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Mathematical model for analysing mobile phone addiction using an empirical approach

Swapnil Talele and Ravi Gor

Abstract

Mobile phone addiction is becoming a serious problem as more people spend excessive time on their devices every day. This behaviour can affect health, sleep, productivity and social life. This paper presents a mathematical model to understand how mobile phone addiction spreads and persists among students. The model employs a compartmental approach, consisting of six non-linear differential equations, to analyze these dynamics. The movement between these compartments depends on factors such as peer influence, awareness and personal efforts to reduce phone use. Both the local and global stability of the model are investigated. Additionally, the next-generation matrix technique is utilized to perform an in-depth analysis based on the reproduction number R_0, which is calculated using Python. Empirical data was gathered through a structured questionnaire distributed to students across various colleges, focusing on phone usage patterns, addictive behaviours and awareness levels. Several numerical simulations are conducted to illustrate the findings, highlighting the impact of various parameters on Mobile phone dynamics and the effectiveness of intervention strategies.

Keywords: Mathematical model, reproduction number, local and global stability, local and global sensitivity analysis, Lyapunov function

Introduction

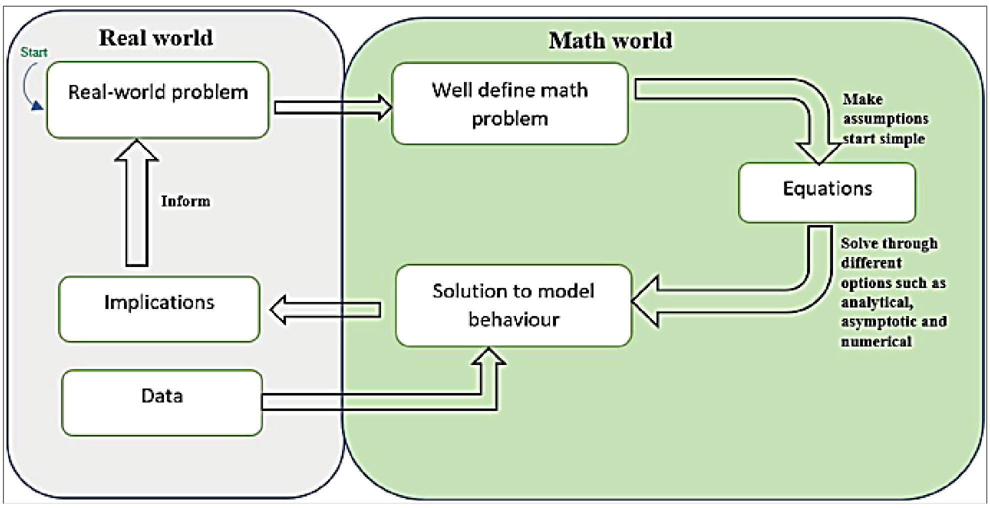

Fig 1: Scientific Process to connect real world problem with Mathematics

A mathematical model is a representation of a real-world system or phenomenon using mathematical concepts, equations and structures. It serves as a simplified abstraction of complex processes, allowing us to analyse, understand and predict their behaviour under various conditions. Mathematical models are widely used in diverse fields such as physics, biology, economics, engineering and social sciences to solve problems, make informed informed decisions and develop theories. The process of converting a real word problem into the language of mathematics is described in figure 1. [5]

Mathematical Model approach is particularly valuable in the field of epidemiology, where it is used to study the spread and control of diseases within populations. By applying mathematical models to epidemics, we can simulate disease transmission, estimate key epidemiological parameters and evaluate the potential impact of various public health interventions. There are two primary types of epidemic models: deterministic and stochastic.

A stochastic model integrates the element of randomness or uncertainty. The result of the modelling process is the making of probable predictions. Stochastic models have been especially important in studying the systems affected by random processes or uncertainties, including stock market fluctuations and weather patterns. Deterministic model depends on fixed initial conditions and parameters to predict the outcome with certainty. These models usually divide a system into various compartments, each representing a specific state or category. The transitions between compartments follow predefined rules or equations and do not leave room for randomness. [5]

Mathematical modelling is playing an important role in spread and control of many addictions including mobile phone addiction. Mobile phones have become an essential part of daily life due to their mobility, accessibility and convenience. However, excessive use has led to concerns about mobile phone addiction, also known as problematic smartphone use. This issue is particularly prevalent among adolescents, who are more vulnerable to its effects.

Problematic smartphone use includes excessive screen time, preoccupation with mobile communication and using phones in inappropriate situations, such as while driving. Studies show that individuals aged 3 to 11 are at the highest risk, spending an average of 9–12 hours daily on smartphones. Research also indicates that: 70% check their phones within an hour of waking up, 56% check their phones before going to bed, 48% frequently use their phones over the weekend, 51% constantly check their phones during vacations, 44% feel anxious and irritable if unable to access their phones for a week.

Excessive mobile phone use has been linked to negative effects on mental and physical health as well as strained social relationships. Many people use online communication as a substitute for face-to-face interactions. In response to health concerns, the World Health Organization (WHO) has recommended limiting screen time for children under five to one hour per day, while infants under two should have no screen exposure. Additionally, 58% of WHO member countries advise reducing radio frequency exposure by using hands-free kits, limiting call duration and avoiding calls in low-signal areas. Taiwan banned the use of mobile devices for children under two years old in 2015 and France banned Wi-Fi in nurseries.

Psychological studies suggest a link between smartphone overuse and issues like depression, anxiety and social isolation. Support groups and therapies, such as cognitive behavioural therapy (CBT), motivational interviewing and family therapy, have been found to help in managing mobile phone addiction. Other behavioural interventions include setting screen time limits, practicing alternative activities (exercise, reading or art) and gradually reducing app usage.

Addressing this issue requires increased awareness, behavioural changes and responsible digital consumption. While mobile phones provide numerous benefits, mindful usage is essential to prevent addiction and its negative consequences.

The goal of this paper is to create a mathematical model to study mobile phone addiction among college students. By identifying important factors like social influence, academic stress and screen time, the study aims to understand how addiction spreads and how severe it can become. The model will also help predict future trends and test different ways to reduce addiction, such as awareness programs and screen time limits.

2. Literature Review

Li, T., & Guo, Y. (2019) [13] constructed an online game addiction model incorporating four compartments: susceptible, infective, professional and quitting individuals. They analysed equilibrium properties using the basic reproduction number (R_0) and examined stability conditions for different equilibria. The study applied Pontryagin's maximum principle to determine optimal control strategies and conducted numerical simulations to validate the analytical findings.

Seno, H. (2021) [15] explored Internet Gaming Disorder (IGD) in the context of increasing internet users, modelling population dynamics through a system of ordinary differential equations. The study categorized gamers into three stages: moderate, addictive and under treatment, emphasizing the role of social interactions in transitioning to addiction. The findings highlighted the difficulty of self-recovery due to social reinforcement and stressed the need for early intervention to prevent IGD from escalating beyond socially controllable levels.

Alemneh, H. T., & Alemu, N. Y. (2021) [2] developed a deterministic mathematical model for social media addiction (SMA) and analysed equilibrium points, stability and the basic reproduction number (R_0). They found that SMA can be controlled if $R_0 < 1$, while a unique endemic equilibrium exists if $R_0 > 1$. The study also introduced optimal control strategies, applying Pontryagin's maximum principle and numerical simulations via the fourth-order Runge-Kutta method.

Saman, A., *et al.* (2022) [14] applied the SEIR model to predict online game addiction among students, determining equilibrium points and stability. They found that before intervention, R_0 was 0.2221, which decreased to 0.1342 after applying optimal control through guidance and counselling. The results demonstrated that structured interventions can significantly reduce addiction, highlighting the importance of guidance programs in education.

Guo, Y., & Li, T. (2022) [7] developed a mathematical model of online game addiction incorporating incomplete recovery and relapse. They analysed the model's basic properties, derived the expression for the basic reproduction number (R_0) and determined all equilibrium points. Using Lyapunov functions, they proved the global asymptotic stability of these equilibria. Numerical simulations were conducted using the least squares estimation method to fit real data on e-sports users in China from 2010 to 2020, allowing for accurate parameter estimation.

Juhari, J., *et al.* (2024) [10] developed a dynamic model for social media addiction by dividing addiction into mild and severe stages, with six compartments. Stability analysis and numerical simulations showed that at year 4, mildly addicted individuals reached 1.6112, while severely addicted individuals reached 36.542. The model demonstrated stability and highlighted the importance of stage-specific interventions.

3. Mathematical Model
Here, we formulate a model based on the SIRS (Susceptible (S), Infected (I), Recovered (R)) model of mobile phone usage behaviour. The entire population is divided into six compartments based on addiction status, which are referred to as state variables. The compartments are denoted as, Susceptible population (S), Engaged population (E), Addicted population (A), Aware population (W), Interventions population (I), Recovered population (R).

- Let us assume homogeneous population mixing, i.e., each individual can contact any other individual.
- The transitions between the different subpopulations are determined as follows:

a. Newly recruited individuals enter the susceptible subpopulation $S(t)$, meaning they have not yet actively engaged in excessive mobile phone use. A fraction of these individuals will transition to the engaged class $E(t)$ due to peer influence, advertisements, or social trends.

b. From the engaged class $E(t)$, individuals may either move to the addicted class $A(t)$ if their mobile phone usage increases excessively or transition to the aware class $W(t)$ if they recognize the negative effects of mobile phone addiction.

c. From the addicted class $A(t)$, individuals can either move to the aware class $W(t)$ if they become conscious of the negative effects of mobile phone addiction or move to the recovered class $R(t)$ if they are mentally strong enough to reduce their phone usage or transition to external intervention $I(t)$ if they struggle to quit on their own and require support such as parental control, awareness programs, or platform restrictions.

d. If individuals in the aware class $W(t)$ actively reduce their phone usage, they move to the recovered class $R(t)$. Similarly, those receiving external intervention efforts may also transition to the recovered class $R(t)$ if the intervention is effective.

e. Eventually, individuals in the recovered class $R(t)$ who successfully reduce their mobile phone usage may relapse and return to the susceptible subpopulation $S(t)$.

The notations and parametric values used in this model are given in the following Table 1.

Table 1: Variables and its descriptions

Variables/ Parameters	Description
$S(t)$	individuals who are influenced by mobile phones but have not started using them
$E(t)$	Individuals who frequently use mobile phones but do not show addiction symptoms
$A(t)$	Individuals addicted to mobile phone usage
$W(t)$	Individuals aware of the negative effects of mobile phone addiction and actively reducing their usage
$I(t)$	External influences (e.g., parental control, awareness campaigns, platform restrictions) aimed at reducing addiction
$R(t)$	Individuals who have completely stopped using mobile phones or those who now use them in a controlled manner
$N(t)$	Total Population
B	Recruitment rate
μ	Natural death rate.
α	The rate at which susceptible individuals become engaged
β	The rate at which engaged individuals become aware
γ	The rate at which engaged individuals become addicted
η	The rate at which addicted individuals become aware
δ	The rate at which addicted individuals receive external intervention efforts
σ	The rate at which addicted individuals recovered
ρ	The rate at which individuals receiving external intervention efforts successfully recover
λ	The rate at which aware individuals recovered
τ	The rate at which recovered individuals relapse and become susceptible to mobile phone addiction again

The mathematical model, formulated and depicted in Figure 2, utilizes the notations specified in Table 1 to represent the dynamics of mobile phone usage behaviour.

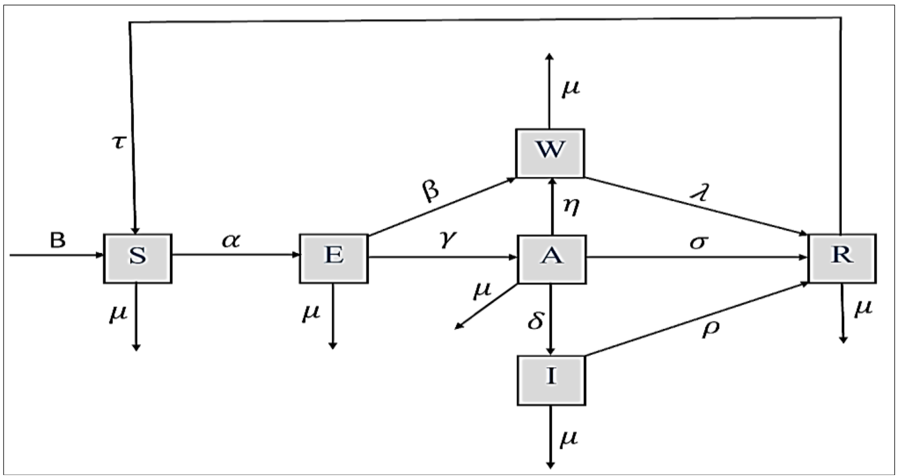

Fig 2: Mathematical Model

By taking the above assumptions into account and the transitions of how people move among classes, we can construct a system of differential equations representing the model of the evolution of mobile phone behaviour as follows:

$$\frac{dS(t)}{dt} = B - \frac{\alpha S(t)E(t)}{N} - \mu S(t) + \tau R(t)$$

$$\frac{dE(t)}{dt} = \frac{\alpha S(t)E(t)}{N} - \beta E(t) - \gamma E(t) - \mu E(t)$$

$$\frac{dA(t)}{dt} = \gamma E(t) - \eta A(t) - \sigma A(t) - \delta A(t) - \mu A(t)$$

$$\frac{dW(t)}{dt} = \eta A(t) + \beta E(t) - \lambda W(t) - \mu W(t)$$

$$\frac{dI(t)}{dt} = \delta A(t) - \rho I(t) - \mu I(t)$$

$$\frac{dR(t)}{dt} = \lambda W(t) + \sigma A(t) + \rho I(t) - \tau R(t) - \mu R(t)$$

With

$$S(t) + E(t) + A(t) + W(t) + I(t) + R(t) = N(t)$$

4. Basic Properties

4.1 Invariant Region

It is necessary to prove that all solutions of system (1) with positive initial data will remain positive for all times $t > 0$. This will be established by the following lemma.

Lemma 1: All feasible solution $S(t), E(t), A(t), W(t), I(t), R(t)$ of system equation (1) are bounded by the region

$$A = \left\{(S, E, A, W, I, R) \in \mathbb{R}^6 : S + E + A + W + I + R \leq \frac{B}{\mu}\right\}$$

Proof. From the system equation (1)

$$\frac{dN(t)}{dt} = \frac{dS(t)}{dt} + \frac{dE(t)}{dt} + \frac{dA(t)}{dt} + \frac{dW(t)}{dt} + \frac{dI(t)}{dt} + \frac{dR(t)}{dt}$$

$$\frac{dN(t)}{dt} = B - \mu\big(S(t) + E(t) + A(t) + W(t) + I(t) + R(t)\big)$$

implies that

$$\frac{dN(t)}{dt} \leq B - \mu N(t)$$

It follows that

$$N(t) \leq \frac{B}{\mu} + N(0)e^{-\mu t}$$

Where $N(0)$ is the initial value of total number of people, thus $\lim_{t\to\infty} \sup N(t) \leq \frac{B}{\mu}$

Then $S(t) + E(t) + A(t) + W(t) + I(t) + R(t) \leq \frac{B}{\mu}$

Hence, for the analysis of model (1), we get the region which is given by the set:

$$A = \left\{(S, E, A, W, I, R) \in \mathbb{R}^6 : S + E + A + W + I + R \leq \frac{B}{\mu}\right\}$$

which is a positively invariant set for (1), so we only need to consider the dynamics of system on the set A, the non-negative sets of solutions.

4.2 Positivity of the solutions of the model

Lemma 2: If $S(0) \geq 0, E(0) \geq 0, A(0) \geq 0, W(0) \geq 0, I(0) \geq 0$ and $R(0) \geq 0$ then the solution of system (1)

$S(t), E(t), A(t), W(t), I(t)$ and $R(t)$ are positive for all $t > 0$.

Proof. From the system equation (1)

$$\frac{dS(t)}{dt} = B - \frac{\alpha S(t)E(t)}{N} - \mu S(t) + \tau R(t)$$

To seeks positivity, we can write

$$\frac{dS(t)}{dt} \geq B - \mu S(t)$$

$$\Rightarrow \frac{dS(t)}{dt} + \mu S(t) \geq B$$

The integrating factor of above equation is given by

$$I.F. = e^{\int \mu dt} = e^{\mu t}$$

Multiplying $e^{\mu t}$ on both side of the equation, we get

$$\frac{d}{dt}\left(e^{\mu t} S(t)\right) \geq B e^{\mu t}$$

Now, by integrating above equation, we have

$$S(t) \geq \frac{B}{\mu} + c e^{-\mu t}$$

where c is an integrating constant.
Considering the initial value at $t = 0, S(t) \geq S(0)$

$$S(0) \geq \frac{B}{\mu} + c \Rightarrow S(0) - \frac{B}{\mu} \geq c$$

Substituting the value of c into above equation, we obtain

$$S(t) \geq \frac{B}{\mu} + \left(S(0) - \frac{B}{\mu}\right) e^{-\mu t}$$

So, at $t = 0$ and $t \longrightarrow \infty, S(t) \geq 0$. By repeating the above procedure, we can prove the positivity of all other state variables. Consequently, it is clear that $\forall\ t \geq 0$.

$$S(t) \geq 0, E(t) \geq 0, A(t) \geq 0, W(t) \geq 0, I(t) \geq 0, R(t) \geq 0$$

5. Equilibria and their stability analysis

5.1 Equilibrium points and reproduction number (R_0)

Now to find equilibrium points of mathematical model, put right hand side equals to zero from equations given in system (1). In this paper python software is used. This analysis helps in understanding the long-term behaviour of mobile phone dynamics under different conditions and interventions.

The mobile phone-free equilibrium $E^0\left(\frac{B}{\mu}, 0,0,0,0,0\right)$ is achieved when there is no active mobile phone user in the population $(E = A = W = I = R = 0)$. The mobile phone present equilibrium $E^*(S^*, E^*, A^*, W^*, I^*, R^*)$ is achieved when mobile phone user exists.

$$S^* = \frac{N(\beta + \gamma + \mu)}{\alpha}$$

$$E^{*}=\frac{(\lambda+\mu)(\mu+\rho)(\mu+\tau)(\delta+\eta+\mu+\sigma)(B\alpha-N\beta\mu-N\gamma\mu-N\mu^{2})}{\begin{gathered}\alpha\mu(\beta\delta\lambda\mu+\beta\delta\lambda\rho+\beta\delta\mu^{2}+\beta\delta\mu\rho+\beta\delta\mu\tau+\beta\delta\rho\tau+\beta\eta\lambda\mu+\beta\eta\lambda\rho+\beta\eta\mu^{2}+\beta\eta\mu\rho+\beta\eta\mu\tau+\\ \beta\eta\rho\tau+\beta\lambda\mu^{2}+\beta\lambda\mu\rho+\beta\lambda\mu\sigma+\beta\lambda\rho\sigma+\beta\mu^{3}+\beta\mu^{2}\rho+\beta\mu^{2}\sigma+\beta\mu^{2}\tau+\beta\mu\rho\sigma+\beta\mu\rho\tau+\beta\mu\sigma\tau\\ +\beta\rho\sigma\tau+\delta\gamma\lambda\mu+\delta\gamma\lambda\rho+\delta\gamma\lambda\tau+\delta\gamma\mu^{2}+\delta\gamma\mu\rho+\delta\gamma\mu\tau+\delta\lambda\mu^{2}+\delta\lambda\mu\rho+\delta\lambda\mu\tau+\delta\lambda\rho\tau+\delta\mu^{3}\\ +\delta\mu^{2}\rho+\delta\mu^{2}\tau+\delta\mu\rho\tau+\eta\gamma\lambda\mu+\eta\gamma\lambda\rho+\eta\gamma\mu^{2}+\eta\gamma\mu\rho+\eta\gamma\mu\tau+\eta\gamma\rho\tau+\eta\lambda\mu^{2}+\eta\lambda\mu\rho+\eta\lambda\mu\tau\\ +\eta\lambda\rho\tau+\eta\mu^{3}+\eta\mu^{2}\rho+\eta\mu^{2}\tau+\eta\mu\rho\tau+\gamma\lambda\mu^{2}+\gamma\lambda\mu\rho+\gamma\lambda\mu\sigma+\gamma\lambda\mu\tau+\gamma\lambda\rho\sigma+\gamma\lambda\rho\tau+\gamma\mu^{3}\\ +\gamma\mu^{2}\rho+\gamma\mu^{2}\sigma+\gamma\mu^{2}\tau+\gamma\mu\rho\sigma+\gamma\mu\rho\tau+\lambda\mu^{3}+\lambda\mu^{2}\rho+\lambda\mu^{2}\sigma+\lambda\mu^{2}\tau+\lambda\mu\rho\sigma+\lambda\mu\rho\tau+\lambda\mu\sigma\tau\\ +\lambda\rho\sigma\tau+\mu^{4}+\mu^{3}\rho+\mu^{3}\sigma+\mu^{3}\tau+\mu^{2}\rho\sigma+\mu^{2}\rho\tau+\mu^{2}\sigma\tau+\mu\rho\sigma\tau)\end{gathered}}$$

$$A^{*}=\frac{\gamma(\lambda+\mu)(\mu+\rho)(\mu+\tau)(B\alpha-N\beta\mu-N\gamma\mu-N\mu^{2})}{\begin{gathered}\alpha\mu(\beta\delta\lambda\mu+\beta\delta\lambda\rho+\beta\delta\mu^{2}+\beta\delta\mu\rho+\beta\delta\mu\tau+\beta\delta\rho\tau+\beta\eta\lambda\mu+\beta\eta\lambda\rho+\beta\eta\mu^{2}+\beta\eta\mu\rho+\beta\eta\mu\tau+\\ \beta\eta\rho\tau+\beta\lambda\mu^{2}+\beta\lambda\mu\rho+\beta\lambda\mu\sigma+\beta\lambda\rho\sigma+\beta\mu^{3}+\beta\mu^{2}\rho+\beta\mu^{2}\sigma+\beta\mu^{2}\tau+\beta\mu\rho\sigma+\beta\mu\rho\tau+\beta\mu\sigma\tau\\ +\beta\rho\sigma\tau+\delta\gamma\lambda\mu+\delta\gamma\lambda\rho+\delta\gamma\lambda\tau+\delta\gamma\mu^{2}+\delta\gamma\mu\rho+\delta\gamma\mu\tau+\delta\lambda\mu^{2}+\delta\lambda\mu\rho+\delta\lambda\mu\tau+\delta\lambda\rho\tau+\delta\mu^{3}\\ +\delta\mu^{2}\rho+\delta\mu^{2}\tau+\delta\mu\rho\tau+\eta\gamma\lambda\mu+\eta\gamma\lambda\rho+\eta\gamma\mu^{2}+\eta\gamma\mu\rho+\eta\gamma\mu\tau+\eta\gamma\rho\tau+\eta\lambda\mu^{2}+\eta\lambda\mu\rho+\eta\lambda\mu\tau\\ +\eta\lambda\rho\tau+\eta\mu^{3}+\eta\mu^{2}\rho+\eta\mu^{2}\tau+\eta\mu\rho\tau+\gamma\lambda\mu^{2}+\gamma\lambda\mu\rho+\gamma\lambda\mu\sigma+\gamma\lambda\mu\tau+\gamma\lambda\rho\sigma+\gamma\lambda\rho\tau+\gamma\mu^{3}\\ +\gamma\mu^{2}\rho+\gamma\mu^{2}\sigma+\gamma\mu^{2}\tau+\gamma\mu\rho\sigma+\gamma\mu\rho\tau+\lambda\mu^{3}+\lambda\mu^{2}\rho+\lambda\mu^{2}\sigma+\lambda\mu^{2}\tau+\lambda\mu\rho\sigma+\lambda\mu\rho\tau+\lambda\mu\sigma\tau\\ +\lambda\rho\sigma\tau+\mu^{4}+\mu^{3}\rho+\mu^{3}\sigma+\mu^{3}\tau+\mu^{2}\rho\sigma+\mu^{2}\rho\tau+\mu^{2}\sigma\tau+\mu\rho\sigma\tau)\end{gathered}}$$

$$W^{*}=\frac{(\mu+\rho)(\mu+\tau)(B\alpha-N\beta\mu-N\gamma\mu-N\mu^{2})(\beta\delta+\beta\eta+\beta\mu+\beta\sigma+\eta\gamma)}{\begin{gathered}\alpha\mu(\beta\delta\lambda\mu+\beta\delta\lambda\rho+\beta\delta\mu^{2}+\beta\delta\mu\rho+\beta\delta\mu\tau+\beta\delta\rho\tau+\beta\eta\lambda\mu+\beta\eta\lambda\rho+\beta\eta\mu^{2}+\beta\eta\mu\rho+\beta\eta\mu\tau+\\ \beta\eta\rho\tau+\beta\lambda\mu^{2}+\beta\lambda\mu\rho+\beta\lambda\mu\sigma+\beta\lambda\rho\sigma+\beta\mu^{3}+\beta\mu^{2}\rho+\beta\mu^{2}\sigma+\beta\mu^{2}\tau+\beta\mu\rho\sigma+\beta\mu\rho\tau+\beta\mu\sigma\tau\\ +\beta\rho\sigma\tau+\delta\gamma\lambda\mu+\delta\gamma\lambda\rho+\delta\gamma\lambda\tau+\delta\gamma\mu^{2}+\delta\gamma\mu\rho+\delta\gamma\mu\tau+\delta\lambda\mu^{2}+\delta\lambda\mu\rho+\delta\lambda\mu\tau+\delta\lambda\rho\tau+\delta\mu^{3}\\ +\delta\mu^{2}\rho+\delta\mu^{2}\tau+\delta\mu\rho\tau+\eta\gamma\lambda\mu+\eta\gamma\lambda\rho+\eta\gamma\mu^{2}+\eta\gamma\mu\rho+\eta\gamma\mu\tau+\eta\gamma\rho\tau+\eta\lambda\mu^{2}+\eta\lambda\mu\rho+\eta\lambda\mu\tau\\ +\eta\lambda\rho\tau+\eta\mu^{3}+\eta\mu^{2}\rho+\eta\mu^{2}\tau+\eta\mu\rho\tau+\gamma\lambda\mu^{2}+\gamma\lambda\mu\rho+\gamma\lambda\mu\sigma+\gamma\lambda\mu\tau+\gamma\lambda\rho\sigma+\gamma\lambda\rho\tau+\gamma\mu^{3}\\ +\gamma\mu^{2}\rho+\gamma\mu^{2}\sigma+\gamma\mu^{2}\tau+\gamma\mu\rho\sigma+\gamma\mu\rho\tau+\lambda\mu^{3}+\lambda\mu^{2}\rho+\lambda\mu^{2}\sigma+\lambda\mu^{2}\tau+\lambda\mu\rho\sigma+\lambda\mu\rho\tau+\lambda\mu\sigma\tau\\ +\lambda\rho\sigma\tau+\mu^{4}+\mu^{3}\rho+\mu^{3}\sigma+\mu^{3}\tau+\mu^{2}\rho\sigma+\mu^{2}\rho\tau+\mu^{2}\sigma\tau+\mu\rho\sigma\tau)\end{gathered}}$$

$$I^{*}=\frac{\delta\gamma(\lambda+\mu)(\mu+\tau)(B\alpha-N\beta\mu-N\gamma\mu-N\mu^{2})}{\begin{gathered}\alpha\mu(\beta\delta\lambda\mu+\beta\delta\lambda\rho+\beta\delta\mu^{2}+\beta\delta\mu\rho+\beta\delta\mu\tau+\beta\delta\rho\tau+\beta\eta\lambda\mu+\beta\eta\lambda\rho+\beta\eta\mu^{2}+\beta\eta\mu\rho+\beta\eta\mu\tau+\\ \beta\eta\rho\tau+\beta\lambda\mu^{2}+\beta\lambda\mu\rho+\beta\lambda\mu\sigma+\beta\lambda\rho\sigma+\beta\mu^{3}+\beta\mu^{2}\rho+\beta\mu^{2}\sigma+\beta\mu^{2}\tau+\beta\mu\rho\sigma+\beta\mu\rho\tau+\beta\mu\sigma\tau\\ +\beta\rho\sigma\tau+\delta\gamma\lambda\mu+\delta\gamma\lambda\rho+\delta\gamma\lambda\tau+\delta\gamma\mu^{2}+\delta\gamma\mu\rho+\delta\gamma\mu\tau+\delta\lambda\mu^{2}+\delta\lambda\mu\rho+\delta\lambda\mu\tau+\delta\lambda\rho\tau+\delta\mu^{3}\\ +\delta\mu^{2}\rho+\delta\mu^{2}\tau+\delta\mu\rho\tau+\eta\gamma\lambda\mu+\eta\gamma\lambda\rho+\eta\gamma\mu^{2}+\eta\gamma\mu\rho+\eta\gamma\mu\tau+\eta\gamma\rho\tau+\eta\lambda\mu^{2}+\eta\lambda\mu\rho+\eta\lambda\mu\tau\\ +\eta\lambda\rho\tau+\eta\mu^{3}+\eta\mu^{2}\rho+\eta\mu^{2}\tau+\eta\mu\rho\tau+\gamma\lambda\mu^{2}+\gamma\lambda\mu\rho+\gamma\lambda\mu\sigma+\gamma\lambda\mu\tau+\gamma\lambda\rho\sigma+\gamma\lambda\rho\tau+\gamma\mu^{3}\\ +\gamma\mu^{2}\rho+\gamma\mu^{2}\sigma+\gamma\mu^{2}\tau+\gamma\mu\rho\sigma+\gamma\mu\rho\tau+\lambda\mu^{3}+\lambda\mu^{2}\rho+\lambda\mu^{2}\sigma+\lambda\mu^{2}\tau+\lambda\mu\rho\sigma+\lambda\mu\rho\tau+\lambda\mu\sigma\tau\\ +\lambda\rho\sigma\tau+\mu^{4}+\mu^{3}\rho+\mu^{3}\sigma+\mu^{3}\tau+\mu^{2}\rho\sigma+\mu^{2}\rho\tau+\mu^{2}\sigma\tau+\mu\rho\sigma\tau)\end{gathered}}$$

$$R^{*}=\frac{\begin{gathered}(B\alpha-N\beta\mu-N\gamma\mu-N\mu^{2})(\beta\delta\lambda\mu+\beta\delta\lambda\rho+\beta\eta\lambda\mu+\beta\eta\lambda\rho+\beta\lambda\mu^{2}+\beta\lambda\mu\rho+\beta\lambda\mu\sigma+\beta\lambda\rho\sigma+\\ \delta\gamma\lambda\rho+\delta\gamma\mu\rho+\eta\gamma\lambda\mu+\eta\gamma\lambda\rho+\gamma\lambda\mu\sigma+\gamma\lambda\rho\sigma+\gamma\mu^{2}\sigma+\gamma\mu\rho\sigma)\end{gathered}}{\begin{gathered}\alpha\mu(\beta\delta\lambda\mu+\beta\delta\lambda\rho+\beta\delta\mu^{2}+\beta\delta\mu\rho+\beta\delta\mu\tau+\beta\delta\rho\tau+\beta\eta\lambda\mu+\beta\eta\lambda\rho+\beta\eta\mu^{2}+\beta\eta\mu\rho+\beta\eta\mu\tau+\\ \beta\eta\rho\tau+\beta\lambda\mu^{2}+\beta\lambda\mu\rho+\beta\lambda\mu\sigma+\beta\lambda\rho\sigma+\beta\mu^{3}+\beta\mu^{2}\rho+\beta\mu^{2}\sigma+\beta\mu^{2}\tau+\beta\mu\rho\sigma+\beta\mu\rho\tau+\beta\mu\sigma\tau\\ +\beta\rho\sigma\tau+\delta\gamma\lambda\mu+\delta\gamma\lambda\rho+\delta\gamma\lambda\tau+\delta\gamma\mu^{2}+\delta\gamma\mu\rho+\delta\gamma\mu\tau+\delta\lambda\mu^{2}+\delta\lambda\mu\rho+\delta\lambda\mu\tau+\delta\lambda\rho\tau+\delta\mu^{3}\\ +\delta\mu^{2}\rho+\delta\mu^{2}\tau+\delta\mu\rho\tau+\eta\gamma\lambda\mu+\eta\gamma\lambda\rho+\eta\gamma\mu^{2}+\eta\gamma\mu\rho+\eta\gamma\mu\tau+\eta\gamma\rho\tau+\eta\lambda\mu^{2}+\eta\lambda\mu\rho+\eta\lambda\mu\tau\\ +\eta\lambda\rho\tau+\eta\mu^{3}+\eta\mu^{2}\rho+\eta\mu^{2}\tau+\eta\mu\rho\tau+\gamma\lambda\mu^{2}+\gamma\lambda\mu\rho+\gamma\lambda\mu\sigma+\gamma\lambda\mu\tau+\gamma\lambda\rho\sigma+\gamma\lambda\rho\tau+\gamma\mu^{3}\\ +\gamma\mu^{2}\rho+\gamma\mu^{2}\sigma+\gamma\mu^{2}\tau+\gamma\mu\rho\sigma+\gamma\mu\rho\tau+\lambda\mu^{3}+\lambda\mu^{2}\rho+\lambda\mu^{2}\sigma+\lambda\mu^{2}\tau+\lambda\mu\rho\sigma+\lambda\mu\rho\tau+\lambda\mu\sigma\tau\\ +\lambda\rho\sigma\tau+\mu^{4}+\mu^{3}\rho+\mu^{3}\sigma+\mu^{3}\tau+\mu^{2}\rho\sigma+\mu^{2}\rho\tau+\mu^{2}\sigma\tau+\mu\rho\sigma\tau)\end{gathered}}$$

To determine the mobile phone generation number R_0 using the next generation matrix method, we first identify the addicted compartment in our model. The matrices F and V are constructed based on the rates of transition between different mobile phone states. Let,

$$F=\begin{bmatrix}\frac{\alpha\beta}{N\mu} & 0\\ \gamma & 0\end{bmatrix} \text{ and } V=\begin{bmatrix}\beta+\gamma+\mu & 0\\ 0 & \eta+\sigma+\delta+\mu\end{bmatrix}$$

Thus, the next generation matrix is

$$FV^{-1}=\begin{bmatrix}\frac{\alpha B}{N\mu(\beta+\gamma+\mu)} & 0\\ \frac{\gamma}{\eta+\sigma+\delta+\mu} & 0\end{bmatrix}$$

The mobile phone generation number R_0 is found by calculating the spectral radius $\rho(FV^{-1})$, which simplifies to:

$$R_0 = \frac{\alpha B}{N\mu(\beta+\gamma+\mu)}$$

This R_0 value represents the average number of new mobile phone users that one mobile phone user would generate in a fully susceptible population. It serves as a crucial metric for understanding the potential spread and persistence of mobile phone usage behaviors within the community, influenced by various transition and treatment rates.

5.2 Local Stability Analysis

5.2.1 Local Stability at E^0

Evaluating the Jacobian matrix of system (1) at E^0 gives

$$J(E^0) = \begin{bmatrix} -\mu & -\frac{\alpha B}{N\mu} & 0 & 0 & 0 & \tau \\ 0 & \frac{\alpha B}{N\mu}-\beta-\gamma-\mu & 0 & 0 & 0 & 0 \\ 0 & \gamma & -\delta-\eta-\mu-\sigma & 0 & 0 & 0 \\ 0 & \beta & \eta & -\lambda-\mu & 0 & 0 \\ 0 & 0 & \delta & 0 & -\mu-\rho & 0 \\ 0 & 0 & \sigma & \lambda & \rho & -\mu-\tau \end{bmatrix}$$

The eigen values are given by

$$\lambda_1 = -\mu, \lambda_2 = \frac{B\alpha}{N\mu}-\beta-\gamma-\mu, \lambda_3 = -\delta-\eta-\sigma-\mu, \lambda_4 = -\lambda-\mu, \lambda_5 = -\mu-\rho, \lambda_6 = -\mu-\tau$$

Hence E^0 is locally asymptotically stable if $R_0 < 1$. For $R_0 = 1$, if $\lambda_i < 0$ for $i = 1,3,4,5,6$ and $\lambda_2 = 0$, E^0 is locally stable. If $R_0 > 1$, the characteristic equation has a real positive eigenvalue, and therefore E^0 is unstable.

5.2.2 Local Stability at E^*

Evaluating the Jacobian matrix of system (1) at E^*gives

$$J(E^*) = \begin{bmatrix} -\frac{\alpha E^*}{N}-\mu & -\frac{\alpha S^*}{N} & 0 & 0 & 0 & \tau \\ \frac{\alpha E^*}{N} & \frac{\alpha S^*}{N}-\beta-\gamma-\mu & 0 & 0 & 0 & 0 \\ 0 & \gamma & -\delta-\eta-\mu-\sigma & 0 & 0 & 0 \\ 0 & \beta & \eta & -\lambda-\mu & 0 & 0 \\ 0 & 0 & \delta & 0 & -\mu-\rho & 0 \\ 0 & 0 & \sigma & \lambda & \rho & -\mu-\tau \end{bmatrix}$$

$$J(E^*) = \begin{bmatrix} b_{11} & b_{12} & 0 & 0 & 0 & b_{16} \\ b_{21} & b_{22} & 0 & 0 & 0 & 0 \\ 0 & b_{32} & b_{33} & 0 & 0 & 0 \\ 0 & b_{42} & b_{43} & b_{44} & 0 & 0 \\ 0 & 0 & b_{53} & 0 & b_{55} & 0 \\ 0 & 0 & b_{63} & b_{64} & b_{65} & b_{66} \end{bmatrix}$$

where,

$$b_{11} = \frac{-\alpha E^*}{N}-\mu,\ b_{12} = \frac{-\alpha S^*}{N}, b_{16} = \tau, b_{21} = \frac{\alpha E^*}{N},\ b_{22} = -\beta-\gamma-\mu+\frac{\alpha S^*}{N}, b_{32} = \gamma$$

$$b_{33} = -\delta-\eta-\mu-\sigma, b_{42} = \beta, b_4 = \eta, b_{44} = -\lambda-\mu, b_{53} = \delta, b_{55} = -\mu-\rho, b_{63} = \sigma$$

$$b_{64} = \lambda, b_{65} = \rho, b_{66} = -\mu-\tau$$

The characteristic Polynomial of E^* is given by

$$\lambda^6 + B_1\lambda^5 + B_2\lambda^4 + B_3\lambda^3 + B_4\lambda^2 + B_5\lambda + B_6 = 0$$

where,

$$B_1 = -(b_{11} + b_{22} + b_{33} + b_{44} + b_{55} + b_{66})$$

$B_2 = b_{11}b_{22} + b_{11}b_{33} + b_{11}b_{44} + b_{11}b_{55} + b_{11}b_{66} - b_{12}b_{21} + b_{22}b_{33} + b_{22}b_{44} + b_{22}b_{55} + b_{22}b_{66} + b_{33}b_{44} + b_{33}b_{55} + b_{33}b_{66} + b_{44}b_{55} + b_{44}b_{66} + b_{55}b_{66}$

$B_3 = -b_{11}b_{22}b_{33} - b_{11}b_{22}b_{44} - b_{11}b_{22}b_{55} - b_{11}b_{22}b_{66} - b_{11}b_{33}b_{44} - b_{11}b_{33}b_{55} - b_{11}b_{33}b_{66} - b_{11}b_{44}b_{55} - b_{11}b_{44}b_{66} - b_{11}b_{55}b_{66} + b_{12}b_{21}b_{33} + b_{12}b_{21}b_{44} + b_{12}b_{21}b_{55} + b_{12}b_{21}b_{66} - b_{22}b_{33}b_{44} - b_{22}b_{33}b_{55} - b_{22}b_{33}b_{66} - b_{22}b_{44}b_{55} - b_{22}b_{44}b_{66} - b_{22}b_{55}b_{66} - b_{33}b_{44}b_{55} - b_{33}b_{44}b_{66} - b_{33}b_{55}b_{66} - b_{44}b_{55}b_{66}$

$B_4 = b_{11}b_{22}b_{33}b_{44} + b_{11}b_{22}b_{33}b_{55} + b_{11}b_{22}b_{33}b_{66} + b_{11}b_{22}b_{44}b_{55} + b_{11}b_{22}b_{44}b_{66} + b_{11}b_{22}b_{55}b_{66} + b_{11}b_{33}b_{44}b_{55} + b_{11}b_{33}b_{44}b_{66} + b_{11}b_{33}b_{55}b_{66} + b_{11}b_{44}b_{55}b_{66} - b_{12}b_{21}b_{33}b_{44} - b_{12}b_{21}b_{33}b_{55} - b_{12}b_{21}b_{33}b_{66} - b_{12}b_{21}b_{44}b_{55} - b_{12}b_{21}b_{44}b_{66} - b_{12}b_{21}b_{55}b_{66} - b_{16}b_{21}b_{32}b_{63} - b_{16}b_{21}b_{42}b_{64} + b_{22}b_{33}b_{44}b_{55} + b_{22}b_{33}b_{44}b_{66} + b_{22}b_{33}b_{55}b_{66} + b_{22}b_{44}b_{55}b_{66} + b_{33}b_{44}b_{55}b_{66}$

$B_5 = -b_{11}b_{22}b_{33}b_{44}b_{55} - b_{11}b_{22}b_{33}b_{44}b_{66} - b_{11}b_{22}b_{33}b_{55}b_{66} - b_{11}b_{22}b_{44}b_{55}b_{66} - b_{11}b_{33}b_{44}b_{55}b_{66} + b_{12}b_{21}b_{33}b_{44}b_{55} + b_{12}b_{21}b_{33}b_{44}b_{66} + b_{12}b_{21}b_{33}b_{55}b_{66} + b_{12}b_{21}b_{44}b_{55}b_{66} - b_{16}b_{21}b_{32}b_{43}b_{64} + b_{16}b_{21}b_{32}b_{44}b_{63} - b_{16}b_{21}b_{32}b_{53}b_{65} + b_{16}b_{21}b_{32}b_{55}b_{63} + b_{16}b_{21}b_{33}b_{42}b_{64} + b_{16}b_{21}b_{42}b_{55}b_{64} - b_{22}b_{33}b_{44}b_{55}b_{66}$

$B_6 = b_{11}b_{22}b_{33}b_{44}b_{55}b_{66} - b_{12}b_{21}b_{33}b_{44}b_{55}b_{66} + b_{16}b_{21}b_{32}b_{43}b_{55}b_{64} - b_{16}b_{21}b_{32}b_{44}b_{53}b_{65} - b_{16}b_{21}b_{32}b_{44}b_{55}b_{63} - b_{16}b_{21}b_{33}b_{42}b_{55}b_{64}$

To determine the local stability of the equilibrium point E^* using the Routh-Hurwitz criteria, we evaluate the polynomial equation associated with the system dynamics. The roots of this polynomial will have negative real parts if the following conditions are met:

$$B_1 > 0, \begin{vmatrix} B_1 & 1 \\ B_3 & B_2 \end{vmatrix} > 0, \begin{vmatrix} B_1 & 1 & 0 \\ B_3 & B_2 & B_1 \\ B_5 & B_4 & B_3 \end{vmatrix} > 0, \begin{vmatrix} B_1 & 1 & 0 & 0 \\ B_3 & B_2 & B_1 & 1 \\ B_5 & B_4 & B_3 & B_2 \\ 0 & B_6 & B_5 & B_4 \end{vmatrix} > 0,$$

$$\begin{vmatrix} B_1 & 1 & 0 & 0 & 0 \\ B_3 & B_2 & B_1 & 1 & 0 \\ B_5 & B_4 & B_3 & B_2 & B_1 \\ 0 & B_6 & B_5 & B_4 & B_3 \\ 0 & 0 & 0 & B_6 & B_5 \end{vmatrix} > 0, \begin{vmatrix} B_1 & 1 & 0 & 0 & 0 & 0 \\ B_3 & B_2 & B_1 & 1 & 0 & 0 \\ B_5 & B_4 & B_3 & B_2 & B_1 & 1 \\ 0 & B_6 & B_5 & B_4 & B_3 & B_2 \\ 0 & 0 & 0 & B_6 & B_5 & B_4 \\ 0 & 0 & 0 & 0 & 0 & B_6 \end{vmatrix} > 0$$

Hence, the equilibrium point E^* is locally asymptotically stable if all the above conditions are satisfied.

5.3 Global Stability Analysis

5.3.1. Global Stability at E^0

Consider Lyapunov function as given below

$$Y = S + E + A + W + I + R$$

$$\frac{dY}{dt} = \frac{dS}{dt} + \frac{dE}{dt} + \frac{dA}{dt} + \frac{dW}{dt} + \frac{dI}{dt} + \frac{dR}{dt}$$

$$= B - \mu S - \mu E - \mu A - \mu W - \mu I - \mu R$$

$$= B - \mu\left(\frac{B}{\mu}\right) - \mu(E + A + W + I + R)$$

$$= -\mu(E + A + W + I + R)$$

We have $\frac{dY}{dt} \le 0$ with $\frac{dY}{dt} = 0$ only if $E = A = W = I = R = 0$. This condition indicates that the Lyapunov function Y is non-increasing over time and only remains constant when the system reaches the mobile phone-free equilibrium.

By LaSalle's Invariance Principle, all system trajectories will ultimately converge to the largest invariant set where $\frac{dY}{dt} = 0$, which in this case is the equilibrium point $E^0\left(\frac{B}{\mu}, 0,0,0,0,0\right)$. Hence, every solution of the system converges to E^0 as time progresses, confirming that the mobile phone-free equilibrium is globally asymptotically stable.

5.3.2 Global Stability at E^*

Consider Lyapunov function as given below

$$Y(t) = \frac{1}{2}[(S - S^*) + (E - E^*) + (A - A^*) + (W - W^*) + (I - I^*) + (R - R^*)]^2$$

$$Y'(t) = [(S - S^*) + (E - E^*) + (A - A^*) + (W - W^*) + (I - I^*) + (R - R^*)]$$

$$(S' + E' + A' + W' + I' + R')$$

$$= [(S - S^*) + (E - E^*) + (A - A^*) + (W - W^*) + (I - I^*) + (R - R^*)](B - \mu S - \mu E -$$

$$\mu A - \mu W - \mu I - \mu R)$$

$$= [(S - S^*) + (E - E^*) + (A - A^*) + (W - W^*) + (I - I^*) + (R - R^*)](\mu S^* + \mu E^* + \mu A^*$$

$$+\mu W^* + \mu I^* + \mu R^* - \mu S - \mu E - \mu A - \mu W - \mu I - \mu R)$$

$$= -\mu[(S - S^*) + (E - E^*) + (A - A^*) + (W - W^*) + (I - I^*) + (R - R^*)]^2$$

$$\leq 0$$

where $B = \mu S^* + \mu E^* + \mu A^* + \mu W^* + \mu I^* + \mu R^*$
Therefore, based on the Lyapunov function $Y(t)$ and its derivative $Y'(t)$, which satisfies $Y'(t) \leq 0$ indicating that $Y(t)$ is non-increasing, we conclude that the unique positive equilibrium point E^* is globally asymptotically stable.

6. Data Collection

To collect data on mobile phone addiction among college students, we designed a structured questionnaire covering aspects of mobile phone use, addiction-related behaviours and awareness. The questionnaire included demographic items (age and gender), daily screen time, frequently used applications and phone-checking habits (e.g., usage upon waking, during meals, studying or working). It also addressed the impact of phone use on academics, sleep and social life. Students were asked about their awareness of phone addiction, emotional responses such as anxiety or guilt and behavioural indicators like distraction, time distortion and the urge to check phones during social interactions. Questions explored their past efforts to reduce phone usage, methods adopted (e.g., screen time apps, disabling notifications, limiting data) and the success of those attempts. The questionnaire also gathered information on participation in digital detox programs, perceptions of external interventions and current self-control over phone use. The survey was distributed online across various colleges to ensure a diverse sample.
The collected data was then used to estimate the values of different parameters and transition rates in our mathematical model. This helped us understand how mobile phone addiction spreads and how effective various intervention strategies might be.
The value of different parameters was extracted from the collected data, and initial conditions were determined for numerical simulations. The initial values represent different groups within the population and are as follows: $S(t) = 511$, $E(t) = 492$, $A(t) = 247$, $W(t) = 368$, $I(t) = 102$, $R(t) = 456$. Here, total population$\big(N(t)\big)$ is 2176.

Table 2: Parameters and its value

Parameters	Values	Description
B	0.016	Recruitment rate.
μ	0.009	Natural death rate.
α	0.96	The rate at which susceptible individuals become engaged.
β	0.49	The rate at which engaged individuals become aware.
γ	0.5	The rate at which engaged individuals become addicted.
η	0.52	The rate at which addicted individuals become aware.
δ	0.41	The rate at which addicted individuals receive external intervention efforts
σ	0.05	The rate at which addicted individuals recovered.
ρ	0.77	The rate at which individuals receiving external intervention efforts successfully recover
λ	0.98	The rate at which aware individuals recovered.
τ	0.89	The rate at which recovered individuals relapse and become susceptible to mobile phone addiction again.

7. Sensitivity Analysis

Sensitivity indices of R_0 to all the different parameters tell us that how crucial each parameter is to the addiction spread. This helps us choose the right parameters responsible for making the scenario endemic.

7.1 Local Sensitivity Analysis

To assess the influence of individual parameters on the basic reproduction number R_0, a local sensitivity analysis was performed. Local sensitivity indices were derived using the normalized forward sensitivity index:

$$(R_0)_p = \frac{\partial R_0}{\partial p} \cdot \frac{p}{R_0}$$

where p denotes a parameter. This index measures the relative change in R_0 with respect to a relative change in parameter p.

We calculate the sensitivity indices for those parameters on which the value of basic reproduction depends. The results are given in Table 3.

Table 3: Local Sensitivity indices of R_0 to the parameters

Parameters	Sign	Value
N	-	1
B	+	1
α	+	1
β	-	0.49
γ	-	0.50
μ	-	1.008

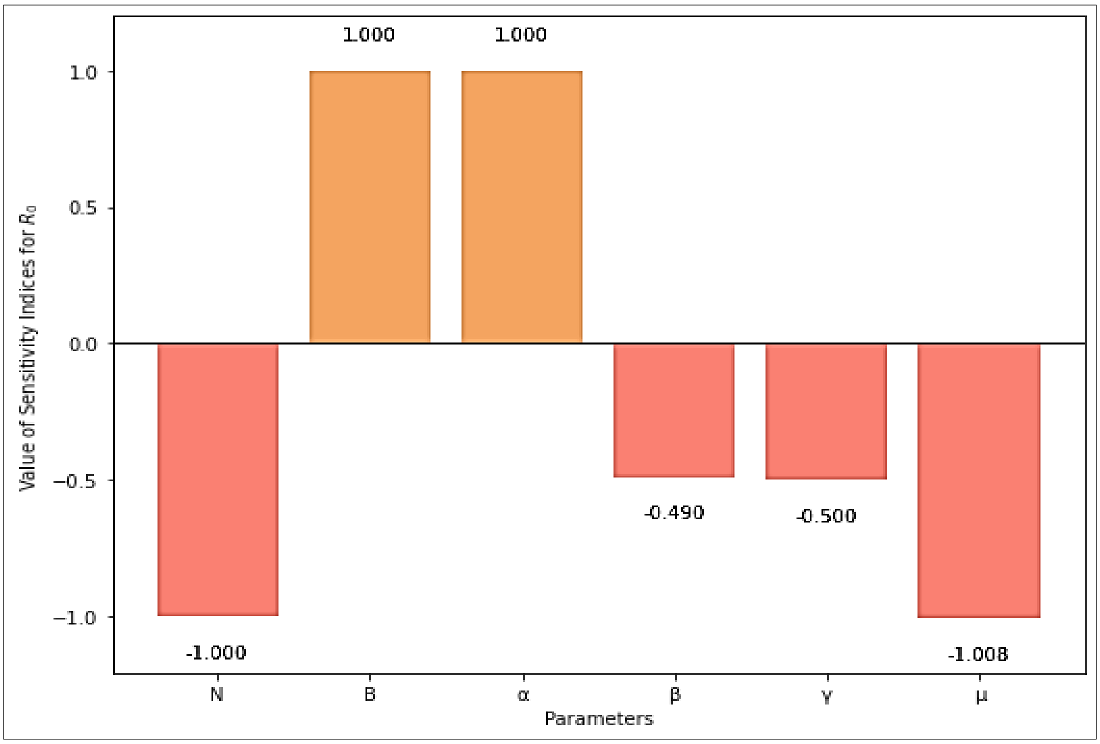

Fig 3: Local Sensitivity analysis for reproduction number

7.2. Global Sensitivity Analysis

Global sensitivity analysis approach was adopted, as it accounts for the simultaneous variation of all parameters across their possible ranges, thereby providing a comprehensive understanding of parameter importance.

Since local sensitivity only reflects behaviour near baseline values, we also performed a global sensitivity analysis using Partial Rank Correlation Coefficients (PRCCs). This approach considers the simultaneous variation of all parameters across their plausible ranges, capturing nonlinearities and interactions.

For global sensitivity analysis, each parameter was varied within $\pm 10\%$ of its nominal (baseline) value. A total of 1,000 parameter sets were generated using Latin Hypercube Sampling (LHS) within biologically probable range. For each set, R_0 was calculated. PRCC values were then estimated between each parameter and R_0, with significance tested using corresponding P - values. PRCC values close to $+1$ (or -1) indicate a strong positive (or negative) monotonic relationship.

Table 4: Global Sensitivity indices of R_0 to the parameters

Parameters	Sign	Value	P value
N	-	0.0355	< 0.001
B	+	0.9580	< 0.001
α	+	0.9322	< 0.001
β	-	0.7622	< 0.001
γ	-	0.7819	< 0.001
μ	-	0.9432	< 0.001

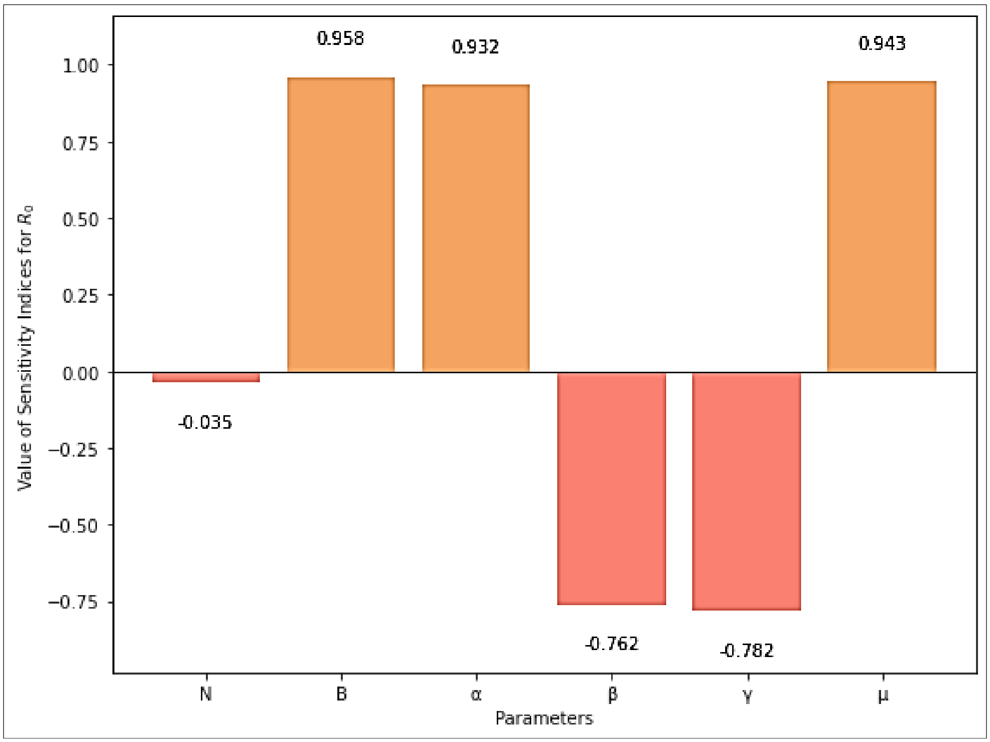

Fig 4: Global Sensitivity analysis for reproduction number

Our analysis shows that the effect of parameters on R_0 depends on whether we look at local or global sensitivity. In the local case, when we study small changes around the baseline values, the progression rate (α) and natural death rate (μ) have the strongest impact on R_0. However, when we consider global sensitivity using Latin Hypercube Sampling with $\pm 10\%$ variation in all parameters, the birth rate (B) and natural death rate (μ) emerge as the most influential. This means that while α directly affects R_0 at the baseline, in a wider range of scenarios the demographic factors $(B \text{ and } \mu)$ play a more dominant role in shaping the overall behaviour of the system.

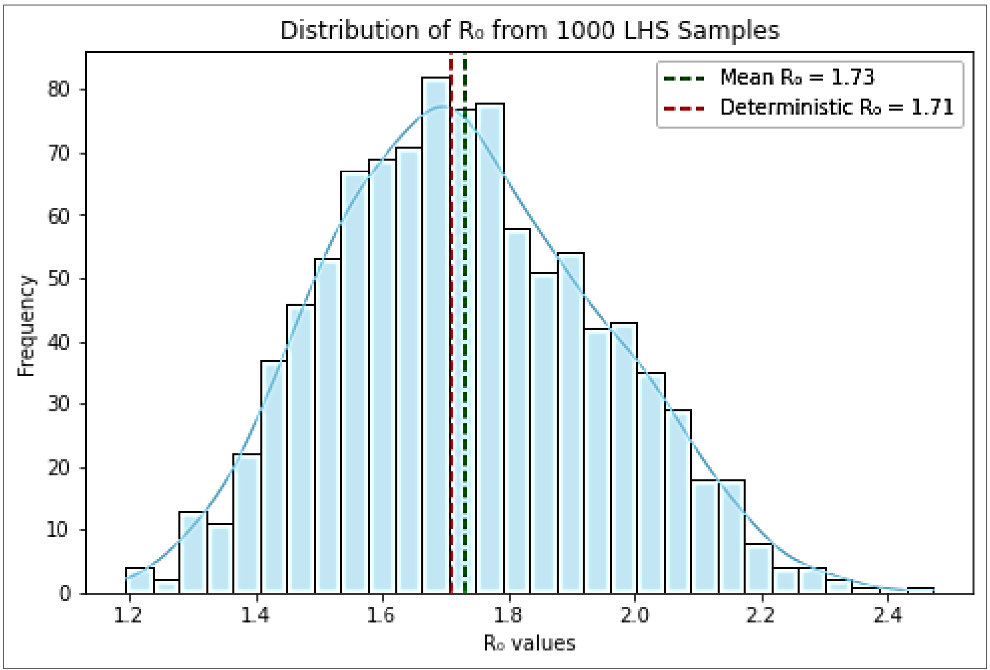

Fig 5: Distribution of R_0 from 100 LHS Samples

'Figure 5 shows the spread of R_0 values from 1000 samples with $\pm 10\%$ change in parameters. The values form a single peak, close to a bell shape, with an average of 1.73. The fixed (deterministic) value of 1.71 is almost the same as the average, which means the chosen parameters represent the system well. Since most R_0 values are above 1, the results suggest that the addiction is likely to continue spreading even when parameters change within this range.

8. Result and Discussion

To predict future trends of the addiction, we conducted simulations using the extracted parameters and initial values of table 2.

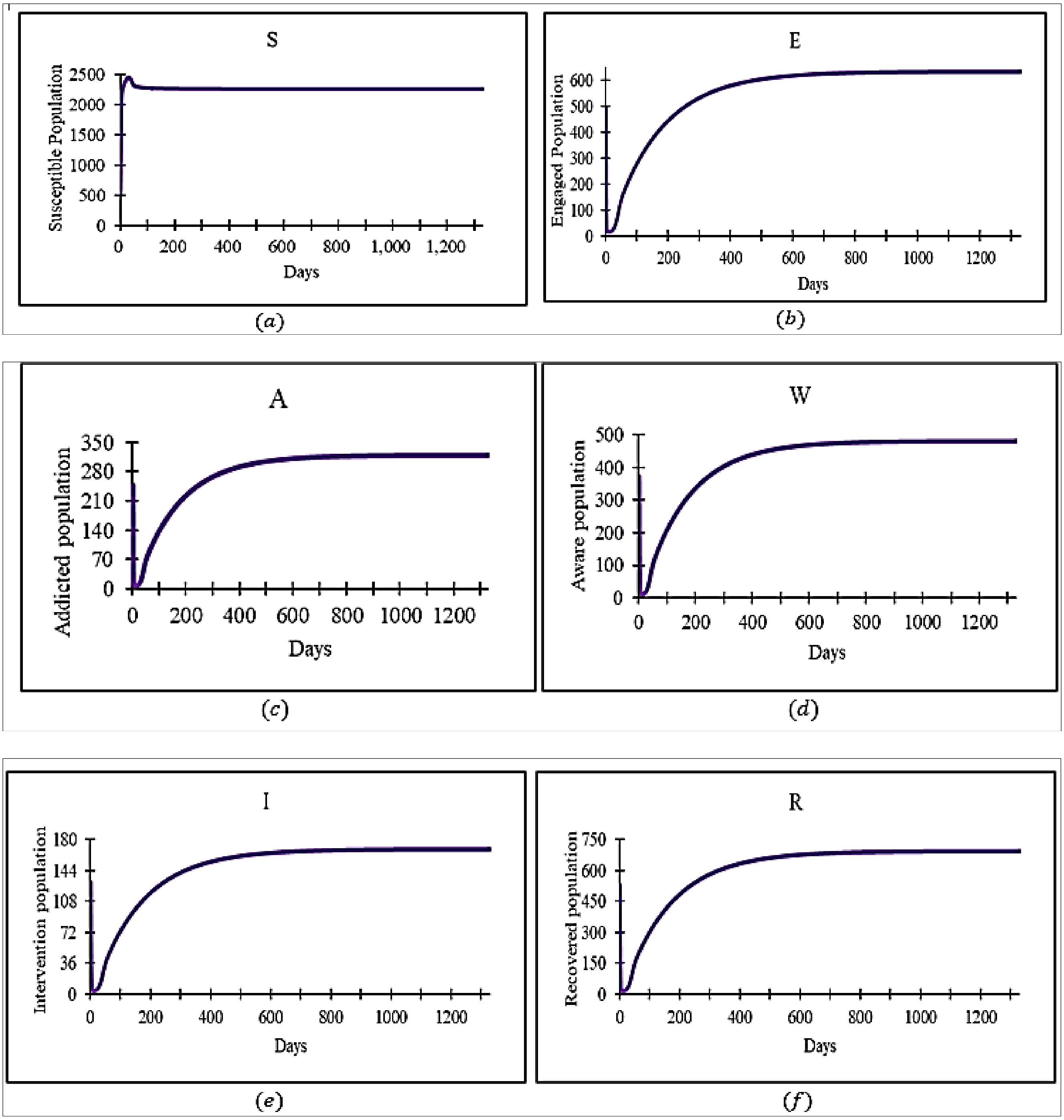

Fig 6: Graph of different compartments (population) vs. days based on the initial and parameter values: (a) Susceptible population vs. Days, (b) Engaged population vs. Days, (c) Addicted population vs. Days, (d) Aware population vs. Days, (e) Interventions population vs. Days, (f) Recovered population vs. Days. Parameters: [Table 2]

Figure 6(a) shows a declining trend as students, initially not engaged in excessive mobile phone use, gradually become influenced by social and environmental factors. As they transition into the Engaged (E) state, the number of susceptible individuals decreases.

Figure 6(b) might first increase as students start using their phones more frequently. However, as time progresses, some of these students will transition into the addicted (A) state, leading to a gradual decline in engagement.

Figure 6(c) is crucial as it highlights the peak of mobile phone dependency. Initially, the number of addicted individuals rises as engagement deepens and awareness of the problem remains low. However, with the introduction of awareness campaigns and interventions, some addicted students may transition into the Aware (W) or Intervention (I) states.

Figure 6(d) shows the population of students who recognize their excessive mobile usage and attempt to regulate it. This graph might show fluctuations as some students successfully move toward intervention and recovery, while others relapse into addiction.

Figure 6(e) reflects the effect of corrective measures such as counselling, or educational campaigns. If interventions are effective, this graph may show a rising trend, ultimately leading to an increase in the Recovered (R) population.

Figure 6(f) represents students who have successfully overcome their addiction. Over time, with sustained intervention and support, the number of recovered students should steadily rise, demonstrating the effectiveness of the intervention strategies.

9. Conclusion

The study highlights that mobile phone addiction is not just a personal issue but a social one that can quickly spread among students if left unchecked. By using a mathematical model, the research tracks how students move through different stages - from being curious about phone use, becoming engaged, developing addiction and eventually reaching awareness, getting help and recovering. The calculated basic reproduction number (R_0) of 1.70 shows that one addicted student can influence more than one.

The analysis of the model's graphs reveals that while mobile phone use initially increases among students, well-planned awareness campaigns and intervention programs play a crucial role in reducing addiction rates and increasing the number of recovered students over time. Initially, the susceptible and engaged populations increase as students adopt mobile phone habits but with time, intervention strategies and awareness efforts contribute to a decline in addiction and an increase in the recovered population. The findings suggest that early intervention, digital detox programs and educational awareness campaigns are crucial in mitigating mobile phone addiction and promoting healthier usage habits among students.

Future work can extend the model by adding factors like mental health effects, peer pressure and school rules. Expanding the study to different age groups, such as high school students or working professionals, can also provide broader insights into the long-term effects of mobile phone addiction.

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