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Garch modeling of Nigerian stock exchange market capitalization

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Abstract

The contribution of stock exchange market to the growth of the Nigeria economy is never in doubt. However, it has been seen that volatility in stock exchange market may trigger in increase in cost of capitalization which is capable of affecting economic growth in Nigeria. This may have implication for portfolio allocation and market risk measure. But this research is based on GARCH Modelling of Nigerian Stock Exchange Market Capitalization¹. GARCH and ARIMA models were chosen to model the Nigeria Stock Exchange Market Capitalization Series. Model selection techniques were applied to obtain the best fit model for the series. GARCH (1,1) and ARIMA (3,1,3) models were identified as better fitted models to the data sets and diagnostic test was carried to ascertain the adequacy of the fitted model. The experimental analysis was performed in different algorithm using ARIMA (3,1,3) and GARCH (1,1) models respectively and results were obtained. The study proves that GARCH (1,1) estimate were found more appropriate and better fit model to describe the volatility of the Nigeria Stock Exchange Market Capitalization, since it has lower Akaike Information Criterion (AIC) and Schwarz Information Criterion (SIC).

Keywords: Rate of metabolism, blood mass stream rate, warm conductivity, warm era, limited component method, Pennes Bio - Heat Model

Introduction

The stock exchange is a well-organized marketplace where investors can purchase and sell securities such as shares, stocks, and debentures. It is a market where those interested in purchasing stocks are brought into direct touch with the seller. It is a crucial component of the capital market, serving as a source of cash for publicly traded firms and government institutions, as well as a platform for individual financial participation. The market exclusively deals with existing shares; no new ones are exchanged there. The stock exchange operates a daily auction market in which specialist traders known as stock brokers purchase and sell shares of quoted firms both for their own account and on behalf of their clients.

Nigeria only has one stock exchange market. The Nigerian stock exchange market, which has seven branches in seven states of the federation plus Abuja, is linked to its headquarters in Lagos via computer networks. The exchange runs an auction market, which is a system in which securities are bought and sold through a large number of brokers at prices determined by competitive bidding. Stock brokers dealing on their own account and on behalf of their clients enter their bids to buy and sell securities in their computer system on the trading floor. Some stock broking firms now have remote trading tools and trade from their offices during trading hours. The Nigeria Stock Exchange was established on June 5, 1961, when the Lagos Stock Exchange Act was enacted by the Federal Government of Nigeria. The business began officially on August 25th, 1961, with 19 stocks registered for trading, while informal operations began earlier in June 1961. The operation was initially handled inside the central bank (CBN) premises, with four firms serving as market dealers: Inlaks, John Holt, Bowring, and Icon (Investment Company of Nigeria). According to Marc Reinganum (1999)^[13], market capitalization is a major driver of portfolio performance and policy decisions.

Gopinath (2005)^[11] investigates market capitalization based on the patterns of share price fluctuation and the behavior of ten (10) companies, revealing the adequate performance of share price movements and the performance of the same company were cyclical.

The study also identifies a significant relationship between share price and price earnings ratio, market capitalizations, turnover ratio, book value, and market returns.

In this study, we continue our investigation into stock exchange market capitalization movement by identifying different aspects in model and time frames, as well as the models that can be used to capture stock exchange market capitalization volatility using the ARIMA and GARCH models, as described by Bollerslev (1986) ^[4] and Eagle (1982) ^[9], respectively. As a result, we strive to discover the model that best captures the features of Nigeria's stock exchange market capitalization. GARCH and ARIMA models were chosen to compare the volatility clustering and capitalization of the Nigeria stock exchange market capitalization. Model selection approaches such as Augmented Dickey Fuller (ADF) and Akaike Information Criterion (AIC) were used to assess the volatility performance of the Nigeria stock exchange market capitalization in order to determine the basis for claiming its model performances.

GARCH is a statistical model that may be used to examine a variety of financial data, including macroeconomic data.

Related Literatures

GARCH is a statistical model that may be used to analyze various types of financial data, including macroeconomic data.

However, Goudazri and Ramanarayanan (2010) studied the volatility of the Indian stock market over ten years using the BSE 500 stock index as a proxy. ARCH and GARCH models were estimated, and the best model was chosen based on the model selection criteria Viz, Akaike Information Criterion (AIC), and Schwarz Information Criterion (SIC). The analysis discovered that GARCH (1,1) was the best model for explaining volatility clustering and mean reversion in the series over the study period.

Usman *et al.* (2017) ^[14] studied and captured the fitting of the Nigeria stock market returns series using GARCH model, the performance of eleven competing time series GARCH model for fitting the rate of returns data, and monthly observations on the market index returns series from January 1996 to December 2015. According to their findings, the log likelihood (Log L), Schwartz's Bayesian Criterion (SBC), and Akaike information criterion (AIC) values indicated that the models discovered were not the same. CGARCH (1,1) and EGARCH (1,1) were used for training, and ARCH (1) and GARCH (2,1) for testing.

Dallah and Ade (2010) ^[7] investigated the volatility of daily stock returns in Nigerian insurance stocks using daily data from 26 insurance companies from December 15, 2000 to June 9, 2008 as training data and from June 10, 2008 to September 9, 2008 as out-of-sample data. The results of ARCH (1), GARCH (1,1), TARCH (1,1), and EGARCH (1,1) reveal that EGARCH is better suited to modeling stock price returns because it beats the other models in model evaluation and out-of-sample forecasting.

Model Equations

According to Eagle R. F. (1982) ^[9], the general GARCH model in terms of (p,q) is written as

$$\delta_t^2 = \alpha_0 + \alpha_1 \varepsilon_t^2 - 1 + \alpha_2 \varepsilon_t^2 - 2 + \dots + \alpha_p \varepsilon_t^2 - p + \beta_1 \delta_t^2 - 2 + \dots + \beta_q \delta_t^2 - q \quad 3.1$$

Where

$\delta_t^2 - 1$ and $\alpha^2 - 1$ = explanatory variable and independent

α_0, α_1 and β_1 = parameters to be estimated

GARCH (1,1) Model is given by

$$\delta_t^2 = \alpha_0 + \alpha_1 \varepsilon_t^2 - 1 + \beta_1 \delta_t^2 - 1 \quad 3.2$$

Where

δ_t^2 = The variance and the dependent variable with time (t)

ε_t = Measure the effect of error term (white noise) when $0 \leq \alpha_i \leq 1$

α_0, α_1 and β_1 = Non-negative parameters to be estimated, it measures the variance of past volatility and past residual of the current volatility.

δ_t^2 = The explanatory variable and independent variable at time(t).

We must note that GRACH is a model for variance. Another proposed model called ARIMA model was adopted based on if an autoregressive moving average (ARIMA) is assumed for the error variance, the model is generalized autoregressive conditional heteroscedasticity (GARCH) model.

Hence, the general ARIMA model is written as

$$x_t = \phi_1 x_{t-1} + \phi_2 x_{t-2} \phi_3 x_{t-p} + \phi_1 \varepsilon_{t-1} + \phi_2 \varepsilon_{t-2} \phi_3 \varepsilon_{t-p} \quad 3.3$$

ARIMA (3,1,3) Model

$$x_t = \phi_1 x_{t-1} + \phi_2 x_{t-2} \phi_3 x_{t-3} + \phi_1 \varepsilon_{t-1} + \phi_2 \varepsilon_{t-2} \phi_3 \varepsilon_{t-3} \quad 3.4$$

Where

X_t = Measures the volatility in the stock exchange market capitalization

ε_t = Control the error term (white noise)

ϕ = The AR Parameters to be estimated which measure the Coefficient of the effect of variation in the past volatility

θ = The MA Parameters to be estimated which measures the coefficient of the past residual of the current volatility
 $\times_{t-2} \times_{t-3}$ & $\varepsilon_{t-1}, \varepsilon_{t-2}, \varepsilon_{t-3}$ are lag order often referred to AR and MA respectively

Analysis and Estimation

Statistical estimations were analyzed accordingly.

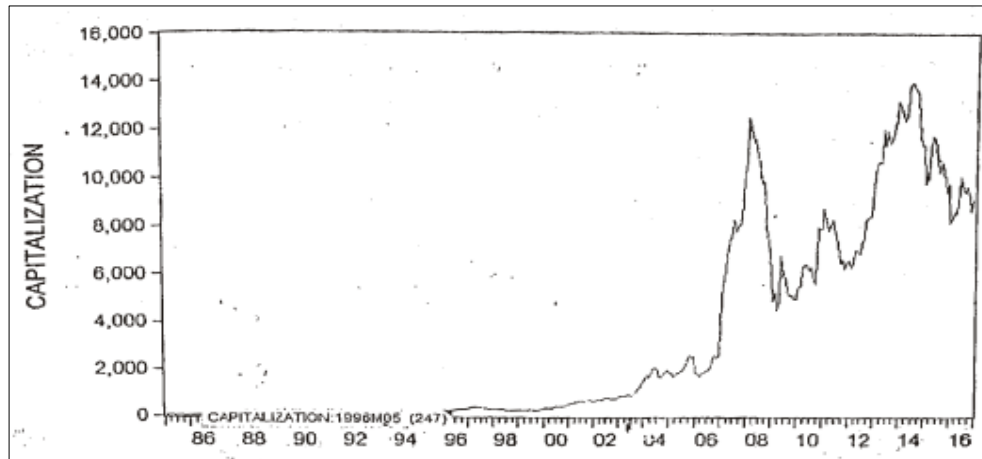


Fig 1: Time Plot of Capitalization without differencing

Time

The figure 1, this the time plot of capitalization, it identified linear pattern and the series and non-stationary. It is a time plot without differencing.

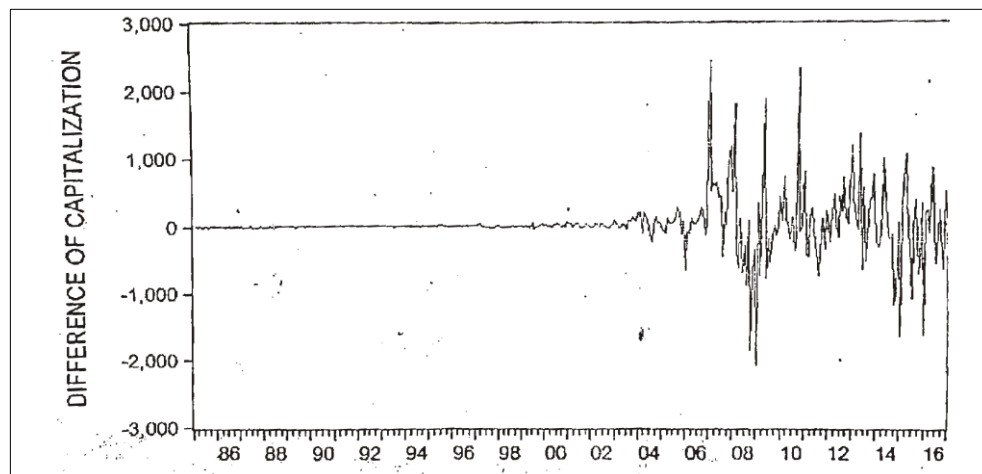


Fig 2: Volatility and heteroscedasticity observed in capitalization data.

Time

Figure 2, is the first differencing of capitalization data (DCPT), it identified heteroscedasticity and volatility. It describes irregular pattern of variation of an error term, hence the series are stationary. The series was stable over some years between 1986-2000 therefore the series are not volatile, the mean and variances are not constant through time therefore there are no stationary. Then between 2002-2016, the series identified heteroscedasticity and stationarity occurs.

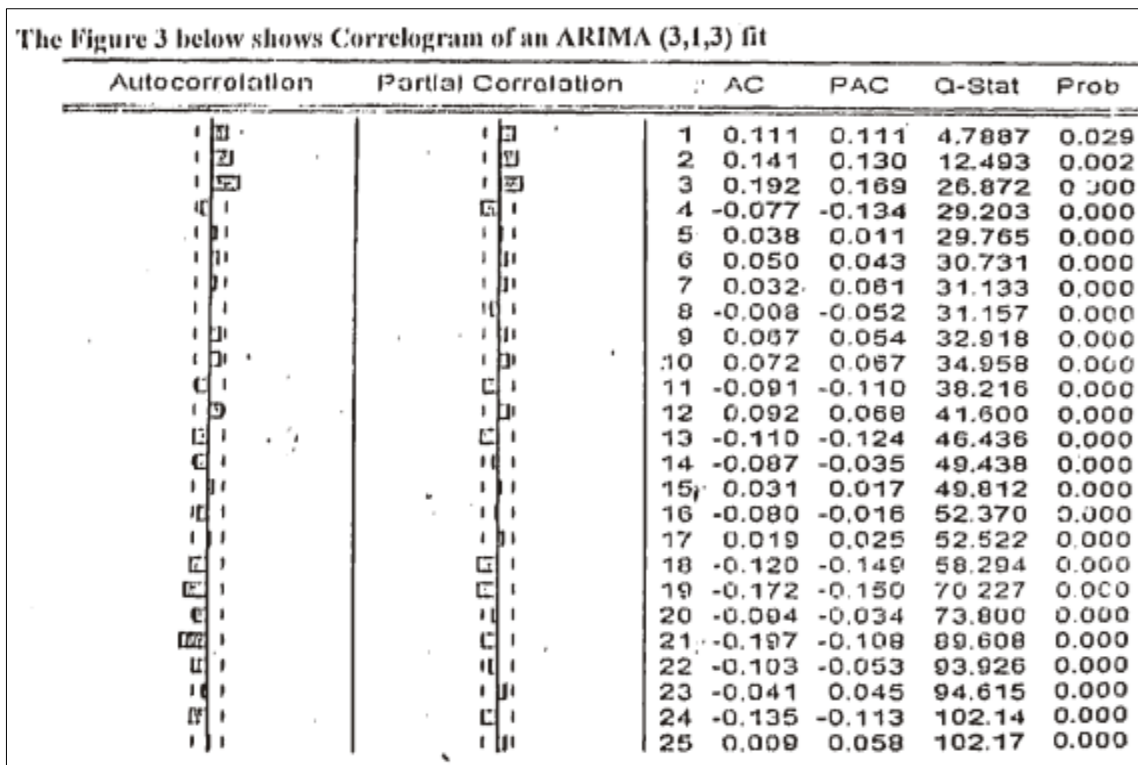
Table 2: Stationarity Test for DCPT

Null Hypothesis: DCPT has a unit root Exogenous: Constant Lng Length: 3(Automatic-based on SIC,maxlag=16)			
		t-Statistic	Prob*
Augmented Dickey-Fulle test statistic		-8.662209	0.0000
Test critical values:	1% level	-3.447395	
	5% level	-2.868948	
	10% level	-2.570783	

*Mackinnon (1996) one-sided p-values.
 Augmented Dickey-Fuller Test Equation
 Dependent Variable: D (DCPT)
 Method: Least Squares
 Date:04/27/18Time00:51
 Sample (adjusted): 1985M06 2016M12

Table 3: Included observations: 379 after adjustments

R-squared	0.478307	Mean de	pendent var	1.388127
Variable	Coefficient	Std.Error	t-Statistic	Prob.
DCPT(-1)	-0.728268	0.084074	-8.662209	0.0000
(DCPT)(-1))	-0.175290	0.079742	-2.198209	0.0285
D(DCPT(-2))	-0.044252	0.069691	-0.634964	0.5258
D(DCPT(-3))	0.137753	0.051643	2.667403	0.0080
C	17.70783	20.21475	0.875986	0.3816
Adjusted R-squared	0.472727	S.D.dependent var		539.

**Fig 3:** ACF and PACF of DCPT**Table 3:** Estimation of ARIMA (3,1,3) Model for Capitalization Dependent Variable: DCPT

Method: ARMA Maximum Likelihood (OPG-BHHH) Date:04/27/18 Time: 00:56				
Sample: 19985 M02 2016M12				
Included observations: 383				
Convergence achieved after 61 iterations				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
AR(1)	-0.221570	0.096068	-2.306389	0.0216
AR(2)	-0.381250	0.090507	-4.212397	0.0000
AR(3)	0.376163	0.110730	3.397115	0.0008
MA(1)	0.315608	0.099937	3.158068	0.0017
MA(2)	-0.570587	0.088029	6.481826	0.0000
MA(3)	-0.125556	0.125020	-1.004292	0.3159
SIGMASQ	148957.9	4330.020	34.40120	0.0000
R-squared	0.079206	Mean dependent Var	24.13081	
Adjusted R-squared	0.064513	S.D. dependent Var	402.7339	
S.E. of regression	389.5266	Akaike info criterion	14.78665	
Sum squared residue	57050857	Schwarz criterion	14.85881	
Log likelihood	-2824.644	Hannan-Quinn criter.	14.81528	
Durbin-Watson stat	1.993652			
Inverted AR Roots	50	-36.781	-36.781	
Inverted MA Roots	19	-25.781	-25.781	

Table 4: Estimation of GARCH (1,1) Model for DCPT

Dependent Variable: DCPT Method: ML ARCH-Normal distribution (BFGS/Marquardt steps) Date: 04/27/18 Time: 00:54 Sample (adjusted): 1985M02 2016M12 Included observations: 383 after adjustments Convergence achieved after 62 iterations Coefficient covariance computed using outer product of gradients Resamples variance: back cast (parameter=0.7) GARCH = C(1) + C(2)*RESID(1-1Y2 + C(3)*GRACH(-1)				
Variable	Coefficient	Std. Error	z-Statistic	Prob.
Variance Equation				
C	0.006446	0.001057	6.101525	0.0000
RESID(-1)^2	0.170195	0.023361	7.285346	0.0000
GARCH(-1)	0.924814	0.012337	74.96129	0.0000
R-squared	-0.003600	Mean dependent Var	24.13081	
Adjusted R-squared	-0.000979	S.D. dependent Var	402.7339	
S.E. of regression	402.9310	Akaike info criterion	9.900357	
Sum squared resid	62181357	Schwarz criterion	9.931282	
Log likelihood	-1892.918	Hannan-Quinn criter.	9.912624	
Durbin-Watson stat	1.766803			

Discussion of Results

A Time plot of capitalization

Dallah and Ade (2010) ^[7] investigated the volatility of daily stock returns in Nigerian insurance stocks using daily data from 26 insurance companies from December 15, 2000 to June 9, 2008 as training data and from June 10, 2008 to September 9, 2008 as out-of-sample data. The results of ARCH (1), GARCH (1,1), TARCH (1,1), and EGARCH (1,1) reveal that EGARCH is better suited to modeling stock price returns because it beats the other models in model evaluation and out-of-sample forecasting. The change has resulted in the series appearing as white noise and being normally distributed. Visual inspection and interpretation of the plot above suggest that the Nigerian stock exchange market capitalization series was effective from 1986 to 2006, when the global market crisis reached Nigeria. It is a clear indication that the market is not tied or experiencing a crisis.

Stationarity Test and Volatility Presence

Figure 2 depicts the time plot of the first difference in capitalization data (DCPT). There is evidence of heteroscedasticity. Essentially, when there is heteroscedasticity, observations tend to cluster rather than corroborate a linear trend. Heteroscedasticity refers to an irregular pattern of variation in an error term or variable in a statistical model. We required stationarity in the series, but if the series is not stationary, the essential test statistics used to evaluate the results become incorrect. The research above revealed that the mean and variance are stable and near, and that the mean, variance, and autocorrelation structure did not vary over time. We also detected upward and downward volatility as the series increased. Without a trend, stable variance over time, a consistent autocorrelation structure over time, and non-periodic fluctuations, the test statistics used to evaluate this estimation become valid. The stationarity test was carried out using the Augmented Dickey-Fuller Test (ADF) method, and the results reveal the Nigeria stock series. Only after the first Differencing of determining if the variable is stationary can the exchange market capitalization series be considered stationary.

The series was stable for a few years from 1986 until it increased slightly, and it continued at this level until about 2006, when the variance suddenly rose, and then the variance decreased from about 2013 until 2015, before increasing slightly until 2016, indicating that there is volatility clustering in the series. The main reason could be traced back to market events such as economic liberalization, enterprise promotion, policies, financial crisis, political instability, and national economic reforms such as Nigeria's Economic Empowerment Development Strategy (NEEDS), Structural Adjustment Programme (SAP), and an alarming decline in Nigerian economic activity in the Nigerian capital market.

It provides a foundation for estimating the time series model's performance. Table 2 shows the test critical values at the 1%, 5%, and 10% levels of significance.

The Augmented Dickey-Fuller (ADF) test result of -8.6622 is smaller than all critical values. Thus, the test is significant ($p=0.0000 < 0.05$). As a result, the DCPT is considered stationary.

Summary Statistics for the Estimated Series of the ARIMA Model.

ARIMA (p, d, q) model with AR (1), AR (2), AR (3) and MA (1), MA (2) and MA (3). Then the diagnostic results for the several ARIMA models estimated are presented in table 2 below. Such as;

AR (1) = 0.221570

AR (2) = 0.381250

AR (3) = 0.376163

Then the moving average (MA)

MA (1) = 0.315608

MA (2) = 0.570587

Fitting the model according to their respective coefficient, we have as follows;

$$\times_t = -0.221570 \times_{t-1} - 0.381250 \times_{t-2} + 0.376163 \times_{t-3} + 0.315608 \varepsilon_{t-1} + 0.570587 \varepsilon_{t-2}$$

The above is ARIMA (3,1,2) model the proposed model to model the Nigeria Stock Exchange Market Capitalization.

Summary Statistics for the Estimated Series for the GARCH (1,1) Model

Considering the ascertainment of the presence of volatility clustering and GARCH effect in the Nigeria stock exchange market capitalization series, the GARCH (1,1) is estimated and the diagnostic results are presented accordingly in table 4 above. Purposely to determine the most efficient model that can effectively model Nigeria stock exchange market capitalization using ARIMA (3,1,2) and GARCH (1,1) models are estimated. The essence of building volatility model for risk modelling on the Nigeria Stock Exchange Market Capitalization Series is to provide investors and policy makers information regarding the future performance of the Nigeria Stock Exchange Market Capitalization.

The GARCH (1,1) model is thought to be the most efficient and capable of accurately modeling the Nigerian stock exchange market capitalization since the test statistic is reliable and bigger than the coefficient, and the probability is zero.

The Akaike Information and Schwarz criterion are lower than those for the ARMA (3,1,2) model. Model and shows that the GARCH (1,1) model is more efficient than the ARIMA (3,1,2) model. Based on the Akaike Information and Schwarz criterion.

The GARCH (1,1) model is deemed more efficient and better to represent the volatility of the Nigerian stock exchange market capitalization series since the model has the lowest AIC and SIC criteria, and most importantly, it is homoscedastic and free from serial correlation.

The GARCH model is of the form

$$\delta \frac{2}{t} = a_0 + a_1 \varepsilon \frac{2}{t} - 1 + \beta_1 \varepsilon \frac{2}{t} - 1$$

From the analysis below the values of those parameters are

$$a_0 = c = 0.006446$$

$$a_0 = \text{Residual}(-1)^2 = 0.170195$$

$$a_0 = \text{Residual}(-1) = 0.924814$$

The proposed GARCH (1,1) model is represented as

$$\delta \frac{2}{t} = 0.006446 + 0.1701953 \varepsilon \frac{2}{t} + 0.924814 \delta \frac{2}{t-1}$$

Where,

$a_0 = C = 0.006446$, Non-negative constant

$a_1 = \text{Residual}(-1)^2 = 0.170195$, measures the effect of previous residual of Nigeria stock exchange market capitalization.

$\beta_1 = \text{GARCH}(-1) = 0.924814$, the effect of change in its lagged value of the Nigeria stock exchange market capitalization and

$\delta \frac{2}{t}$ = the dependence capitalization of variance over time

The first coefficients (a_0) the usual constant in the model, the second coefficient measures the effect of previous year residual of variance on current variance. It shows positive correlation between past residual and current variance in the stock exchange market capitalization series, the third coefficient measures the correlation between the past value and current value which found to be positive.

The implication of the statistic findings shows average risk in Nigeria stock exchange market capitalization of the current period regarding investment risk and previous investment risk in the stock exchange market in capitalization in Nigeria till date.

Conclusion and Policy Implication

Assessment of volatility and estimation in the Nigeria stock exchange market capitalization series was conducted and the presence of volatility was evident, therefore, this presented justification to proceed to modelling of the volatility in the Nigeria stock exchange market capitalization series.

Two major models, the autoregressive integrated moving average (ARIMA) model and the generalized autoregressive conditional heteroscedasticity (GARCH) model, were objectively estimated for comparison in order to develop an efficient and better model to effectively model the volatility of Nigeria's stock exchange market capitalization. Therefore, the result obtained from the statistical estimation revealed clearly that the ARIMA (3, 1, 2) model is efficient but not better in constructing a volatility to find efficient and better for the series to model the volatility of the Nigerian stock exchange market. The Akaike Information Criterion is 14.78665, while the Schwarz Criterion is 14.85881. Similarly, the Criterion is 9.900357 and the Schwarz Criterion is 9.931282.

According to the statistical estimation above, (1,1) is an efficient and better model for the series since it has lower Akaike Information, Schwarz capitalization is variable over time, and trading is not continuous (increasing and decreasing).

The policy implications of the findings in the Nigerian economy show that the stock exchange market capitalization is not stable, that the volatility in the Nigerian stock exchange market capitalization is gradually increasing, and that trading in the Nigerian stock exchange market capitalization tends to become more risky over time.

We propose that both indigenous and foreign investors develop their own investment policies based on this to direct their trading and, ultimately, improve the Nigerian economy through stock exchange market capitalization. This is a good tool for anticipating trade and returns on investments in Nigeria's stock exchange market capitalization.

As a result, GARCH (1,1), which can compensate for dynamism in variance, was discovered to be the better and more efficient model for the Nigeria stock exchange market capitalization series. Because GARCH (1,1) has a lower Akaike Information and Schwarz Criteria value, it indicates that the stock exchange market capitalization is volatile (up and down), implying that the stock market rises and falls according to economic conditions.

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