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A theorem on the generalized Laplace transform

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Abstract
In this paper a theorem on the generalized Laplace transform is proved. It generalizes the result proved by Samar, Gupta [4, p. 245] and Saxena [3, pp. 154-157]. A special case of the theorem is to get the Mellin –transform of the H- function of 'r' variables having an arbitrary power of $(p_i + b_i t_i + c_i t_i^{h_i})$ in the argument.

Keywords: Generalized Laplace transform, Mellin transform, h-function of several variables, integral transforms, mathematical theorems

Introduction

The H- function of two variables defined by Mittal and Gupta has been generalized to the H- function of several complex variables [or the multi variable H- function] $x_1,\dots,x_r$ by srivastava and Panda [5] in terms of multiple contour integral as:

$$H[x_1,\dots,x_r] = H^{0,n:m_1,n_1;\dots;m_r,n_r}_{p,q:p_1,q_1;\dots;p_r,q_r}\left[\begin{matrix} x_1 \\ \cdot \\ \cdot \\ x_r \end{matrix}\left|\begin{matrix} (a_j,\alpha_j^{(1)},\dots,\alpha_j^{(r)})_{1,p} : (c_j^{(1)},C_j^{(1)})_{1,p_1};\dots;(c_j^{(r)},C_j^{(r)})_{1,p_r} \\ (b_j,\beta_j^{(1)},\dots,\beta_j^{(r)})_{1,q} : (d_j^{(1)},D_j^{(1)})_{1,q_1};\dots;(d_j^{(r)},D_j^{(r)})_{1,q_r} \end{matrix}\right.\right]$$

$$= \frac{1}{(2\pi i)^r}\int_{L_1}\dots\int_{L_r}\theta(s_1,\dots,s_r)\prod_{k=1}^{r}[\phi_k(s_k)x_k^{s_k}]ds_1\dots ds_r \tag{1.1}$$

where

$$\theta(s_1,\dots,s_r) = \frac{\prod_{j=1}^{n}\Gamma(1-a_j+\sum_{k=1}^{r}\alpha_j^{(k)}s_k)}{\prod_{j=n+1}^{p}\Gamma(a_j-\sum_{k=1}^{r}\alpha_j^{(k)}s_k)\prod_{j=1}^{q}\Gamma(1-b_j+\sum_{k=1}^{r}\beta_j^{(k)}s_k)} \tag{1.2}$$

k= 1, 2,….,r

$$\phi_k(s_k) = \frac{\prod_{j=1}^{m_k}\Gamma(d_j^{(k)}-D_j^{(k)}s_k)\prod_{j=1}^{n_k}\Gamma(1-c_j+C_j^{(k)}s_k)}{\prod_{j=m_k+1}^{q_k}\Gamma(1-d_j^{(k)}+D_j^{(k)}s_k)\prod_{j=n_k+1}^{p_k}\Gamma(c_j^{(k)}-C_j^{(k)}s_k)} \tag{1.3}$$

k = 1, 2,…,r and an empty product is interpreted as unity; n, p, q, m_k, n_k, q_k are nonnegative integers such that: $0 \le n \le p,\quad q \ge 0,\quad 0 \le m_k \le q_k,\quad 0 \le n_k \le p_k,\quad (k = 1,2,\dots,r)$ and $\alpha_j^{(k)}, \beta_j^{(k)}, C_j^{(k)}, D_j^{(k)}$ are all positive. The contour L_k in the complex S_k plane is of the Mellin –Barness type which runs from $-i\infty$ to $+i\infty$ with indentations, if necessary, to ensure that all the poles of $\Gamma(d_j^{(k)} - D_j^{(k)} s_k),\ j = 1,\dots,m_k$ are to the right of the path, and those of $\Gamma(1 - c_j^{(k)} - C_j^{(k)} s_k),\ j = 1,\dots,n_k$ and $\Gamma(1 - a_j - \sum_{k=1}^{r} \alpha_j^{(k)} s_k),\ j = 1,\dots,n$ are to the left of L_k. The various parameters are so restricted that the poles of all the Gamma functions that involved in (2.1.2) are simple and none of them coincide.

By applying the method of Braaksma, it can be proved that the H- function of several variables (or multivariable H- function)is an analytic function, if

$$A_k = \sum_{j=1}^{p} \alpha_j^{(k)} - \sum_{j=1}^{q} \beta_j^{(k)} + \sum_{j=1}^{p_k} C_j^{(k)} - \sum_{j=1}^{q_K} d_j^{(k)} \le 0, (j = 1,\dots,r) \tag{1.4}$$

The integral (8.1.1) converges absolutely if $|\arg x| < \frac{1}{2}\pi\Delta_k,) k = 1,2,..,r)$ where

$$\Delta_k = -\sum_{j=n+11}^{p} \alpha_j^{(k)} + \sum_{j=1}^{n_k} C_j^{(k)} - \sum_{j=n_k+1}^{p_k} C_j^{(k)} - \sum_{j=1}^{q} \beta_j^{(k)} + \sum_{j=1}^{m_k} D_j^{(k)} - \sum_{j=m_k+1}^{q_k} D_j^{(k)} > 0 \tag{1.5}$$

The Laplace transform

$F(p) = \int_0^\infty e^{-pt} f(t)\,dt$, Re(p) > 0 is represented by $F(p) \doteqdot f(t)$

$$\int_0^\infty \cdots \int_0^\infty e^{-(s_1 x_1 + \dots + s_r x_r)} x_1^{\rho_1 - 1} \dots x_r^{\rho_r - 1} \times H^{0,0:m_1,n_1;\dots;m_r,n_r}_{p,q:p_1,q_1;\dots;p_r,q_r} \left[\begin{matrix} z_1 x_1^{\lambda_1} \\ \cdot \\ z_r x_r^{\lambda_r} \end{matrix} \left| \begin{matrix} (a_j;\alpha_j^{(1)},\dots,\alpha_j^{(r)})_{1,p} : (c_j^{(1)},\gamma_j^{(1)})_{1,p_1};\dots;(c_j^{(r)},\gamma_j^{(r)})_{1,p_r} \\ \\ (b_j;\beta_j^{(1)},\dots,\beta_j^{(r)})_{1,q} : (d_j^{(1)},\delta_j^{(1)})_{1,q_1};\dots;(d_j^{(r)},\delta_j^{(r)})_{1,q_r} \end{matrix} \right. \right] dx_1 \dots dx_r$$

$$= s_1^{-\rho_1} \dots s_r^{-\rho_r} \times H^{0,0:m_1,n_1+1;\dots;m_r,n_r+1}_{p,q:p_1+1,q_1;\dots;p_r+1,q_r} \left[\begin{matrix} z_1 s_1^{-\lambda_1} \\ \cdot \\ z_r s_r^{-\lambda_r} \end{matrix} \left| \begin{matrix} (a_j;\alpha_j^{(1)},\dots,\alpha_j^{(r)})_{1,p} : (1-\rho_1,\lambda_1),(c_j^{(1)},\gamma_j^{(1)})_{1,p_1};\dots;(1-\rho_r,\lambda_r),(c_j^{(r)},\gamma_j^{(r)})_{1,p_r} \\ \\ (b_j;\beta_j^{(1)},\dots,\beta_j^{(r)})_{1,q} : (d_j^{(1)},\delta_j^{(1)})_{1,q_1};\dots;(d_j^{(r)},\delta_j^{(r)})_{1,q_r} \end{matrix} \right. \right], \tag{1.6}$$

provided that

i) $\min\{\lambda_i, \mathrm{Re}(s_i)\} > 0$,

ii) $\mathrm{Re}(\rho_i) + \lambda_i \min_{1 \le j \le m_i} \mathrm{Re}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right) > 0$,(i = 1,…,r).

$$\int_0^\infty \cdots \int_0^\infty e^{-(s_1 x_1 + \dots + s_r x_r)} x_1^{\rho_1 - 1} \dots x_r^{\rho_r - 1} \times H^{0,0:m_1,n_1;\dots;m_r,n_r}_{p,q:p_1,q_1;\dots;p_r,q_r}$$

$$\left[\begin{array}{l|l} z_1 x_1^{-\lambda_1} & (a_j;\alpha_j^{(1)},...,\alpha_j^{(r)})_{1,p} : (c_j^{(1)},\gamma_j^{(1)})_{1,p_1};...;(c_j^{(r)},\gamma_j^{(r)})_{1,p_r} \\ . & \\ z_r x_r^{-\lambda_r} & (b_j;\beta_j^{(1)},...,\beta_j^{(r)})_{1,q} : (d_j^{(1)},\delta_j^{(1)})_{1,q_1};...;(d_j^{(r)},\delta_j^{(r)})_{1,q_r} \end{array}\right] dx_1...dx_r$$

$$= s_1^{-\rho_1}...s_r^{-\rho_r} \times H^{0,0:m_1+1,n_1;...;m_r+1,n_r}_{p,q:p_1,q_1+1;...;p_r,q_r+1}$$

$$\left[\begin{array}{l|l} z_1 s_1^{\lambda_1} & (a_j;\alpha_j^{(1)},...,\alpha_j^{(r)})_{1,p} : (c_j^{(1)},\gamma_j^{(1)})_{1,p_1};...;(c_j^{(r)},\gamma_j^{(r)})_{1,p_r} \\ . & \\ z_r s_r^{\lambda_r} & (b_j;\beta_j^{(1)},...,\beta_j^{(r)})_{1,q} : (\rho_1,\lambda_1),(d_j^{(1)},\delta_j^{(1)})_{1,q_1};...;(\rho_r,\lambda_r),(d_j^{(r)},\delta_j^{(r)})_{1,q_r} \end{array}\right], \tag{1.7}$$

provided that

i) $\min\{\lambda_i, \mathrm{Re}(s_i)\} > 0$

ii) $\mathrm{Re}(\rho_i) - \lambda_i \min\limits_{1\le j\le m_i} \mathrm{Re}\left(\dfrac{c_j^{(i)}-1}{\gamma_j^{(i)}}\right) > 0$,(i = 1,…,r).

$$H^{1,0}_{0,1}\left[x\Big|\begin{matrix}- \\ (l,p)\end{matrix}\right]) = p^{-1} x^{\frac{i}{p}} e^{-x} \tag{1.8}$$

$$H^{1,0}_{0,1}\left[x\Big|\begin{matrix}- \\ (0,1)\end{matrix}\right] = e^{-x} \tag{1.9}$$

2. Main Result

If

$$\exp\left(-\sum_{i=1}^{r} q_i \phi(t_i)\right) f(t_1,...,t_r) \doteqdot F(p_1,...,p_r;q_1,...,q_r), \tag{2.1}$$

And

$$g(t_1,...,t_r) \doteqdot G(p_1,...,p_r), \tag{2.2}$$

then

$$F(b_1t_1,...,b_rt_r;c_1t_1,...,c_rt_r)g(t_1,...,t_r)$$

$$\doteqdot \int_0^\infty ... \int_0^\infty f(t_1,...,t_r) G(p_1+b_1t_1+c_1\phi(t_1),...,p_r+b_rt_r+c_r\phi(t_r)dt_1...dt_r \tag{2.3}$$

provided that the integrals involved are absolutely convergent.

Proof:

$$\int_0^\infty ... \int_0^\infty f(t_1,...,t_r) G(p_1+b_1t_1+c_1\phi(t_1),...,p_r+b_rt_r+c_r\phi(t_r)dt_1...dt_r$$

$$= \int_0^\infty ... \int_0^\infty f(t_1,...,t_r) \left\{ \int_0^\infty ... \int_0^\infty \exp\left(-\sum_{i=1}^{r}(p_i+b_it_i+c_i\phi(t_i))x_i\right) g(x_1,...,x_r)dx_1...dx_r \right\} dt_1...dt_r$$

$$= \int_0^\infty \ldots \int_0^\infty \exp(-\sum_{i=1}^r (p_i x_i) g(x_1,\ldots,x_r)$$

$$\times \left\{ \int_0^\infty \ldots \int_0^\infty \exp\left(-\sum_{i=1}^r (b_i x_i t_i + c_i x_i \phi(t_i)) \right) f(t_1,\ldots,t_r) dt_1 \ldots dt_r \right\} dx_1 \ldots dx_r, \qquad (2.4)$$

Using (21) and (2.2) in (2.4), the right hand side of (2.4) becomes:

$$\doteq g(x_1,\ldots,x_r) F(b_1 x_1,\ldots,b_r x_r; c_1 x_1,\ldots,c_r x_r)$$

$$\doteq g(t_1,\ldots,t_r) F(b_1 t_1,\ldots,b_r t_r; c_1 t_1,\ldots,c_r t_r).$$

The change of order of integration is justified by Delavalle Foussins theorem.

Provided that the integrals

$$\int_0^\infty \ldots \int_0^\infty \exp\left(-\sum_{i=1}^r (p_i x_i + b_i x_i t_i + c_i x_i \phi(t_i) \right) g(x_1,\ldots,x_r) dx_1 \ldots dx_r$$ and

$$\int_0^\infty \ldots \int_0^\infty \exp\left(-\sum_{i=1}^r (b_i x_i t_i + c_i x_i \phi(t_i) \right) f(t_1,\ldots,t_r) dt_1 \ldots dt_r$$ are both convergent and the Laplace transform of $g(t_1,\ldots,t_r)$ and $F(b_1 t_1,\ldots,b_r t_r; c_1 t_1,\ldots,c_r t_r)$ exist.

Special Cases

Let in (2.1) put $\phi(t_i) = t_i^{h_i}$ and $f(t_1,\ldots,t_r) = t_1^{s_1-1} \ldots t_r^{s_r-1}$, then

$$\exp\left(-\sum_{i=1}^r q_i \phi(t_i) \right) f(t_1,\ldots,t_r) =$$

$$t_1^{s_1-1} \ldots t_r^{s_r-1} \times H^{0,0:1,0;\ldots;1,0}_{0,0:0,1;\ldots;0,1} \left[\begin{matrix} q_1 t_1^{h_1} \\ \cdot \\ q_r t_r^{h_r} \end{matrix} \middle| \begin{matrix} - : -;\ldots;- \\ \\ - : (0,1);\ldots;(0,1) \end{matrix} \right], \qquad (2.5)$$

expressing $\exp\left(-q_i t_i^{h_i}\right)$ as an H- function of several variables.

$$\doteq p_1^{-s_1} \ldots p_r^{-s_r} \times H^{0,0:1,1;\ldots;1,1}_{0,0:1,1;\ldots;1,1} \left[\begin{matrix} q_1 p_1^{-h_1} \\ \cdot \\ q_r p_r^{-h_r} \end{matrix} \middle| \begin{matrix} - : (1-s_1,h_1);\ldots;(1-s_r,h_r) \\ \\ - : \quad (0,1);\ldots; \quad (0,1) \end{matrix} \right].$$

Now let

$g(t_1,\ldots,t_r) \doteq G(p_1,\ldots,p_r)$, then the theorem gives:

$$g(t_1,\ldots,t_r) F(b_1 t_1,\ldots,b_r t_r; c_1 t_1,\ldots,c_r t_r) = g(t_1,\ldots,t_r)(b_1 t_1)^{-s_1} \ldots (b_r t_r)^{-s_r}$$

$$\times H^{0,0:1,1;\ldots;1,1}_{0,0:1,1;\ldots;1,1}\left[\begin{array}{c} c_1 t_1 (b_1 t_1)^{-h_1} \\ \cdot \\ c_r t_r (b_r t_r)^{-h_r} \end{array}\left|\begin{array}{l} -:(1-s_1,h_1);\ldots;(1-s_r,h_r) \\ \\ -:\quad (0,1);\ldots;\quad (0,1) \end{array}\right.\right]$$

$$\doteq \int_0^\infty\ldots\int_0^\infty t_1^{s_1-1}\ldots t_r^{s_r-1} G\left(p_1+b_1t_1+c_1t_1^{h_1},\ldots,p_r+b_rt_r+c_rt_r^{h_r}\right)dt_1\ldots dt_r$$

$$\doteq M\left[G\left(p_1+b_1t_1+c_1t_1^{h_1},\ldots,p_r+b_rt_r+c_rt_r^{h_r}\right)\right]. \tag{2.6}$$

Thus we have

If $g(t_1,\ldots,t_r)\doteq G(p_1,\ldots,p_r)$, then

$$M\left[G\left(p_1+b_1t_1+c_1t_1^{h_1},\ldots,p_r+b_rt_r+c_rt_r^{h_r}\right)\right]\doteq$$

$$g(t_1,\ldots,t_r)(b_1t_1)^{-s_1}\ldots(b_rt_r)^{-s_r} \times H^{0,0:1,1;\ldots;1,1}_{0,0:1,1;\ldots;1,1}\left[\begin{array}{c} c_1 b_1^{-h_1} t_1^{1-h_1} \\ \cdot \\ c_r b_r^{-h_r} t_r^{1-h_r} \end{array}\left|\begin{array}{l} -:(1-s_1,h_1);\ldots;(1-s_r,h_r) \\ \\ -:\quad (0,1);\ldots;\quad (0,1) \end{array}\right.\right],\quad h_i>0 \tag{2.7}$$

Or

$$M\left[G\left(p_1+b_1t_1+c_1t_1^{h_1},\ldots,p_r+b_rt_r+c_rt_r^{h_r}\right)\right]\doteq$$

$$g(t_1,\ldots,t_r)(b_1t_1)^{-s_1}\ldots(b_rt_r)^{-s_r}$$

$$\times H^{0,0:2,0;\ldots;2,0}_{0,0:0,2;\ldots;0,2}\left[\begin{array}{c} c_1 b_1^{-h_1} t_1^{1-h_1} \\ \cdot \\ c_r b_r^{-h_r} t_r^{1-h_r} \end{array}\left|\begin{array}{l} -:\quad -;\ldots\ldots;\quad - \\ \\ -:(s_1,h_1),(0,1);\ldots;(s_r,h_r),(0,1) \end{array}\right.\right],\quad h_i<0. \tag{2.8}$$

Provided

$\mathrm{Re}(s_i)>0$, $\mathrm{Re}(p_i)>0$, $\mathrm{Re}(b_i)>0$ for $(i=1,\ldots,r)$ and

$\left|\arg(c_i b_i^{-h_i})\right|<\dfrac{\pi}{2},(i=1,\ldots,r)$ and the integrals are absolutely convergent.

Application:

Let $g(t_1,\ldots,t_r)$ and $F(p_1,\ldots,p_r)$ represents the left hand side and right hand side of (2.3), further let

$$\phi(t_1,\ldots t_r;p_1,\ldots,p_r)=(p_1+b_1t_1+c_1t_1^{h_1})^{-\rho_1}\ldots\ldots(p_r+b_rt_r+c_rt_r^{h_r})^{-\rho_r}$$

$$\times H^{0,0:m_1,n_1+1;\ldots;m_r,n_r+1}_{0,0:p_1+1,q_1;\ldots;p_r+1,q_r}\left[\begin{array}{l} z_1(p_1+b_1t_1+c_1t_1^{h_1})^{-\lambda_1} \\ \cdot \\ z_r(p_r+b_rt_r+c_rt_r^{h_r})^{-\lambda_r} \end{array}\right.$$

$$\left|\begin{array}{l} -:(1-\rho_1,\lambda_1),(c_j^{(1)},\gamma_j^{(1)})_{1,p_1};\ldots;(1-\rho_r,\lambda_r),(c_j^{(r)},\gamma_j^{(r)})_{1,p_r} \\ \\ -:\quad (d_j^{(1)},\delta_j^{(1)})_{1,q_1};\ldots;\quad (d_j^{(r)},\delta_j^{(r)})_{1,q_r} \end{array}\right],$$

then (2.7) and (2.8) give the following results.

$$M\{\phi(t_1,\dots,t_r;p_1,\dots,p_r\}\doteq t_1^{\rho_1-1}\dots t_r^{\rho_r-1}$$

$$\times H^{0,0:m_1,n_1;\dots;m_r,n_r}_{p,q:p_1,q_1;\dots;p_r,q_r}\left[\begin{matrix} z_1t_1^{\lambda_1} \\ \cdot \\ z_rt_r^{\lambda_r}\end{matrix}\left|\begin{matrix}(a_j,\alpha_j^{(1)},\dots,\alpha_j^{(r)})_{1,p}:(c_j^{(1)},\gamma_j^{(1)})_{1,p_1};\dots;(c_j^{(r)},\gamma_j^{(r)})_{1,p_r} \\ \\ (b_j,\beta_j^{(1)},\dots,\beta_j^{(r)}):\quad (d_j^{(1)},\delta_j^{(1)})_{1,q_1};\dots;(d_j^{(r)},\delta_j^{(r)})_{1,q_r}\end{matrix}\right.\right]$$

$$\times(b_1t_1)^{-s_1}\dots(b_rt_r)^{-s_r}$$

$$\times H^{0,0:1,1;\dots;1,1}_{0,0:1,1;\dots;1,1}\left[\begin{matrix} c_1b_1^{-h_1}t_1^{1-h_1} \\ \cdot \\ c_rb_r^{-h_r}t_r^{1-h_r}\end{matrix}\left|\begin{matrix}-:(1-s_1,h_1);\dots;(1-s_r,h_r) \\ \\ -:\quad (0,1);\dots;\quad (0,1)\end{matrix}\right.\right]$$

when $h_i>0$.

$$M\{\phi(t_1,\dots,t_r;p_1,\dots,p_r\}\doteq t_1^{\rho_1-1}\dots t_r^{\rho_r-1}$$

$$\times H^{0,0:m_1,n_1;\dots;m_r,n_r}_{p,q:p_1,q_1;\dots;p_r,q_r}\left[\begin{matrix} z_1t_1^{\lambda_1} \\ \cdot \\ z_rt_r^{\lambda_r}\end{matrix}\left|\begin{matrix}(a_j,\alpha_j^{(1)},\dots,\alpha_j^{(r)})_{1,p}:(c_j^{(1)},\gamma_j^{(1)})_{1,p_1};\dots;(c_j^{(r)},\gamma_j^{(r)})_{1,p_r} \\ \\ (b_j,\beta_j^{(1)},\dots,\beta_j^{(r)}):\quad (d_j^{(1)},\delta_j^{(1)})_{1,q_1};\dots;(d_j^{(r)},\delta_j^{(r)})_{1,q_r}\end{matrix}\right.\right]$$

$$\times(b_1t_1)^{-s_1}\dots(b_rt_r)^{-s_r}$$

$$\times H^{0,0:2,0;\dots;2,0}_{0,0:0,2;\dots;0,2}\left[\begin{matrix} c_1b_1^{-h_1}t_1^{1-h_1} \\ \cdot \\ c_rb_r^{-h_r}t_r^{1-h_r}\end{matrix}\left|\begin{matrix}-:\quad -;\dots\dots;\quad - \\ \\ -:(s_1,-h_1),(0,1);\dots;(s_r,-h_r),(0,1)\end{matrix}\right.\right]$$

when $h_i<0$.

provided

i) $\mathrm{Re}(s_i)>0$, $\mathrm{Re}(p_i)>0$, $\mathrm{Re}(t_i)>0$, (i=1,…,r)

ii) $\left|\arg c_it_i^{-h_i}\right|<\frac{\pi}{2}\Delta_i,\Delta_i>0$, $\left|\arg z_i\right|<\frac{\pi}{2}\theta_i,\theta_i>0$,

iii) $\mathrm{Re}\left(\rho_r+t_i\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)>0,(j=1,\dots,m_i),(i=1,\dots,r)$.

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