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Optimizing dental healthcare services: A queueing theory approach for Ahmedabad's dental hospital

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Abstract
Long patient waiting times and inefficient service delivery are common challenges in dental healthcare systems, leading to dissatisfaction and operational inefficiencies. This study employs queueing theory to analyze and optimize patient flow at a dental hospital in Ahmedabad, India. By modeling the arrival and service patterns of patients, we evaluate key performance metrics such as average waiting time, queue length, server utilization, and system efficiency. Data was collected through direct observation and hospital records to fit appropriate queueing models (e.g., M/M/1 or M/M/c). The analysis identifies bottlenecks in the current system and proposes data-driven recommendations for resource allocation, staff scheduling, and process improvements to minimize delays and enhance service quality. The findings provide actionable insights for dental hospitals to improve patient satisfaction and operational performance while maintaining cost-effectiveness.

Keywords: Queueing theory, dental healthcare, patient waiting time, service optimization, healthcare management

1. Introduction
Govt. Dental College & Hospital, Ahmedabad is one of the oldest dental institutes imparting its services to the people of Gujarat and its neighboring states. Established in 1963, the college has garnered a reputation for academic and scientific excellence at national and international level. The hospital has a daily OPD of more than 400 patients from Gujarat as well as neighboring states providing quality treatment to them. The Department of Conservative Dentistry and Endodontic has adequate number of electrically operated programmable dental chairs and units which cater to more than 100 patients per day coming from different walks of life [5].

The department has different clinical service departments which includes:
1. **Endodontics:** Focuses on diagnosing and treating diseases and injuries of the dental pulp and surrounding tissues, including root canals.
2. **Oral & Maxillofacial Surgery:** Surgical treatment for conditions affecting the mouth, jaws and face, including extractions, repairs, and reconstructive surgeries.
3. **Prosthodontics:** Specializes in restoring and replacing damaged or missing teeth with crowns, bridges, dentures, and implants for functional restoration.
4. **Periodontology:** Concerned with the prevention, diagnosis and treatment of gum diseases and the structures supporting teeth, like bones and ligaments.
5. **Orthodontics:** Deals with diagnosing, preventing and correcting misaligned teeth and jaws using braces, retainers and other corrective appliances.
6. **Paedodontics:** Focuses on dental care for children, including preventive, diagnostic and therapeutic treatments tailored to young patients.
7. **Pathology (Laboratory):** Involves the study and diagnosis of diseases through laboratory tests, including tissue biopsies and analysis of dental conditions.

One of the fundamental aspects of queueing theory is the modelling of arrival and service processes. Arrival processes describe how customers or entities enter the system, while service processes determine how these entities are served and ultimately exit the system.

By mathematically representing these processes, queueing models can be developed to analyze the behaviour and performance of queues under different scenarios and assumptions [9].

The efficient delivery of healthcare services presents a significant operational challenge in urban environments, where growing patient demand frequently strains available medical resources. Empirical observation reveals systemic inefficiencies in dental care facilities, where certain departments such as radiology and patient registration experience persistent congestion and service delays, while other clinical units remain underutilized. This imbalance in resource allocation results in suboptimal patient flow and inefficient use of medical staff and equipment.

This study addresses these operational inefficiencies by examining patient flow management and resource allocation within a dental hospital in Ahmedabad. The research focuses specifically on quantifying and optimizing the performance of interconnected service nodes through queueing theory, aiming to identify data-driven solutions for improving service delivery while maintaining operational feasibility.

2. Literature Review

Curry *et al.* [3] (2021) addressed the critical challenge of capacity planning in hospitals under transient conditions like pandemics by developing a queueing methodology to analyze performance metrics such as patient throughput, waiting times, and resource utilization. Their model incorporated realistic constraints including blocking, transient arrivals, and dynamic capacity adjustments, while also examining the effects of adding resources on system performance. Their approach provided a framework for optimizing emergency department management through probabilistic system analysis.

Lade *et al.* [6] (2013) applied queuing theory and simulation techniques to analyze and optimize patient waiting times in a hospital's Outpatient Department (OPD). Their study focused on modeling patient arrival and service times, identifying that inter-arrival times followed an exponential distribution while service times approximated a normal distribution. Using Monte Carlo simulation, the authors evaluated the impact of resource allocation—specifically, adding an extra doctor—on reducing average waiting times. Their results demonstrated that increasing the number of doctors from four to five reduced the average waiting time by 40.97%, from 7.20 minutes to 4.25 minutes per patient.

Lee *et al.* [7] (2018) developed an innovative queueing network model to analyze patient transitions among ED, ICU, and general wards. The study addressed critical points by incorporating feedback loops, blocking effects and general arrival/service distributions. The authors proposed an iterative solution method that demonstrated computational efficiency while maintaining accuracy, with validation showing less than 10% error in performance estimates. Their analysis revealed that increasing ED capacity could paradoxically worsen delays when utilization exceeded 80%. The model provided hospital administrators with quantitative tools for resource planning and congestion management.

Palvannan and Teow [8] (2012) applied M/M/c queueing model to healthcare, showing how variability in patient arrivals and service times creates queues even with adequate capacity. Their analysis revealed key trade-offs: reducing wait times requires underutilized systems, while pooling resources improves efficiency compared to partitioning. Case studies demonstrated practical applications, including endoscopy recovery bed planning and infection control cohorting. The authors highlighted queueing models as valuable for initial capacity planning despite limitations like assuming Poisson arrivals.

3. Methodology

3.1 Establishment of hospital Queueing model

The patient flow in a dental hospital is conceptualized as a two-node queueing network. This model simulates the process of patient arrival, waiting, and receiving treatment across different departments. The process for each node is defined as follows:

Table 3.1.1: Notations

λ_i	Arrival rate of node i (per minute)
μ_i	Service rate of node i (per minute)
ρ_i	Utilization of node i (per minute)
L_i	Average number of patients in node i
L_{qi}	Average number of patients waiting in line of node i
W_i	Average time spent by patients in system of node i
W_{qi}	Average time spent by passenger in waiting line of node i
P_{0i}	Probability that no passenger in system of node i

Node A: Case Registration Desk

This node represents the patient registration counter. Patients arrive at the hospital following a stochastic process with a mean arrival rate(λ_A), modeled as a Poisson process. Upon arrival, patients join a queue for case registration. The time taken to complete registration for a single patient is the service time, which follows an exponential distribution. The service rate μ_A is the mean number of patients served per five minutes by one registration clerk. The hospital has a one case registration desk. Therefore, an M/M/1 queueing model is appropriate for Node A. This node may be bypassed by old patients who already registered their case.

Node B: Clinical Service Departments

This node encompasses various specialized clinical units including Endodontics, Paedodontics, Prosthodontics, Orthodontics, Periodontology, and the X-ray department. The X-ray department operates independently within this node, as patients typically visit their respective clinical departments first based on their dental needs before being referred for radiographic examination based on professional diagnosis.

Arrival rates for the X-ray department were determined through direct observation, while arrival rates for other clinical departments were estimated using probability distributions derived from patient routing patterns. All clinical departments maintain a capacity of ten operational beds and corresponding medical staff, resulting in ten parallel service channels (C = 10) for each department. However, the X-ray department operates with a single radiographic unit, establishing a single-channel service configuration (C = 1) for this unit.

The M/M/C queueing model is appropriately applied to each department, where service channels represent either dental treatment stations or radiographic equipment, enabling precise performance evaluation across all clinical service units.

3.1.1 Assumptions

- The arrival of patients at each node follows a Poisson process, and the corresponding service times are exponentially distributed.

- The hospital is assumed to have infinite capacity, meaning no patient is turned away due to space constraints.
- Phenomena such as balking, reneging and jockeying are assumed not to occur. System failures are also not considered.
- The performance of each clinical service department is evaluated in isolation, without accounting for potential interdependencies or resource sharing.
- External disruptions, such as equipment breakdowns or unforeseen events, are excluded from the model.
- Pre-appointment processes and departments are excluded from the analysis, with focus limited to in-clinic patient flow.
- The arrival rate calculations consider only new patients; returning patients are not included in the analysis.

3.12 Queueing Model [4]

Using the balance equations of birth-death processes, the steady state probabilities (p_n) of M/M/C model is given by:

$$p_n = \begin{cases} \dfrac{\lambda^n}{n!\,\mu^n}\, p_0 \ (0 \le n < c) \\ \dfrac{\lambda^n}{c^{n-c}c!\,\mu^n} p_0 \ (n \ge c) \end{cases}$$

$$p_0 = \left(\sum_{n=0}^{c-1} \frac{\lambda^n}{n!\,\mu^n} + \sum_{n=c}^{\infty} \frac{\lambda^n}{c^{n-c}c!\,\mu^n}\right)^{-1}$$

Let $r = {}^{\lambda}/_{\mu}$ and $\rho = {}^{\lambda}/_{c\mu}$, then we have

$$p_0 = \left(\sum_{n=0}^{c-1} \frac{r^n}{n!} + \sum_{n=c}^{\infty} \frac{r^n}{c^{n-c}c!}\right)^{-1}$$

Consider infinite series term and rearrange it with suitable formula:

$$\sum_{n=c}^{\infty} \frac{r^n}{c^{n-c}c!} = \frac{r^c}{c!}\frac{1}{1-{}^{r}/_{c}} \ ({}^{r}/_{c} = \rho < 1)$$

Therefore,

$$p_0 = \left(\sum_{n=0}^{c-1} \frac{r^n}{n!} + \frac{r^c}{c!}\frac{1}{1-\rho}\right)^{-1}$$

Now, the performance measures of M/M/C model by means of the following formulas:

1) $L_q = \sum_{n=c+1}^{\infty}(n-c)p_n = \left(\frac{r^c\rho}{c!(1-\rho)^2}\right)p_0$

2) $L = L_q + \frac{\lambda}{\mu} = r + \left(\frac{r^c\rho}{c!(1-\rho)^2}\right)p_0$

3) $W_q = \frac{L_q}{\lambda} = \left(\frac{r_A{}^c}{c!(c\mu)(1-\rho)^2}\right)p_0$

4) $W = \frac{1}{\mu} + W_q = \frac{1}{\mu} + \left(\frac{r^c}{c!(c\mu)(1-\rho)^2}\right)p_0$

3.2 Data Collection

Primary data was collected through direct observation at the case registration desk over a one-week period, focusing specifically on peak operational hours.

Arrival Rate

The arrival rate for Node A (case registration) was calculated by recording the number of patients entering the registration area per five-minute interval. For Node B (clinical departments), directly measuring the arrival rate per department proved challenging. To address this, the probability of a patient proceeding to each department was first estimated using historical visitation data. The departure rate from Node A was subsequently distributed to each department in Node B based on these calculated probabilities.

Service Rate

For Node A, the service rate was determined by measuring the time taken to complete the registration process for each patient once they reached the window. This rate remained consistent across individuals. For Node B, the service rate for each department was defined as the average time required to complete the respective treatment procedures.

4. Results and Discussion

Node A: Case Registration Desk ($M/M/1$)

Table 4.1 presents the performance measures for the case registration desk, calculated using the standard M/M/1 queueing equations and primary data on arrival and service rates.

Table 4.1: Operating characteristics of case registration desk

Time	λ_A	μ_A	ρ_A	L_{qA}	L_A	W_{qA}	W_A	P_{0A}
10:30-10:35	4	5	0.80	3.2	4	4	5	0.2
10:35-10:40	4.4	5	0.88	6.45	7.33	7.33	8.34	0.12
10:40-10:45	4.8	5	0.96	23.04	24	24	25	0.04
10:45-10:50	4.8	5	0.96	23.04	24	24	25	0.04
10:50-10:55	4	5	0.80	3.2	4	4	5	0.2
10:55-11:00	5.6	5	1.12					
11:00-11:05	5.4	5	1.08					
11:05-11:10	4.6	5	0.92	10.58	11.50	11.50	12.50	0.08
11:10-11:15	3	5	0.60	0.90	1.50	1.50	2.5	0.40
11:10-11:20	4.6	5	0.92	10.58	11.50	11.50	12.5	0.08
11:20-11:25	4.4	5	0.88	6.45	7.33	7.33	8.34	0.12
11:25-11:30	2.8	5	0.56	0.71	1.27	1.27	2.28	0.44
11:30-11:35	3.6	5	0.72	1.85	2.57	2.57	3.57	0.28
11:35-11:40	3	5	0.60	0.90	1.5	1.5	2.5	0.4
11:40-11:45	3.2	5	0.64	1.14	1.78	1.78	2.78	0.36
11:45-11:50	3.8	5	0.76	2.41	3.17	3.17	4.17	0.24
11:50-11:55	3.8	5	0.76	2.41	3.17	3.17	4.17	0.24
11:55-12:00	3.6	5	0.72	1.85	2.57	2.57	3.57	0.28
12:00-12:05	2.2	5	0.44	0.35	0.79	0.79	1.79	0.56
12:05-12:10	2.6	5	0.52	0.56	1.08	1.08	2.09	0.48
12:10-12:15	2	5	0.40	0.27	0.67	0.67	1.67	0.6
12:15-12:20	1.8	5	0.36	0.20	0.56	0.56	1.57	0.64
12:20-12:25	1.8	5	0.36	0.20	0.56	0.56	1.57	0.64
12:25-12:30	1.6	5	0.32	0.15	0.47	0.47	1.47	0.68

As shown in Table 4.1, the desk operated from a single window, leading to a high utilization rate. This initially resulted in a large number of individuals within the system. Furthermore, it was observed that the system periodically deviated from a steady state (i.e., $\lambda > \mu$) due to spikes in the arrival rate.

Figure 4.1 presents the transition probability distribution for patients departing Node A and entering various departmental queues. These probabilities, estimated from previous arrival data for the respective departments, were used to apportion Node A's departure rate into the arrival rates for downstream nodes (such as Node B). The resultant arrival rates for each department are measured in Table 4.2.

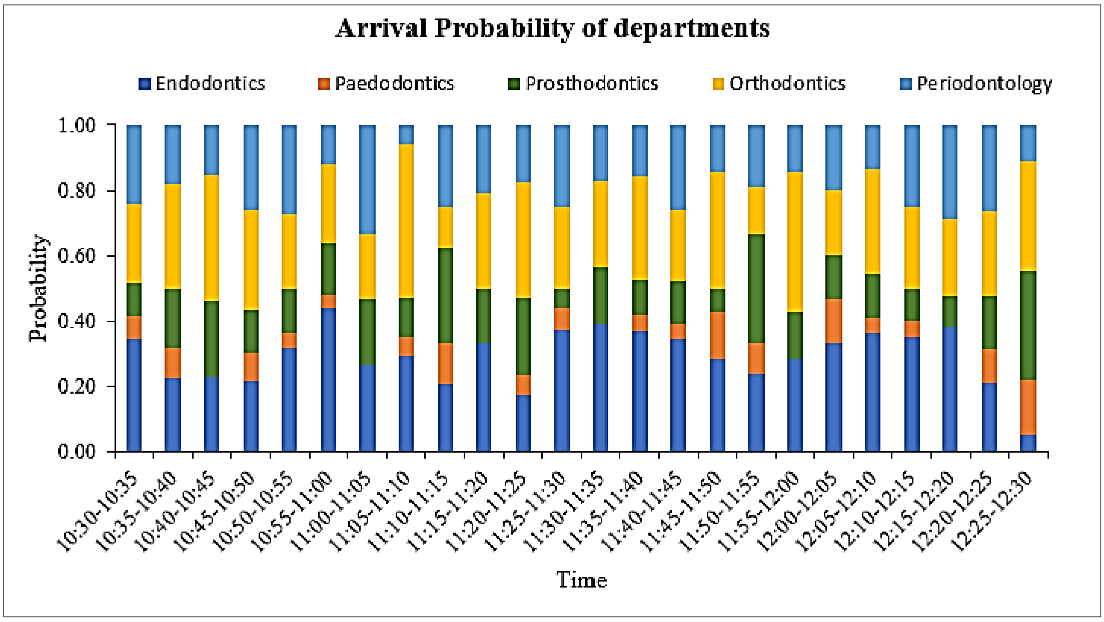

Fig 1: Arrival Probability distribution based on observations

Table 4.2: Operating characteristics of departments

Department	λ_B	μ_B	C	ρ_B	L_B	L_{qB}	W_B	W_{qB}	P_{0B}
Endodontics	29.20	3.4	10	0.85	11.95	3.36	49.12	13.82	0.01
Paedodontics	5.80	8	10	0.07	0.73	00	15	00	0.48
Prosthodontics	16	2.66	10	0.60	6.17	0.16	46.28	1.16	0.24
Orthodontics	27.20	3	10	0.90	15.75	6.69	69.51	29.51	0.01
Periodontology	19.80	2.66	10	0.74	8.31	0.86	50.34	5.23	0.05
X-ray	100	60	1	1.67					

The calculated operating characteristics for each departmental queueing system are summarized in Table 4.2. This includes the independent X-ray department. Unlike the case registration desk, the service times across these departments were substantially higher and thus were not subdivided into 5-minute intervals for analysis. Critical observations from the data indicate that the X-ray department consistently failed to reach a steady state during the observation period. Furthermore, the Orthodontics department exhibited critically high utilization, leading to higher patient waiting times.

5. Proposed System

Node A: Case Registration Desk

The analysis of the current single-server system at the Case Registration Desk (Table 4.1) confirms its over-utilization, which is the direct cause of patient congestion and significant service delays. To mitigate this, a two-server system was evaluated. The results (Figures 5.1 and 5.2) confirm that adding a second server during peak demand periods drastically reduces queue length and waiting times, thereby restoring system efficiency.

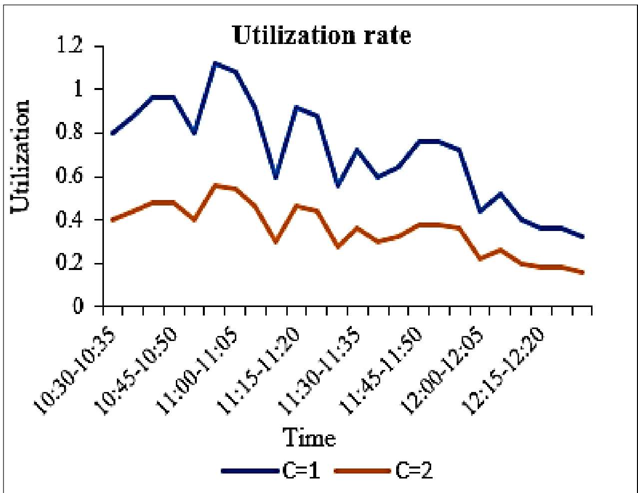

Fig 5.1: Case registration desk

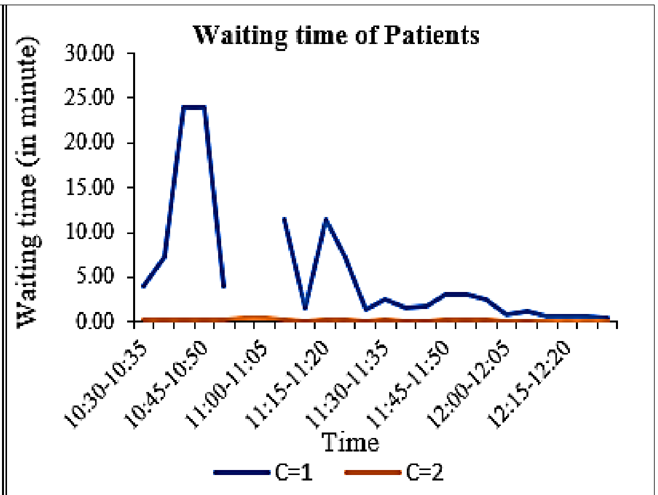

Fig 5.2: Case registration desk

Crucially, the analysis indicates that a second server is only essential during the initial peak period (approximately 10:30 AM-11:25 AM). Beyond this window, a single server is sufficient to maintain stable operations without congestion. Therefore, to optimize costs, it is recommended to deploy a second server dynamically only during this specific high-demand interval. Unnecessary operation of a second server post-11:25 AM would incur avoidable staffing and maintenance costs. Implementing a flexible shift schedule is proposed as a cost-effective strategy to achieve this dynamic resource allocation.

Node B: Clinical Service Departments

Table 5.1: Effects of adding extra X ray channel

λ_A	μ_A	C	ρ_A	L_A	L_{qA}	W_A	W_{qA}	P_{0A}
100	60	1	1.67					
100	60	2	0.83	5.45	3.78	6.6	4.56	0.09
100	60	3	0.55	2.041	0.375	2.4	0.48	0.17

Based on the numerical results for the X-ray department (Table 4.2), the addition of an X-ray channel is required to achieve steady-state condition. As shown in Table 5.1, incorporating one additional channel stabilizes the system. While adding a second extra channel would further enhance operational smoothness, but this approach raises cost concerns. Therefore, if the expense of a three-channel configuration is manageable for the hospital, it represents the preferable option for smooth system performance.

6. Conclusion

This study employed queueing theory to analyze and optimize patient flow processes within a dental hospital system in Ahmedabad. The investigation revealed critical operational bottlenecks and proposed data-driven solutions to enhance service efficiency and patient satisfaction. Key findings include:

7. Department-Specific Solutions

- Quantitative assessment confirmed unsustainable utilization levels at the patient registration desk during peak hours. Modeling demonstrated that strategic deployment of additional service capacity during specified high-demand periods (10:30-11:25 AM) would reduce average waiting time while maintaining cost-efficiency through targeted resource allocation.
- Performance metrics identified distinct operational challenges across clinical departments. The Orthodontics unit showed critical over-utilization ($\rho > 85\%$) requiring capacity expansion, while the X-Ray department operated in non-steady state conditions ($\lambda > \mu$) necessitating service rate adjustments.

8. Resource Allocation Adjustments

- Dynamic staffing models—such as flexible shift scheduling for registration clerks and dental staff—were shown to effectively match resource availability with patient inflow patterns.
- Reallocating existing resources during off-peak hours to high-demand departments can alleviate congestion without significant additional investment.

9. Recommendations

- Implement dynamic resource allocation for registration and clinical services based on hourly and daily demand variations.
- Expand capacity in high-utilization departments like Orthodontics and X-Ray to reduce waiting times and improve patient satisfaction.
- Introduce digital pre-registration and appointment systems to decentralize arrival patterns and reduce peak-time burden.
- Continuously monitor performance metrics such as average waiting time, queue length, and server utilization to iteratively refine strategies.

10. Future work

- Conduct a detailed cost-benefit analysis of adding servers (e.g., staff, equipment) versus patient wait-time reductions.
- Extend the model to simulate interconnected departments with patient feedback loops.
- Integrate digital tools like AI-driven scheduling for dynamic resource allocation.
- Validate the framework in multi-specialty hospitals to assess scalability.

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