)})'$ and $B^{(p)}=((\beta_{r,s:m(p)}))$ be the vector of means and dispersion matrix of the RSS arising from $f_{j0}(z_j)$, $j=1,2,...,t.$ Then the BLUE of μ and its variance is given by

$$\tilde{\mu}=\frac{(c\underline{\alpha}^{(p)}+\underline{1})'(B^{(p)})^{-1}}{(c\underline{\alpha}^{(p)}+\underline{1})'(B^{(p)})^{-1}(c\underline{\alpha}^{(p)}+\underline{1})}\underline{X}^{(p)} \tag{5}$$

And

$$V(\tilde{\mu})=\frac{c^2\mu^2}{(c\underline{\alpha}^{(p)}+\underline{1})'(B^{(p)})^{-1}(c\underline{\alpha}^{(p)}+\underline{1})} \tag{6}$$

Proof:

Let $\underline{X}^{(p)}=(X_{(1:m(p))1}, X_{(2:m(p))2},..., X_{(m:m(p))m})'$ be the vector of RSS arising from (4). Define $Z_{(r:m(p))r}=(\frac{X_{(r:m(p))r}-\mu}{c\mu}), r=1,2,...,m.$ Then $(Z_{(1:m(p))1}, Z_{(2:m(p))2},\ldots, Z_{(m:m(p))m})'$ be the RSS arising from $f_{j0}(z_i)$, $j=1,2,..,t$. Now clearly

$$(Z_{(1:m(p))1}, Z_{(2:m(p))2},..., Z_{(m:m(p))m})' \stackrel{d}{=} (\frac{X_{(1:m(p))1}-\mu}{c\mu},\frac{X_{(2:m(p))2}-\mu}{c\mu},...,\frac{X_{(m:m(p))m}-\mu}{c\mu})'.$$

Let

$E(Z_{(r:m(p))r})=\alpha_{r:m(p)}$ and $V(Z_{(r:m(p))r})=\beta_{r,r:m(p)}$, r =1,2,…,m. Also clearly $Cov(Z_{(r:m(p))r}, Z_{(s:m(p))s})=0$ for $1<r<s<m$.Now we have the following

$$E(X_{(r:m(p))r})=(c\alpha_{r:m(p)}+1)\mu\ , r=1,2,..,m \tag{7}$$

$$V(X_{(r:m(p))r})=c^2\mu^2\beta_{r,r:m}\ , r=1,2,...,m \tag{8}$$

Also clearly

$$Cov(X_{(r:m(p))r}, X_{(s:m(p))s}) = 0\,,\, 0 < r < s < m \tag{9}$$

Using (7) to (9) we can write

$$E(\underline{X}^{(p)}) = (c\underline{\alpha}^{(p)} + \underline{1})\mu \tag{10}$$

And

$$D(\underline{X}^{(p)}) = B^{(p)} c^2 \mu^2\,, \tag{11}$$

Where $\underline{1}$ is a column vector of m ones, $\underline{\alpha}^{(p)} = (\alpha_{1:m(p)}, \alpha_{2:m(p)}, \ldots, \alpha_{m:m(p)})'$ and $B^{(p)} = ((\beta_{r,s:m(p)}))$.Then by Generalized Gauss Markov theorem, we got the required result.

Remark: $\underline{\alpha}^{(p)}$ defined in equations (2) and (3) is same as in defined in equations (5) and (6), while $V^{(p)}$ defined in equations (2) and (3) is not same as that of $B^{(p)}$ defined in equations (5) and (6).In $B^{(p)}$ all the elements except the diagonal elements are zeros.

4. Use of Ranked Set Sampling to estimate the common mean of normal and double exponential distributions with known coefficients of variation

The probability density function of a Normal distribution with location parameter μ and scale parameter $c\mu$, is given by

$$f(x_1; \mu, c\mu) = \frac{1}{c\mu\sqrt{2\pi}} \exp\{-\frac{(x_1 - \mu)^2}{2c^2\mu^2}\},\; x_1 \in R\,, \mu > 0\,, c > 0. \tag{12}$$

Also the probability density function of the Double Exponential distribution with location parameter μ and scale parameter $c\mu$, is given by

$$g(x_2; \mu, c\mu) = \frac{\exp\{-\sqrt{2}\left|\frac{x_2 - \mu}{c\mu}\right|\}}{c\mu\sqrt{2}},\; x_2 \in R\,, \mu > 0\,, c > 0. \tag{13}$$

Clearly both the populations defined in (12) and (13) have common mean μ and common standard deviation c μ.

Simulation Study.

We take sample of size m_1 from (12) and take sample of size m_2 taken from (13), such that $m = m_1 + m_2$. Using the means, variances and covariance's of order statistics of inid random variables of standard normal and standard double exponential distributions available in Sajeevkumar (2005) [13, 16-17], we have calculated the BLUE $\tilde{\mu}$ of μ using RSS of inid random variables and calculate the relative efficiency of our estimator $\tilde{\mu}$ using RSS of inid random variables with that of the estimator $\hat{\mu}$ using order statistics of inid random variables(defined in (2) and Sajeevkumar and Thomas(2006)) [16-17] were calculated (for $1 \le m_1 < 5\,, 1 \le m_2 < 5$ such that $m = m_1 + m_2 \le 5$ and for c=0.1(0.05)0.3), "Using R software" and are presented in Table 4.1 and Table 4.2 respectively.

Table 4.1: Coefficients of the BLUE $\tilde{\mu}$ for c=0.1

m_1	m_2	m	Coefficients				
			b_1	b_2	b_3	b_4	b_5
1	1	2	0.47107	0.52591			
1	2	3	0.23871	0.47674	0.28109		
2	1	3	0.25995	0.42898	0.30714		
1	3	4	0.13415	0.34035	0.35829	0.16379	
2	2	4	0.15069	0.32153	0.33941	0.18448	
3	1	4	0.16892	0.30090	0.31849	0.20728	
1	4	5	0.08309	0.23149	0.32585	0.25195	0.10434
2	3	5	0.09398	0.22903	0.30486	0.25019	0.11824
3	2	5	0.10637	0.22496	0.28379	0.24664	0.13404
4	1	5	0.12050	0.21908	0.26262	0.24105	0.15205

Table 4.1: Coefficients of the BLUE $\tilde{\mu}$ for c=0.15

m_1	m_2	m	Coefficients				
			b_1	b_2	b_3	b_4	b_5
1	1	2	0.45564	0.53760			
1	2	3	0.22714	0.47469	0.29043		
2	1	3	0.24694	0.42689	0.31738		
1	3	4	0.12621	0.33444	0.36123	0.17048	
2	2	4	0.14155	0.31553	0.34221	0.19200	
3	1	4	0.15846	0.29488	0.32111	0.21568	
1	4	5	0.07746	0.22545	0.32452	0.25602	0.10921
2	3	5	0.08751	0.22270	0.30346	0.25430	0.12370
3	2	5	0.09894	0.21840	0.28232	0.25075	0.14023
4	1	5	0.11196	0.21233	0.26108	0.24510	0.15901

Table 4.1: Coefficients of the BLUE $\tilde{\mu}$ for c=0.2

m_1	m_2	m	Coefficients				
			b_1	b_2	b_3	b_4	b_5
1	1	2	0.43967	0.54837			
1	2	3	0.21529	0.47185	0.29918		
2	1	3	0.23360	0.42399	0.32689		
1	3	4	0.11812	0.32803	0.36354	0.17681	
2	2	4	0.13225	0.30899	0.34432	0.19905	
3	1	4	0.14778	0.28830	0.32301	0.22350	
1	4	5	0.07176	0.21910	0.32268	0.25963	0.11385
2	3	5	0.08095	0.21604	0.30151	0.25790	0.12893
3	2	5	0.09139	0.21147	0.28028	0.25429	0.14605
4	1	5	0.10327	0.20519	0.25896	0.24853	0.16550

Table 4.1: Coefficients of the BLUE $\tilde{\mu}$ for c=0.25

m_1	m_2	m	Coefficients				
			a_1	b_2	b_3	b_4	b_5
1	1	2	0.42323	0.55821			
1	2	3	0.20324	0.46825	0.30750		
2	1	3	0.22002	0.42032	0.33562		
1	3	4	0.10995	0.32115	0.36520	0.18276	
2	2	4	0.12284	0.30196	0.34575	0.20563	
3	1	4	0.13698	0.28120	0.32417	0.23071	
1	4	5	0.06601	0.21248	0.32034	0.26278	0.11825
2	3	5	0.07434	0.20908	0.29904	0.26098	0.13383
3	2	5	0.08378	0.20422	0.27770	0.25726	0.15147
4	1	5	0.09451	0.19771	0.25629	0.25133	0.17149

Table 4.1: Coefficients of the BLUE $\tilde{\mu}$ for c=0.3

m_1	m_2	m	Coefficients				
			b_1	b_3	b_4	b_5	b_5
1	1	2	0.40641	0.56707			
1	2	3	0.19105	0.46392	0.31477		
2	1	3	0.20627	0.41592	0.34354		
1	3	4	0.10174	0.31386	0.36624	0.18830	
2	2	4	0.11337	0.29449	0.34650	0.21169	
3	1	4	0.12611	0.27366	0.32462	0.23727	
1	4	5	0.06026	0.20563	0.31753	0.26546	0.12239
2	3	5	0.06771	0.20188	0.29608	0.26353	0.13839
3	2	5	0.07615	0.19670	0.27461	0.25964	0.15648
4	1	5	0.08573	0.18996	0.25310	0.25349	0.17695

Table 4.2: $V_1 = \frac{Var(\tilde{\mu})}{\mu^2}$, $V_2 = \frac{Var(\hat{\mu})}{\mu^2}$ and the relative efficiency e_1 of $\tilde{\mu}$ relative to $\hat{\mu}$ for c=0.1.

m_1	m_2	m	V_1	V_2	e_1
1	1	2	0.00348	0.00496	1.42529
1	2	3	0.00171	0.00312	1.82456
2	1	3	0.00172	0.00325	1.88953
1	3	4	0.00099	0.00222	2.24242
2	2	4	0.00101	0.00235	2.32673
3	1	4	0.00103	0.00244	2.36893
1	4	5	0.00063	0.00169	2.68254
2	3	5	0.00066	0.00180	2.72727
3	2	5	0.00067	0.00189	2.82089
4	1	5	0.00069	0.00195	2.82609

Table 4.2: $V_1 = \frac{Var(\tilde{\mu})}{\mu^2}$, $V_2 = \frac{Var(\hat{\mu})}{\mu^2}$ and the relative efficiency e_1 of $\tilde{\mu}$ relative to $\hat{\mu}$ for c=0.15

m_1	m_2	m	V_1	V_2	e_1
1	1	2	0.00779	0.01106	1.41977
1	2	3	0.00382	0.00695	1.81937
2	1	3	0.00385	0.00722	1.87532
1	3	4	0.00221	0.00493	2.23076
2	2	4	0.00227	0.00521	2.29515
3	1	4	0.00232	0.00540	2.32759
1	4	5	0.00142	0.00377	2.65493
2	3	5	0.00147	0.00400	2.72109
3	2	5	0.00151	0.00419	2.77483
4	1	5	0.00153	0.00432	2.82352

Table 4.2: $V_1 = \frac{Var(\tilde{\mu})}{\mu^2}$, $V_2 = \frac{Var(\hat{\mu})}{\mu^2}$ and the relative efficiency e_1 of $\tilde{\mu}$ relative to $\hat{\mu}$ for c=0.2

m_1	m_2	m	V_1	V_2	e_1
1	1	2	0.01378	0.01941	1.40856
1	2	3	0.00675	0.01217	1.80296
2	1	3	0.00680	0.01262	1.85588
1	3	4	0.00391	0.00864	2.20972
2	2	4	0.00400	0.00911	2.27750
3	1	4	0.00406	0.00941	2.31773
1	4	5	0.00251	0.00661	2.63347
2	3	5	0.00259	0.00700	2.70270
3	2	5	0.00266	0.00731	2.74812
4	1	5	0.00270	0.00751	2.78148

Table 4.2: $V_1 = \frac{Var(\tilde{\mu})}{\mu^2}$, $V_2 = \frac{Var(\hat{\mu})}{\mu^2}$ and the relative efficiency e_1 of $\tilde{\mu}$ relative to $\hat{\mu}$ for c=0.25.

m_1	m_2	m	V_1	V_2	e_1
1	1	2	0.02139	0.02982	1.39411
1	2	3	0.01047	0.01867	1.78319
2	1	3	0.01054	0.01928	1.82922
1	3	4	0.00606	0.01326	2.18812
2	2	4	0.00620	0.01393	2.24677
3	1	4	0.00629	0.01433	2.27822
1	4	5	0.00389	0.01014	2.60668
2	3	5	0.00402	0.01071	2.66418
3	2	5	0.00414	0.01116	2.69565
4	1	5	0.00418	0.01141	2.72967

Table 4.2: $V_1 = \frac{Var(\tilde{\mu})}{\mu^2}$, $V_2 = \frac{Var(\hat{\mu})}{\mu^2}$ and the relative efficiency e_1 of $\tilde{\mu}$ relative to $\hat{\mu}$ for c=0.3.

m_1	m_2	m	V_1	V_2	e_1
1	1	2	0.03055	0.04210	1.37807
1	2	3	0.01494	0.02630	1.76037
2	1	3	0.01501	0.02705	1.80213
1	3	4	0.00864	0.01869	2.16319
2	2	4	0.00884	0.01954	2.21040
3	1	4	0.00895	0.02002	2.23687
1	4	5	0.00555	0.01430	2.57658
2	3	5	0.00573	0.01505	2.62653
3	2	5	0.00586	0.01561	2.66382
4	1	5	0.00595	0.01590	2.67227

5. Concluding Remarks

In all the cases, efficiency of the estimator $\tilde{\mu}$ defined in equation (5), calculated using RSS of inid random variables is much better than that of the estimator $\hat{\mu}$ defined in equation (2) calculated using Order Statistics of inid random variables given in Sajeevkumar and Thomas (2006) [16-17]. Also it is very difficult to calculate the covariance between any two order statistics in the inid case, but in the case of RSS covariance between any two RSS in the inid case is zero.

Hence it is very easy to calculate BLUE of the estimator $\tilde{\mu}$ using RSS of inid random variables than the estimator $\hat{\mu}$ using order statistics of inid random variables. All the computational work in this article are done using "R " software.

6. References

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