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Analyzing topology induced by graph metrics on the vertex set of some special graphs

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Abstract

This paper explores the construction of topologies on a graph's vertex set using open balls and eccentric neighbourhoods defined by graph metrics such as geodesic distance, detour distance, circular distance, and circular D-distance. We examine how these metric-induced topologies behave when applied to specific classes of graphs such as Friendship Graph, Bistar Graph, Pan Graph, Complete Bipartite Graph, Barbell Graph and Helm Graph.

Keywords: Topology, graph metrics, geodesic distance, detour distance, circular distance, circular d-distance, open ball, eccentric neighbourhood

1. Introduction

Graph theory and topology are powerful mathematical tools widely used to model and analyze complex systems across diverse fields such as computer science, engineering, transportation, and the biological sciences. Graphs provide a flexible structure for representing relationships and processes, while graph-based topologies help understand connectivity patterns, enabling advanced modeling in areas like medical diagnostics, biological networks, and infrastructure design. Graph metrics such as geodesic distance, detour distance and circular distance play a critical role in measuring connectivity, separation and reachability within networks. These metrics support essential applications like shortest path algorithms, clustering, optimization and navigation systems. In computer vision and image processing, distance measures guide object detection and pattern recognition, while in social network analysis, they reveal influence and community structures.

A metric space is a non-empty set where a distance function (called a metric) is defined between every pair of its elements. The study of abstract topological spaces requires an understanding of topological features, such as open and closed sets, which are produced by a metric defined on a set. A subbasis for the metric topology can be constructed using the metric. Furthermore, machine learning models performance can be maximized by utilizing the topologies obtained from significant distance measures. Biological systems, often represented as networks, benefit from graph-based models to describe complex interactions between biological entities. Topologies derived from vertex relations aid in analyzing these networks for purposes like disease detection and diagnosis. In logistics, circular distance metrics help optimize delivery routes by balancing coverage and return efficiency. Similarly, detour distance becomes useful in modeling scenarios such as delivery networks, where visiting multiple destinations and minimizing travel costs are crucial.

2. Preliminaries

The following definitions are derived from references [1, 2, 3, 4, 5].

Graph metrics are ways of describing metric spaces that are defined over sets of vertices according to distances in a graph that is defined over those sets of vertices. When the graph is connected, the distance function and the collection of vertices can be transformed into a metric space. Here we are considering different graph metrics such as geodesic distance, detour distance, circular distance and circular D-distance.

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The length of the shortest path between two vertices u and v , in a connected graph G is the geodesic distance $d(u, v)$, where $d(u, v) = 0$ if $u = v$, while the length of the longest simple path between two vertices u and v is the detour distance $D(u, v)$, where $D(u, v) = 0$ if $u = v$. The maximum geodesic distance (or detour distance) from u to any other vertices in the graph is called eccentricity $e_d(u)$ (or $e_D(u)$) of a vertex u . The definitions of geodesic radius $r_d(G)$ (or detour radius $r_D(G)$) and geodesic diameter $diam_d(G)$ (or detour diameter $diam_D(G)$) of a graph G are as follows:

$$r_d(G) = \min \{e_d(u) : u \in V(G)\} \text{ (or } r_D(G) = \min \{e_D(u) : u \in V(G)\}),$$

$$diam_d(G) = \max \{e_d(u) : u \in V(G)\} \text{ (or } diam_D(G) = \max \{e_D(u) : u \in V(G)\}).$$

The circular distance $cd(u, v)$ between two distinct vertices u and v in a connected graph G is defined as the sum of the geodesic distance $d(u, v)$ and the detour distance $D(u, v)$. i. e. $cd(u, v) = d(u, v) + D(u, v)$. The circular eccentricity

$e_{cd}(u)$ of the vertex u is defined as the maximum circular distance to any other vertex from the vertex u and the circular radius $r_{cd}(G)$ and circular diameter $diam_{cd}(G)$ of a graph G are defined as:

$$r_{cd}(G) = \min \{e_{cd}(u) : u \in V(G)\} \text{ and } diam_{cd}(G) = \max \{e_{cd}(u) : u \in V(G)\}.$$

If u, v are vertices of a connected graph G , then the D-length of a $u - v$ path P is defined as $DI(P) = l(P) + deg(u) + deg(v) + \sum deg(w)$ where the summation is taken over all the internal vertices w of P . The geodesic D-distance $d^D(u, v)$ between distinct two vertices u and v of a connected graph G is defined as $d^D(u, v) = \min \{DI(P)\}$ where the minimum is taken over all $u - v$ paths P in G and zero if $u = v$.

The detour D-distance $D^D(u, v)$ between two distinct vertices u and v of a connected graph G is defined as $D^D(u, v) = \max \{DI(P)\}$ where maximum is taken over all $u - v$ paths P in G and zero if $u = v$. The circular D-distance $cd^D(u, v)$ between two distinct vertices u and v in a connected graph G is defined as the sum of the geodesic D-distance $d^D(u, v)$ and the detour D-distance $D^D(u, v)$. i. e. $cd^D(u, v) = d^D(u, v) + D^D(u, v)$. The circular D-radius

$r_{cd}^D(G)$ and circular D-diameter $diam_{cd}^D(G)$ of a graph G are defined as follows:

$$r_{cd}^D(G) = \min \{e_{cd}^D(u) : u \in V(G)\} \text{ and } diam_{cd}^D(G) = \max \{e_{cd}^D(u) : u \in V(G)\}.$$

3. Topology Induced by Open Balls and Eccentric Neighbour- hoods

This section examines how graph metrics such as geodesic, detour, circular, and circular D-distances are used to generate topologies caused by open balls and eccentric neighborhoods on graphs like Friendship, Bistar, Pan, Complete Bipartite, Barbell, and Helm Graphs. References ^[1, 6, 7] provide the definitions in this section.

Definition 1. Let $dis(u, v)$ denote a distance metric. According to metric dis , the open balls in a graph G are defined as

$$B_{dis}(v, r_{dis}(G)) = \{u \in V(G) : dis(u, v) < r_{dis}(G)\}$$

Therefore, to form the topology τ_{dis} on G 's vertex set, $B_{dis}(v, r_{dis}(G))$ forms a subbasis.

Definition 2. The eccentric neighbourhoods in a graph G are defined as follows in terms of the metrics such as geodesic distance $d(u, v)$, detour distance $D(u, v)$, circular distance $cd(u, v)$ and circular D-distance $cd^D(u, v)$:

$$N_d(v) = \{u \in V(G) : d(u, v) = e_d(v)\},$$

$$N_D(v) = \{u \in V(G) : D(u, v) = e_D(v)\},$$

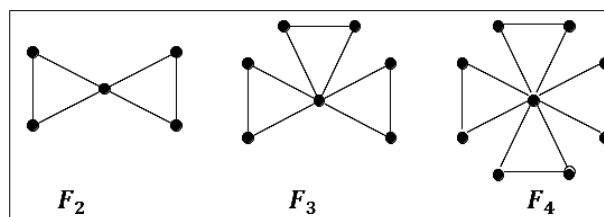
$$N_{cd}(v) = \{u \in V(G) : cd(u, v) = e_{cd}(v)\},$$

$$N_{cd}^D(v) = \{u \in V(G) : cd^D(u, v) = e_{cd}^D(v)\}.$$

The complement of $N_d(v)$, $N_D(v)$, $N_{cd}(v)$ and $N_{cd}^D(v)$ should be $M_d(v)$, $M_D(v)$, $M_{cd}(v)$ and $M_{cd}^D(v)$ respectively. On a vertex set of G , these families of sets constitute subbases for the topologies τ_{M-d} , τ_{M-D} , τ_{M-cd} and τ_{M-cd}^D .

Friendship Graph

Definition 3. A Friendship graph F_n , $n \geq 1$ is a graph made up of n triangles with a common vertex, having $2n + 1$ vertices and $3n$ edges.



Theorem 1. For a friendship graph F_n with $n \geq 2$,

$$B_d(v, r_d(F_n)) = B_D(v, r_D(F_n)) = B_{cd}(v, r_{cd}(F_n)) = \{v\},$$

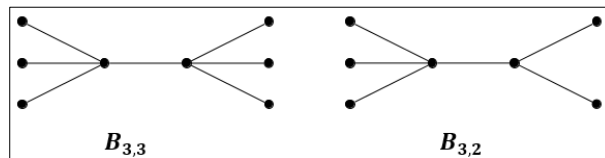
for all $v \in V(F_n)$.

Proof. For any two vertices $u \neq v$ in F_n , $d(u, v) = 1$ or 2 , $D(u, v) = 2$ or 4 and $cd(u, v) = 3$ or 6 . Specifically, for every $v \in V(F_n) - w$, $d(w, v) = 1$ and $cd(w, v) = 3$ if w is the center vertex of F_n . This means that $e_d(w) = 1$, $e_D(w) = 2$ and $e_{cd}(w) = 3$. It follows that $r_d(F_n) = 1$, $r_D(F_n) = 2$ and $r_{cd}(F_n) = 3$. Therefore, for every $v \in V(F_n)$, $B_d(v, r_d(F_n)) = B_D(v, r_D(F_n)) = B_{cd}(v, r_{cd}(F_n)) = v$.

Observation 1. Let $\{v_1, v_2, \dots, v_n\}$ be the outer vertices of a friendship graph F_n where $n \geq 2$ and w be the central vertex of F_n . Then for $i = 1, 2, \dots, n$, $N_d(v_i) = N_D(v_i) = N_{cd}(v_i) = N_{cd}^D(v_i)$, for every $v \in V(P_n)$ and $N_d(w) = N_D(w) = N_{cd}(w) = N_{cd}^D(w) = \{v_1, v_2, \dots, v_n\}$. Also, $\tau_{M-d} = \tau_{M-D} = \tau_{M-cd} = \tau_{M-cd}^D$.

Bistar Graph

Definition 4. When two-star graphs $K_{1,n}$, have their central vertices joined by a single edge, the resulting graph is called a bistar graph $B_{m,n}$.



Theorem 2. For a bistar graph $B_{m,n}$, with $m, n \geq 2$,

$$B_d(v, r_d(B_{m,n})) = B_D(v, r_D(B_{m,n})) = B_{cd}(v, r_{cd}(B_{m,n})),$$

for all $v \in V(B_{m,n})$.

Proof. In a bistar graph $B_{m,n}$, there exists exactly one path between any two vertices. Hence, for any $v \in V(B_{m,n})$, $d(u, v) = D(u, v)$, for all $u \in V(B_{m,n})$. So $r_d(B_{m,n}) = r_D(B_{m,n})$, which implies $B_d(v, r_d(B_{m,n})) = B_D(v, r_D(B_{m,n}))$.

Moreover, $cd(u, v) = 2d(u, v)$ holds for any two vertices $u, v \in V(B_{m,n})$. Hence, $r_{cd}(B_{m,n}) = 2r_d(B_{m,n})$. Now let $u \in B_{cd}(v, r_{cd}(B_{m,n})) \Leftrightarrow cd(u, v) < r_{cd}(B_{m,n}) \Leftrightarrow 2d(u, v) < 2r_d(B_{m,n}) \Leftrightarrow d(u, v) < r_d(B_{m,n}) \Leftrightarrow u \in B_d(v, r_d(B_{m,n}))$.

Therefore, $B_{cd}(v, r_{cd}(B_{m,n})) = B_d(v, r_d(B_{m,n}))$.

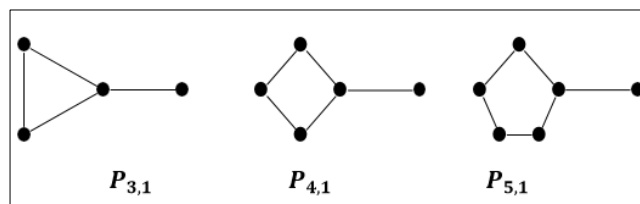
Thus, $B_d(v, r_d(B_{m,n})) = B_D(v, r_D(B_{m,n})) = B_{cd}(v, r_{cd}(B_{m,n}))$, for all $v \in V(B_{m,n})$.

Observation 2. In bistar graph $B_{m,n}$ we have, $N_d(v) = N_D(v) = N_{cd}(v) = N_{cd}^D(v)$.

Also, $\tau_{M-d} = \tau_{M-D} = \tau_{M-cd} = \tau_{M-cd}^D$.

Pan Graph

Definition 5. A Pan graph $P_{(n,1)}$, $n \geq 3$ is a graph formed by created by adding an edge from a cycle graph C_n to a singleton graph K_1 .



Theorem 3. For a pan graph $P_{n,1}$ with $n \geq 3$,

$$B_d(v, r_d(P_{n,1})) \cap B_D(v, r_D(P_{n,1})) = B_{cd}(v, r_{cd}(P_{n,1})), \text{ for all } v \in V(P_{n,1}).$$

Proof. Let $u \in B_d(v, r_d(P_{n,1})) \cap B_D(v, r_D(P_{n,1})) \Rightarrow u \in B_d(v, r_d(P_{n,1}))$ and $u \in B_D(v, r_D(P_{n,1})) \Rightarrow d(u, v) < r_d(P_{n,1})$ and $D(u, v) < r_D(P_{n,1})$.

We know that $r_d(P_{n,1}) < r_D(P_{n,1}) < r_{cd}(P_{n,1})$. So $d(u, v) < r_{cd}(P_{n,1})$ and $D(u, v) < r_{cd}(P_{n,1}) \Rightarrow d(u, v) + D(u, v) < r_{cd}(P_{n,1}) \Rightarrow cd(u, v) < r_{cd}(P_{n,1}) \Rightarrow u \in B_{cd}(v, r_{cd}(P_{n,1}))$.

Therefore, $B_d(v, r_d(P_{n,1})) \cap B_D(v, r_D(P_{n,1})) \subseteq B_{cd}(v, r_{cd}(P_{n,1}))$.

Conversely, let $u \in B_{cd}(v, r_{cd}(P_{n,1})) \Rightarrow cd(u, v) < r_{cd}(P_{n,1}) \Rightarrow d(u, v) + D(u, v) < r_{cd}(P_{n,1}) \Rightarrow d(u, v) + D(u, v) < r_d(P_{n,1}) + r_D(P_{n,1})$, since $r_{cd}(P_{n,1}) < r_d(P_{n,1}) + r_D(P_{n,1}) \Rightarrow d(u, v) < r_d(P_{n,1})$ and $D(u, v) < r_D(P_{n,1}) \Rightarrow u \in B_d(v, r_d(P_{n,1}))$ and $u \in B_D(v, r_D(P_{n,1})) \Rightarrow u \in B_d(v, r_d(P_{n,1})) \cap B_D(v, r_D(P_{n,1}))$.
Therefore, $B_{cd}(v, r_{cd}(P_{n,1})) \subseteq B_d(v, r_d(P_{n,1})) \cap B_D(v, r_D(P_{n,1}))$.

Thus, $B_d(v, r_d(P_{n,1})) \cap B_D(v, r_D(P_{n,1})) = B_{cd}(v, r_{cd}(P_{n,1}))$, for all $v \in V(P_{n,1})$.

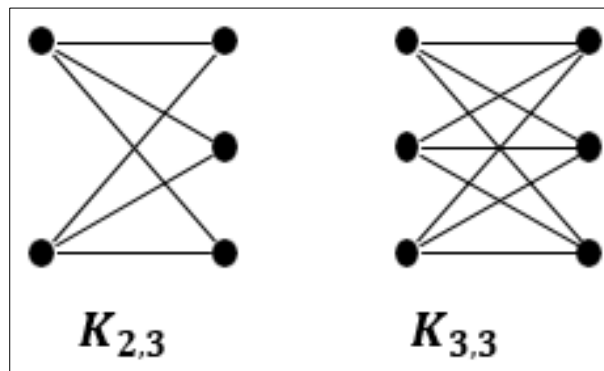
Note 1. We consider the vertices such that neither $d(u, v)$ nor $D(u, v)$ exceeds their respective bounds $r_d(G)$ and $r_D(G)$.

Note 2. For a pan graph P_n , we have $r_{cd}(P_{n,1}) < r_d(P_{n,1}) + r_D(P_{n,1})$.

Observation 3. Let $\{v_1, v_2, \dots, v_n\}$ be the vertices of a pan graph P_n and w be the single vertex. Let v_m be the vertex connected to w . We have, $N_{cd}(v_i) = N_{cd}^D(v_i) = \{w\}$ where $i \neq m$ and $N_{cd}(v_m) = N_{cd}^D(v_m) = N_{cd}(w) = N_{cd}^D(w) = \{v_1, v_2, \dots, v_{m-1}, v_{m+1}, \dots, v_n\}$.

Complete Bipartite Graph

Definition 6. A bipartite graph is defined as a graph whose vertex set can be divided into two subsets, X and Y , such that every edge connects a vertex in X to a vertex in Y . This division (X, Y) is referred to as the bipartition of the graph. Simple bipartite graphs with bipartition (X, Y) in which every vertex of X is connected to every vertex of Y are called complete bipartite graphs. If $|X| = m$ and $|Y| = n$, then the graph is denoted as $K_{m,n}$.



Theorem 4. For a complete bipartite graph $K_{m,n}$,

$$B_{cd}(v, r_{cd}(K_{m,n})) = \{v\}, \text{ for all } v \in V(K_{m,n}).$$

Proof. Case I: When $m = n$, the sum of the length of geodesic path and detour path is $2n$. So $cd(u, v) = 2n$, for all $v \in V(K_{n,n}) \Rightarrow e_{cd}(v) = 2n \Rightarrow r_{cd}(v) = 2n$. Therefore, $B_{cd}(v, r_{cd}(K_{m,n})) = \{v\}$.

Case II: When $m \neq n$, we have $cd(u, v) = 2n$ or $cd(u, v) = 2m$, for all $v \in V(K_{m,n})$. If $m < n$, then $r_{cd}(K_{m,n}) = 2m$ and if $m > n$, then $r_{cd}(K_{m,n}) = 2n$. Therefore, $B_{cd}(v, r_{cd}(K_{m,n})) = \{v\}$.

Theorem 5. For a complete bipartite graph $K_{m,n}$, when $m = n$, we have the following:

$$(i) B_d(v, r_d(K_{n,n})) \cup B_D(v, r_D(K_{n,n})) = V(K_{n,n}), \text{ for all } v \in V(K_{n,n}). B_{cd}^D(v, r_{cd}^D(K_{n,n})) = \{v\}, \text{ for all } v \in V(K_{n,n}).$$

Proof. (i) We have the length of geodesic path + length of detour path = $2n$. Also, $e_d(v) = 2$ and $e_D(v) = 2n - 1$, for all $v \in V(K_{n,n})$. So $r_d(K_{n,n}) = 2$ and $r_D(K_{n,n}) = 2n - 1$. Clearly, $v \in B_d(v, r_d(K_{n,n}))$ and $v \in B_D(v, r_D(K_{n,n}))$, for all $v \in V(K_{n,n})$. Let $w \in V(K_{n,n})$ be arbitrary. If $d(v, w) < 2$, then $w \in B_d(v, r_d(K_{n,n}))$, otherwise $D(v, w) = 2n - 2 < 2n - 1 \Rightarrow w \in B_D(v, r_D(K_{n,n}))$. Thus, for all vertices $w \in V(K_{n,n})$ will either be in $B_d(v, r_d(K_{n,n}))$ or in $B_D(v, r_D(K_{n,n}))$. Hence, $B_d(v, r_d(K_{n,n})) \cup B_D(v, r_D(K_{n,n})) = V(K_{n,n})$, for all $v \in V(K_{n,n})$.

(ii) For any $K_{n,n}$ graph, the degree of vertex is n . Therefore, $cd^D(u, v)$ values are equal for $u \in V(K_{n,n}) - \{v\}$. Let $cd^D(u, v) = k$, for every $u \in V(K_{n,n}) - \{v\}$. Hence, $e_{cd}^D(v) = k$. So, $r_{cd}^D(K_{n,n}) = k$. Therefore, $B_{cd}^D(v, r_{cd}^D(K_{n,n})) = \{v\}$, for all $v \in V(K_{n,n})$.

Theorem 6. In a complete bipartite graph $K_{(m, n)}$, for all $v \in V(K_{(n, n)})$,

$$M_d(v) = B_d(v, r_d(K_{(m,n)}))$$

Proof. In $K_{(m,n)}$, $e_d(v) = 2$, for all $v \in V(K_{(m,n)})$ and $r_d(K_{(m,n)}) = 2$.

Now, $u \in B_d(v, r_d(K_{(m,n)})) \Rightarrow d(u, v) < r_d(K_{(m,n)}) = 2 = e_d(v) \Rightarrow u \notin N_d(v) \Rightarrow u \in M_d(v) \Rightarrow B_d(v, r_d(K_{(m,n)})) \subseteq M_d(v)$.

Now, $u \in M_d(v) \Rightarrow d(u, v) < e_d(v) \Rightarrow d(u, v) < r_d(K_{(m,n)}) \Rightarrow u \in B_d(v, r_d(K_{(m,n)})) \Rightarrow M_d(v) \subseteq B_d(v, r_d(K_{(m,n)}))$.

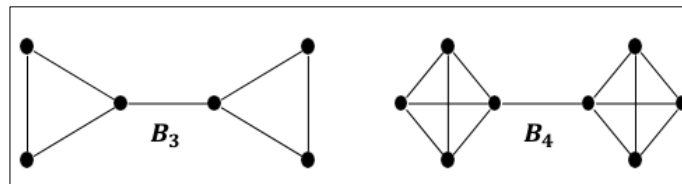
Therefore, $M_d(v) = B_d(v, r_d(K_{(m,n)}))$, for all $v \in V(K_{(m,n)})$.

Observation 4. In a complete bipartite graph $K_{(m,n)}$, we have,

- When $m=n$, $N_{cd}(v) = N_{cd}^D(v) = V(K_{(n,n)}) - \{v\}$ and $M_D(v) = B_D(v, r_D(G))$.
- When $m \neq n$, $N_{cd}(v) = N_{cd}^D(v)$ and $M_{cd}^D(v) = B_{cd}^D(v, r_{cd}^D(G))$.

Barbell Graph

Definition 7. A Barbell graph B_n , is a graph formed by joining two copies of a complete graph K_n by an edge.



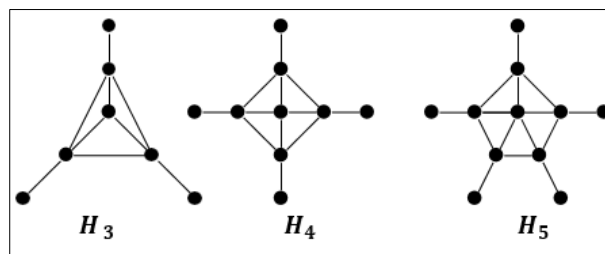
Observation 5. For a barbell graph B_n with $n \geq 3$,

$$B_d(v, r_d(B_n)) = B_D(v, r_D(B_n)) = B_{cd}(v, r_{cd}(B_n)) = B_{cd}^D(v, r_{cd}^D(B_n)), \text{ for all } v \in V(B_n).$$

Observation 6. In a barbell graph B_n with $n \geq 3$, we have $N_d(v) = N_D(v) = N_{cd}(v) = N_{cd}^D(v)$.

Helm Graph

Definition 8. The Helm graph H_n is a graph formed by connecting one vertex and one edge to each vertex of a wheel graph W_n 's outer cycle.



Observation 7. In a helm graph H_n , for all $v \in V(H_n)$,

(i) When $n = 3$,

$$B_d(v, r_d(H_n)) = B_D(v, r_D(H_n)) = B_{cd}(v, r_{cd}(H_n)) = B_{cd}^D(v, r_{cd}^D(H_n)).$$

(ii) When $n = 4$,

$$B_d(v, r_d(H_n)) = B_{cd}^D(v, r_{cd}^D(H_n)).$$

(iii) When $n = 5, 6$,

$$B_{cd}(v, r_{cd}(H_n)) \cap B_{cd}^D(v, r_{cd}^D(H_n)) = B_d(v, r_d(H_n)).$$

Observation 8. In a helm graph H_n with $n \geq 3$, $N_d(v) = N_{cd}(v) = N_{cd}^D(v)$. Also, $\tau_{M-d} =$

$$\tau_{M-d} = \tau_{M-cd}^D.$$

Based on the theorems above, we can conclude that:

1. Except for complete bipartite graph and pan graph, $B_D(u, r_D(G)) = B_{cd}(u, r_{cd}(G))$, for every graph G .

2. $\tau_D = \tau_{cd}$ for any graph G , except for complete bipartite graphs and pan graphs.
3. τ_d, τ_D , and τ_{cd} are discrete topologies for friendship graphs.
4. For a bistar graph, the topologies τ_d, τ_D , and τ_{cd} on its vertex set are discrete.
5. In complete bipartite graph $K_{m,n}$,

(i) When $m \neq n$, $\tau_{cd}, \tau_{cd}^D, \tau_{M-cd}$ and τ_{M-cd}^D are discrete topologies.

(ii) When $m = n$, τ_{cd} and τ_{cd}^D on the vertex set are same and τ_{cd} is discrete topology.

Conclusion

This study investigates the formation of topologies on graph vertex sets induced by open balls and eccentric neighborhoods, utilizing various graph metrics. Specifically, geodesic, detour, circular, and circular D-distances are explored across several special graph classes, including friendship, bistar, pan, complete bipartite, barbell, and helm graphs. The resulting topologies are analyzed to emphasize the distinct topological characteristics inherent to each graph type. For any graph G , $B_D(u, r_D(G)) = B_{cd}(u, r_{cd}(G))$, except for complete bipartite graphs and pan graphs. We also observed that for complete bipartite graph $K_{m,n}$, $M_d(v) = B_d(v, r_d(K_{m,n}))$, for all $v \in V(K_{m,n})$. This work serves as a foundational step in the topological study of graphs through metric-based approaches. This work can be further extended by developing topologies based on alternative graph metrics, such as those induced by closed balls and closed eccentric neighborhoods. These extensions hold promise for practical applications in areas such as network optimization, communication systems, and biological analysis, where understanding the underlying topological structure is crucial.

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