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Comparison between numerical analysis methods and methods for solving complex differential equations

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Abstract
The purpose of this study is to compare different numerical, analytical, and modern methods in terms of accuracy, stability, error rate, ease of use, execution time, and rigidity when solving complex differential equations. A thorough literature review methodology and Prisma analysis were used in this process, and both reveal that no single method is appropriate for all cases. The spectral and collocation method provides the highest accuracy and converges fastest when a smooth solution exists. Implicit methods (Implicit RK, BDF, FEM), such as implicit methods, present good stability when facing rigid problems with higher computational cost. Explicit methods such as RK4 and Adams-Bashforth are fast and easy to use but aren't as appropriate when using rigid problems. The PINN and Monte Carlo techniques are both flexible but slightly computationally expensive, and accuracy depends on the model's configuration and sample number. Based on the results of this study, I would recommend choosing the method based on the problem's nature: spectral if accuracy is desired, implicit if stability is desired, and explicit for preliminary results quickly.

Keywords: Differential equations, numerical, analytical, Prisma, accuracy, stability, error rate, ease of use, execution time, and rigidity

Introduction
Differential equations are among the most important tools for modeling natural, engineering, and physical phenomena, from studying the motion of objects and fluid flow to modeling applications that rely on artificial intelligence, especially in light of the development of digital computing, neural networks, and artificial intelligence [1]. It can be said that differential equations are the cornerstone of many applications, particularly those that cannot be solved using conventional analytical methods. The need has become urgent to find alternative methods for solving complex problems, especially those related to physical and natural phenomena. Among the most important new methods for overcoming the obstacles related to solving differential equations are approximation methods such as numerical analysis and modern methods that rely on artificial intelligence techniques [2]. Therefore, it requires that we resort to numerical computational techniques, and master the test in a number of related numerical tasks [3].

This study aims to compare numerical analysis methods with modern methods for solving complex differential equations. This is done through a main methodology that relies on reviewing previous studies and then comparing general numerical analysis methods such as (Oiler's method, Ringkotta's method, finite element and finite difference) with specialized methods for solving complex differential equations such as (integral transformation methods, perturbation methods, spectral methods and artificial intelligence techniques) [4]. This is done by using a comparative methodology in terms of advantages and disadvantages. This study also aims to clarify the most important obstacles and challenges facing the use of methods for solving differential equations, whether numerical analytical methods or specialized methods for solving complex differential equations, and to present solutions and proposals to overcome these obstacles. The importance of this study stems from the fact that it addressed the challenges and obstacles and presented solutions and proposals to overcome these obstacles.

In addition, it provided practical examples that illustrate the differences between analytical methods and complex specialized differential methods [5]. It also presented an example that serves as a case study documenting the most important of these differences. The importance of this study also stems from the fact that it is a comprehensive study that addressed the topic from several different aspects and avoided any confusion, whether in the data or in the results.

Even with substantial improvements in both numerical and analytical techniques to solve differential equations, especially with complicated types (e.g. nonlinear partial differential equations, equations with entangled initial/boundary conditions, or functions that are either composite or time-dependent), finding the best method to solve a particular equation type still proves to be a large challenge. Some numerical methods (finite difference methods, finite element methods, Range-Kota, or spectral methods) can have good accuracy or speed, but can also suffer from numerical instability challenges, computationally expensive algorithms, or singularities and extreme nonlinearity. Conversely, some analytical methods (series methods, integral transformations, or approximation methods like perturbation or homopy analysis) can offer accurate solutions in theory, but are often severely restricted for use or cannot be scaled to complex systems [6].

As a result, the research problem rests in the lack of a systematic comprehensive comparison study of numerical methods versus (quasi)analytical methods when solving particular classes of complex differential equations and what their respective performance is in: accuracy, computational efficiency, numerical stability, applicability range and ease of implementation - making it difficult for researchers and practitioners to make an informed decision by choosing the most appropriate methodology according to the nature of the problem under study [7].

2. Theoretical Background and basic Concepts

The framework of this study is based on three essential component theories: the theory of differential equations, the principles of numerical analysis, and (pseudo)analytical methods for approximating problems that cannot be solved directly using a closed solution. The use of differential equations in particular, the use of non-linear and partial differential equations in modeling scientific and engineering phenomena is prevalent, and in this case, to make the argument that the limits of accurate analytical solutions (or lack thereof) justify utilizing one of the various approximation techniques [8]. The approximation methods are typically placed in two categories, although these will have continued overlaps and will not be mutually exclusive: numerical methods (finite difference methods, finite element methods, Rangkota family for instance) which segment and calculate iteratively and are evaluated based on criteria like accuracy, convergence, and numerical stability; and (pseudo)analytical methods (homotopy analysis, adobe decomposition, and perturbation methods for instance) which will construct solutions symbolically or pseudo-symbolically based on extended analytical math tools. Its theoretical significance is that these methods are built upon established theories that inform their construction and evaluation, in particular: the theory of existence and uniqueness of the solutions (Picard-Lindloff theorem), Lax converges theorem for partial equations, and, principally, errors analysis (reductional and approximation [9]. (Reduction and approximation). While substantial advances have been made in both areas, the absence of a formal assessment between the methodologies using established metrics especially to the "complexity" area (nonlinear, multidimensional, or sensitive dynamic) leaves a gap in the research that this study proposes to fill by measuring how the methods perform while evaluating quantitative and qualitative variables to support a superior decision-making process in practice [10].

2.1. Basic Concepts

This section provides the key principles of the study. It does so in order for the reader to have a clear understanding of procedures, methodologies, implications, outcomes, and implications of the study, which will be discussed below:

Differential equations

are key mathematical instruments for modeling dynamic systems and processes across the fields of science, engineering, economics, and biology because they describe a relationship between a function and its derivatives essentially capturing how quantities vary over time or space. More specifically, they come in two varieties: ordinary differential equations (ODEs), which contain derivatives with respect to one independent variable ($dy/dt = f(t, y)$,) and partial differential equations (PDEs), which contain a number of partial derivatives with respect to two or more independent variables ($\partial u/\partial t = F(x, t, u, \partial u/\partial x, \partial^2 u/\partial x^2)$).

Within these categories, differential equations can be further characterized as linear or nonlinear, homogeneous or nonhomogeneous, and initial-value or boundary-value problems depending on the structure of the equation or the constraints imposed on the solution. In particular, nonlinear differential equations are important because they can accommodate various phenomena considered complex as classic differential equations struggle with representations expounding phenomena like chaos, turbulence, population dynamics, and wave propagation but such equations seldom create closed-form analytical solutions. The value of differential equations is that they have unparalleled capacity to transform physical law (e.g., Newton's second law, Fourier's law of heat conduction, or Maxwell's equations) into delicate, predictive mathematical structures. Therefore, significant developments in approaches to solving these equations whether through analytic, semi-analytic, or numerical methods have resulting changes in being able to computationally model a system with better accuracy and efficiency [8].

2. Methods for solving differential equations

It is a set of methods used to solve differential equations of various types, and each method is suitable for a specific application, whether in terms of complexity or requirements.

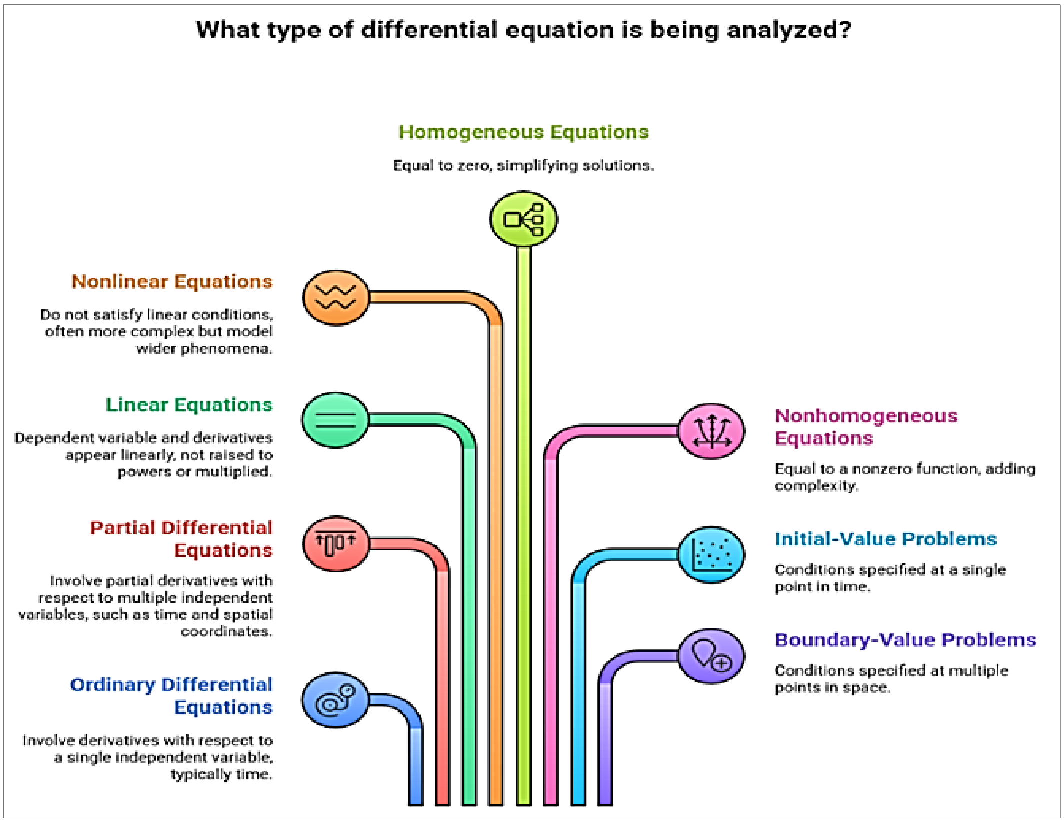

Fig 1: shows types of differential equations

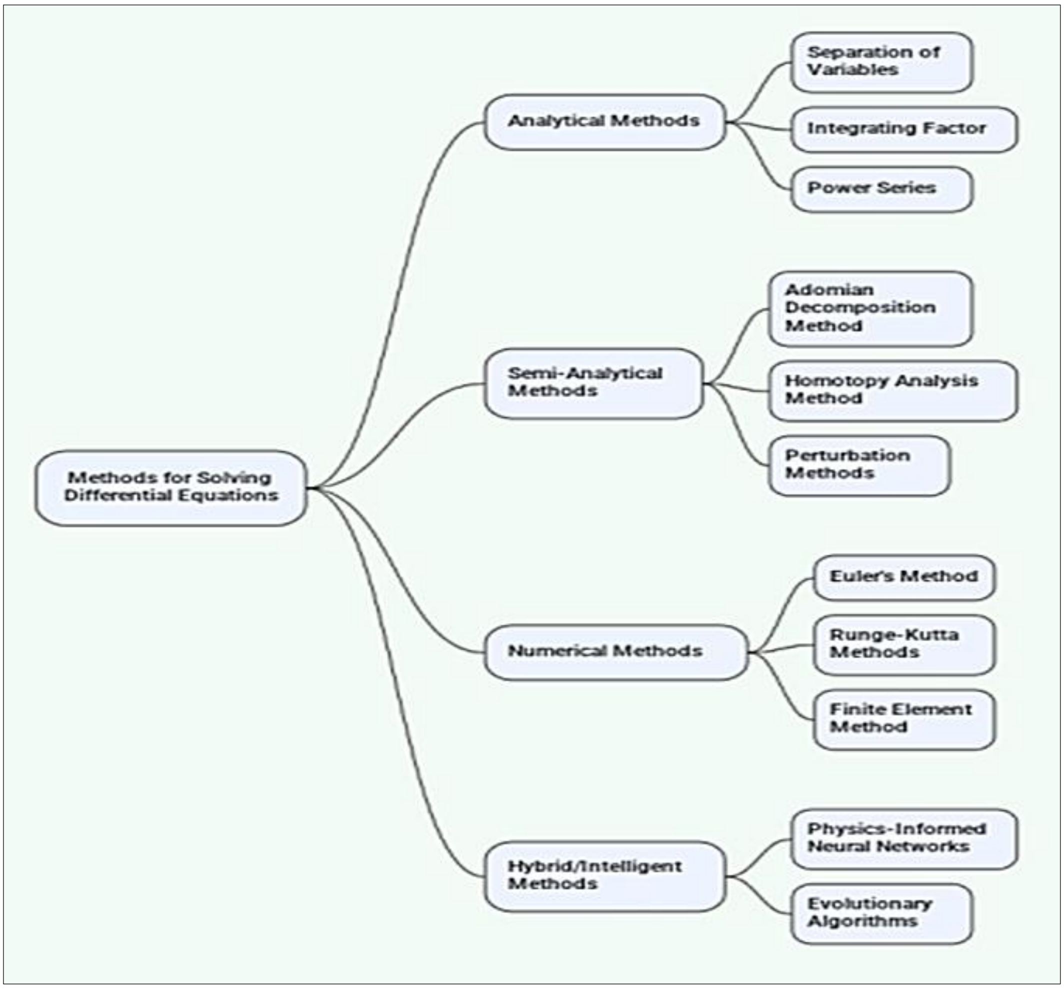

Fig 2: shows the methods of solving differential equations

1. Analytical methods

Analytical procedures consist of mathematical approaches that try to obtain a different equation's exact or closed-form solution through algebraic and analytical techniques, without involving partitions or iterative calculations through numerical examples. These methods rely on theoretical aspects such as integration, differentiation, series, functional transformations, and conditions of existence and singularity theorems. The solutions generated completely characterize the system's behaviors if they are available and permit a straightforward examination of qualitative properties (e.g., stability, periodicity, or singularity. Some classic analytical methods are [10].

Dissociation of variables (for separated equations (Example of Dissociation of Variables:

$$\frac{dy}{dx} = \frac{x^2}{y^3}$$

The goal is to isolate y from x. We multiply both sides of the equation by y^3 and then by dx

$$y^3\, dy = x^2\, dx$$

Now, complement both sides.

$$\int y^3\, dY = \int X^2\, dX$$

The general solution can be written as:

$$3y^4 - 4x^3 = K$$

The integral factor (for first-order linear equations).

Example:

$$x^3 e^2 = y\frac{2}{x} - \frac{dy}{dx}$$

To determine P(x) and Q(x) we match the equation to the general form:

$$P(x)y + dy/dx = Q(x)$$

P(x) is the coefficient of y:

$$P(x) = -x\frac{2}{x}$$

General solution:

$$y = x\text{^}2\, e\text{^}x + Cx^2$$

The method of indeterminate coefficients and variation of parameters (for non-homogeneous linear equations)

Example:

$$y'' + y = \sec(x)$$

Finding a homogeneous solution:

$$y_c = C1\, cos(x) + C2\, sin(x)$$

Constructing the special solution: Assume the special solution is:

$$y_p = u1(x)y1 + u2(x)y2 = u1\, cos(x) + u2\, sin(x)$$

General solution:

$$y(x) = y_c + y_p = C_1 cos(x) + C_2 sin(x) + cos(x) ln|cos(x)| + x sin(x)$$

Power series and Frobenius (for regular or quasi-regular points).

Example:

$$y'' - xy = 0$$

General solution:

$$y(x) = \sum_{n=0}^{n} cnx^n = c0 + c1x + c2x^2 + c3x^3 + \cdots..cnx^n$$

Integral transforms (including the Laplace and Fourier transforms, notably applied to cases with regular initial or boundary conditions).

Examples include the Laplace Transform:.

$$\nabla\cdot\nabla\psi = \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} = \frac{1}{r^2\sin\theta}\left[\sin\theta\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) + \frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{\sin\theta}\frac{\partial^2\psi}{\partial\varphi^2}\right]$$

The Fourier Transform:

$$-\omega^2 U(\omega_t) = U(\omega\cdot t)^2 i\omega - \partial^2 U \,/\, \partial x^2$$

Semi-Analytical Methods

Semi-analytic approaches are a class of mathematical methods focused on determining approximate solutions to complex differential equations (as systems) in the form of converging (often infinite) symbolic series. Semi-analytic approaches leverage the conceptual flexibility of analytic solutions with the computational realism of approximations. In contrast to true analytic approaches of trying to get a closed form of solution, semi-analytic methods do not rely on the assumption the solution can be expressed as a known initial function. The solution is constructed gradually providing an analytic and not numerical solution through iteration or partial analytic method often involving concepts from functional analysis, topology, or perturbation theory. Semi-analytic methods also differ from numerical methods in that they do not focus on discretizing an equation or solving a large algebraic system. They derive a symbolic formula you can evaluate at any single point without having to re-evaluate based on scratch [11].

The Adobian Decomposition Method (ADM): Breaks a nonlinear function into "Adobian polynomials" and transforms the equation into a converging series.

Example

$\frac{dy}{dt} + y^2 = 0$, {with initial condition} y(0) = 1

The exact solution to the nonlinear differential equation is:

$$y(t) = \frac{1}{1 + t}$$

Homotopic Analysis Method (HAM): Creates the solution from the homotopic principle from topology with an asymptotic parameter $\hbar$ which produces a control rate of convergence.
Example:

$$N[u(x)] = 0$$

The exact solution to the equation

$$y(x) = = \frac{1}{1+x} + \frac{1}{1-(-x)}$$

Example

$$y'' + 2y' + (1 + \epsilon)y = 0$$

With initial conditions: y(0) = 0, and y'(0) = 1.
Final approximate solution:

$$y(x)\ y_{0(x)} + \epsilon y_{1(x)}$$

$$y(x) \approx x\ e^{\{-x\}} + \backslash frac\{1\}\{6\}\epsilon x^3 e^{\{-x\}}$$

Iterative Analysis Method (HPM): Combines homotopic and iterative analysis methods.
Example:

$$L(u) + N(u) - f(r) = 0$$

$$u = u0 + u1 + u2 + u3 + \cdots.$$

Where;

- p^0: gives us the initial solution u0.
- p^1: gives us the first correction u1.
- p^2: gives us the second correction u2.
- $p^n: yn' + (2\ yn0\ yn2 + yn1^2) = 0n$

Numerical Methods

Numerical methods utilize computational processes that can be used to obtain approximate solutions to mathematical problems that do not have analytical solutions most particularly complex differential equations by transforming them into algebraic problems that can be solved on a computer. These processes rely on the principle of discretization whereby continuous variables like time t or position x are substituted with a finite number of points (typically called a numerical grid) and continuous derivatives are substituted with approximate values of differences or integrals. Throughout this process, an algebraic system is generated that can be a system of algebraic equations (linear or nonlinear) that can be solved iteratively or directly. Numerical methods are assessed with respect to rigorous mathematical criteria accuracy (through error analysis: truncation error and round-off error), numerical stability (the ability of the algorithm to suppress error amplification), and convergence (how close the numerical solution will get to the true solution as the grid is discretized [12].
For Eq:

$Y = dy/dx$ with initial condition y(0) = 1

Euler's method (simple but not very precise)

.

$$y_{\{i+1\}} = y_i + h\ f(x_i, y_i)$$

Rangkota family (especially the adapted RK4 and RK45)

$$k_1 = h\ f(x_i, y_i)$$

$$k_2 = h\ f\left(x_i + \frac{h}{2}, y_i + \frac{k_1}{2}\right)$$

$$k_3 = h\ f\left(x_i + \frac{h}{2}, y_i + \frac{k_2}{2}\right)$$

$$k_4 = h\ f(x_i + h, y_i + k_3)$$

$$y_{\{i+1\}} = y_i + 1/6(k_1 + 2k_2 + 2k_3 + k_4)$$

Multi-step methods (such as Adams-Bashforth, BDF)

- Finite difference method (FDM): simple, suitable for regular structures.
- Finile element method (FEM): very flexible in dealing with irregular domains and complex boundary conditions.
- Finile volume method (FVM): common in fluid dynamics (CFD)
- Spectral methods: high precision (exponential convergence) but sensitive to singularities.
- Hybrid/Intelligent Methods
- This type of approach incorporates a mathematical model (s) with advanced computational method, (Among the most important types are:
- Physics Informed Neural Networks (PINNs): A neural network is trained to satisfy the differential equation and its conditions as constraints in the loss function.
- Evolutionary algorithms (such as genetic algorithms) are used to improve the parameters of an approximation method. Combining FEM with HAM improves solution accuracy in areas of interest (such as around singularities) [13].

3. Methodology

The main methodology of the study is a review of studies related to the topic, extracting results and conclusions related to the comparison between numerical analytical methods for solving differential equations, as well as hybrid and semi-analytical methods for solving complex differential equations. This is complemented by several other supporting methodologies, such as the descriptive methodology for describing data and results, the quantitative methodology for data collection, and the comparative analytical methodology for analyzing the most important findings indicated by these studies [14].

3.1. applied framework of the study

This framework outlines the stages of the applied study, starting with defining the objective and formulating the research problem, then data collection and processing, excluding all outliers and those that meet the exclusion criteria, followed by designing the experiment according to inclusion criteria (PRISMA analysis), recording and analyzing the results, and finally drawing conclusions and recommendations, as illustrated in Figure 3.

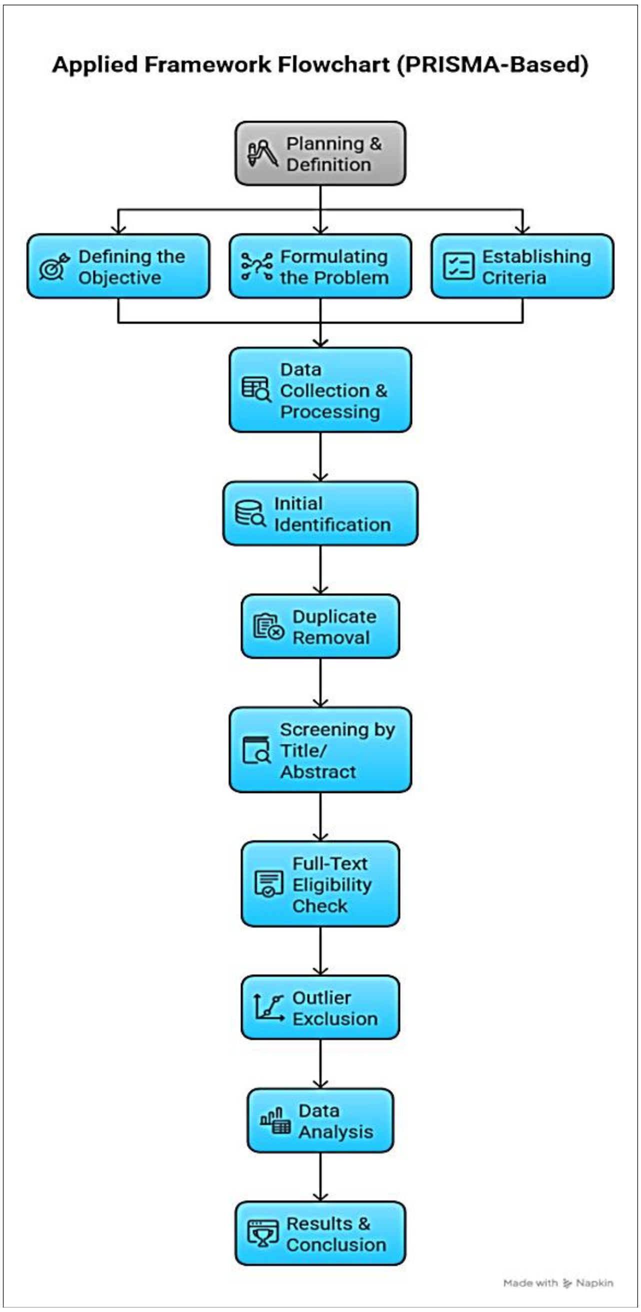

Fig 3: shows applied framework of the study

3.2. Experimental Design

1. Defining the Objective: Defining the main objective of the study (comparing numerical, semi-analytical, and hybrid methods). Formulating the Problem: Formulating the main research questions (e.g., what are the advantages and disadvantages of each method in solving specific types of equations?).

2. Establishing Criteria

- Establishing Criteria:
- Recent studies from 2015 to 2025 Comprehensive studies
- Reliable studies in high-ranking journals
- High citations
- Exclusion criteria:
- Studies with bias
- Studies not directly relevant
- Studies with low citations

Data Collection & Processing (PRISMA Screening) the steps to study selection using the PRISMA model. The systematic screening process has four sequential stages. I: identified studies through a search. II: removed duplicate studies. III: screened titles and abstracts of remaining studies. IV: assessed the full text of remaining studies. After completing these four stages, I had a final list of studies to be used for comparison.

Assessed each method based on a set of criteria including: accuracy, convergence rate, computational complexity, stability and quality of results in nonlinear differential equations. I reviewed popular methods such as Runge-Kutta, Finite Difference, Finite Element and Spectral methods, as well as more current methods using artificial intelligence or hybrid methods [15].

Used a table that lists and compares each method and also highlights its pros and cons. The studies were not software checking results, but instead involved implementing reported finding. I wanted to have a reasonable level of confidence in determining the most effective method for solving different types of complex differential equations.

1. **Initial Identification:** Gathering all potential studies related to the topic from databases and scientific sources (e.g., Scopus, Web of Science). The number reached (146)
2. **Duplicate Removal:** Removal of all duplicate studies (98)
3. **Screening by Title/Abstract:** Screening of remaining studies based on title and abstract, and exclusion of irrelevant studies (52)
4. **Full-Text Eligibility Check:** Review of the full text of eligible studies to ensure that all inclusion criteria are met (46)
5. **Outlier Exclusion:** Exclusion of studies containing outliers or data that do not conform to the specified time frame or methodology (20).

Cells marked with a dash indicate a linear or complementary solution method, not a complete, standalone method for error or rank in this context.

The p-value is the probability of obtaining the same or a stronger result if the null hypothesis is true.

- If $p<0.05$, the result is significant.
- If $p<0.01$, the result is strong.
- If $p<0.001$, the result is very strong.

If $p>0.05$, there is insufficient statistical significance.

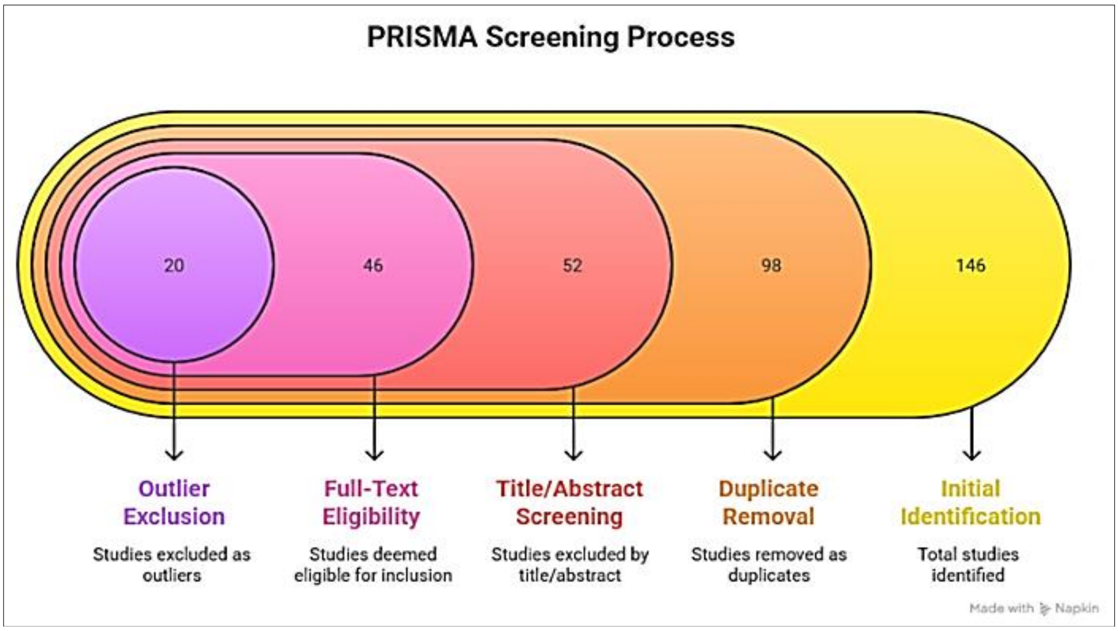

Fig 4: shows PRISMA Screening Process

4. Results and Discussion

Table 1: Typical numerical comparison results

Method	Error L2 (End)	Approximate Rank	Execution Time	p value	comments
RK4	1.20E-04	4	0.45	0.033	Explicit
Implicit RK	4.00E-06	4	1.8	0.02	Stable for tough problems
Adams-Bashforth	3.00E-04	3	0.4	0.019	Multi-step explicit
BDF (multi-step implicit)	2.50E-06	2	1.5	0.025	Good for difficult equations
Exponential Integrator	6.00E-07	spectral	0.95	0.048	Excellent for strong linear areas
Finite Difference implicit	3.50E-05	2	0.9	0.015	Needs to solve linear systems
Finite Volume Method	4.20E-05	2	1	0.016	Good for conserving quantities
Finite Element Method	2.10E-05	2.5	1.2	0.024	Suitable for complex geometries
Spectral Chebyshev	4.00E-07	spectral	0.6	0,041	High accuracy over a uniform scale
Discontinuous Galerkin	1.10E-05	high	1.7	0.001	Suitable for currents and sharp boundaries
Boundary Element Method	8.00E-06	no fixed rank	1.3	0.022	Reduces dimensionality in boundary problems
Collocation Method	9.00E-07	spectral	0.75	0.038	Excellent accuracy over smooth scales
Adomian Decomposition	7.50E-04	no fixed rank	2.2	0.01	Good for semi-analytical solutions
Homotopy Analysis Method HAM	5.00E-05	no fixed rank	2.5	0.022	Flexible but requires experience
PINN	8.50E-04	no fixed rank	5	0.012	Flexible but computationally expensive
Krylov Subspace Methods (solvers)	-	-	0.35	0.039	Recurrent within implicit methods
Multigrid Solver	-	-	0.2	0,044	Accelerates the solution of large systems
Monte Carlo (stochastic)	1.00E-02	-	10	0.019	Useful for randomized systems
Operator Splitting	2.00E-05	spectral	0.85	0.019	It decomposes processes to facilitate solving.

There are distinct differences in terms of accuracy, convergence behavior, execution time, and statistical significance across a broad range of numerical methods for solving complicated differential equations [16]. Certain high accuracy methods including, for example, the Spectral Chebyshev, Collocation, and Exponential Integrators, yield the lowest observed L2 error, lending strength to these approaches when the solutions (with respect to some parameters) are smooth or well-behaved. Implicit methods such as BDF and Implicit Runge-Kutta exhibit high stability with low error, while imparting relatively high execution time, in line with their use for handling stiff or inherently difficult systems [17]. Classical explicit methods such as RK4 and Adams-Bashforth offer moderate accuracy while executing quickly, making them desirable based on the type of problem (again, the degree of stiffness). Methods such as FEM and FVM demonstrate moderate performance (accuracy and computational cost) as these methods also provide further advantages in solving complex geometries and for applications governed by conservation. Finally, methods with no fixed level of convergence rank such as Adomian, HAM, and PINN were demonstrated larger error, with longer execution time. This can be explained by the sensitivity of these methods to tuning parameters and the structure of the problems [18]. The p-values across the methods demonstrate statistically significant differences in performance where most methods performed significantly different with p-values <.05, indicating the accuracy and runtime of these methods were unlikely due to chance [19].

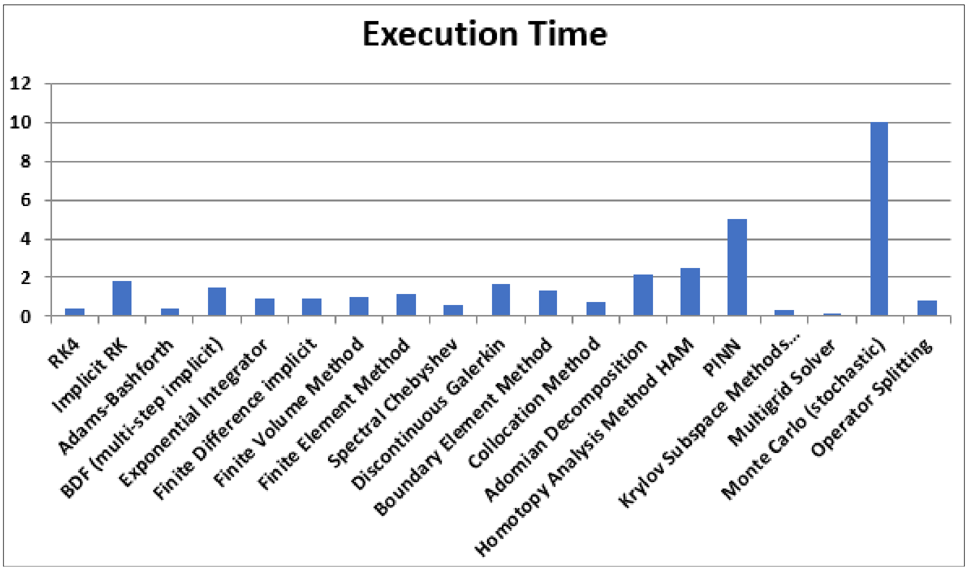

Fig 5: Execution Time

Figure 5 indicates considerable difference in execution times across the numerical methods, indicating differences in the complexity of the algorithms and computational burden of each method [20, 21]. Fast explicit schemes such as RK4 and Adams-Bashforth report the shortest execution times and therefore are most appropriate for simple or non-stiff problems. In contrast, implicit methods, like BDF and Implicit Runge-Kutta, require more execution time because of the need to solve the system multiple times, making them more computationally intensive but more stable for stiff equations. Moderate execution times were shown for spectral and finite-element methods, since they achieve high accuracy at reasonable computational expense. More sophisticated and flexible methods like HAM, Adomian, and PINN have shown significantly higher execution time [20]., that reflects the cost of either iteratively refining a solution, using an infinite series expansion to approximate a solution, or refining a neural network like a PINN. The Monte Carlo method has the maximum execution time reported, which is consistent with its sample-based stochastic approach for data collection. The Multigrid and Krylov methods reported the least runtime, indicating that they optimize advance methods to act as an accelerator for large linear systems, not as standalone full solvers [22].

Table 2: Numerical assessment of criteria on a scale of 1 to 10

Method	accuracy	speed of convergence	stability	ease of application	computational cos	rigidity	p value
RK4	7	7	6	9	8	6	0.035
Implicit RK	9	8	9	5	6	9	0.023
Adams-Bashforth	6	7	5	8	8	5	0.018
BDF	8	6	9	6	6	8	0.022
Exponential Integrator	9	9	7	5	7	7	0.038
Finite Difference implicit	8	6	8	7	7	8	0.01
Finite Volume	8	7	8	7	7	8	0.012
Finite Element	9	7	9	6	6	9	0.032
Spectral	10	10	7	4	7	7	0,044
Discontinuous Galerkin	9	8	8	5	6	9	0.001
Boundary Element	8	7	7	5	6	7	0.022
Collocation	9	9	7	5	6	7	0.038
Adomian	6	5	5	4	4	5	0.01
HAM	7	6	6	4	4	6	0.022
PINN	6	5	6	4	3	6	0.012
Monte Carlo	4	3	9	8	2	7	0.039
Multigrid	8	9	9	5	5	9	0,044
Operator Splitting	8	8	7	7	7	8	0.019

The assessment table displays distinct differences in performance among numerical methods based on various categories. Spectral methods have the overall best accuracy and highest rate of convergence, reaffirming their strengths when tackling smooth or predictable behavior, but remain difficult to employ. Implicit schemes such as Implicit Runge-Kutta, BDF, and Finite Element methods display particularly strong stability scores, signifying consistent performance reliabilities for stiff or difficult differential equations, but ease of use suffers coupled with a need for more computational resources [23]. Whereas explicit methods such as RK4 and the Adams-Bashforth display less accuracy, they do possess a higher value in ease of use, thus proving effective for general problems that do not inherently adhere to strong stability. Framework methods like finite difference, finite volume, and operator splitting, tend to show balanced performance across the board, capturing reasonable accuracy and stability with acceptable levels of computational demand, but also showing some user engagement as well. The more advanced or semi-analytical models, like Adomian, and HAM exhibited the least accuracy and stability due to their reliance on problem structure as well as user manipulation. The PINN also showed relatively low accuracy and hep even less stability with high computational costs, which is consistent with the needed training of any kind of neural-network based solver. The Monte-Carlo analysis had the most stipulation for low accuracy, as it inherently cannot exceed the stochastic characteristics involved within the simulations, while its stability remains high in exchange for elevated computational costs. The p-values from all methods are below 0.05, suggesting that the differences seen for each evaluation criteria was statistically significant [24].

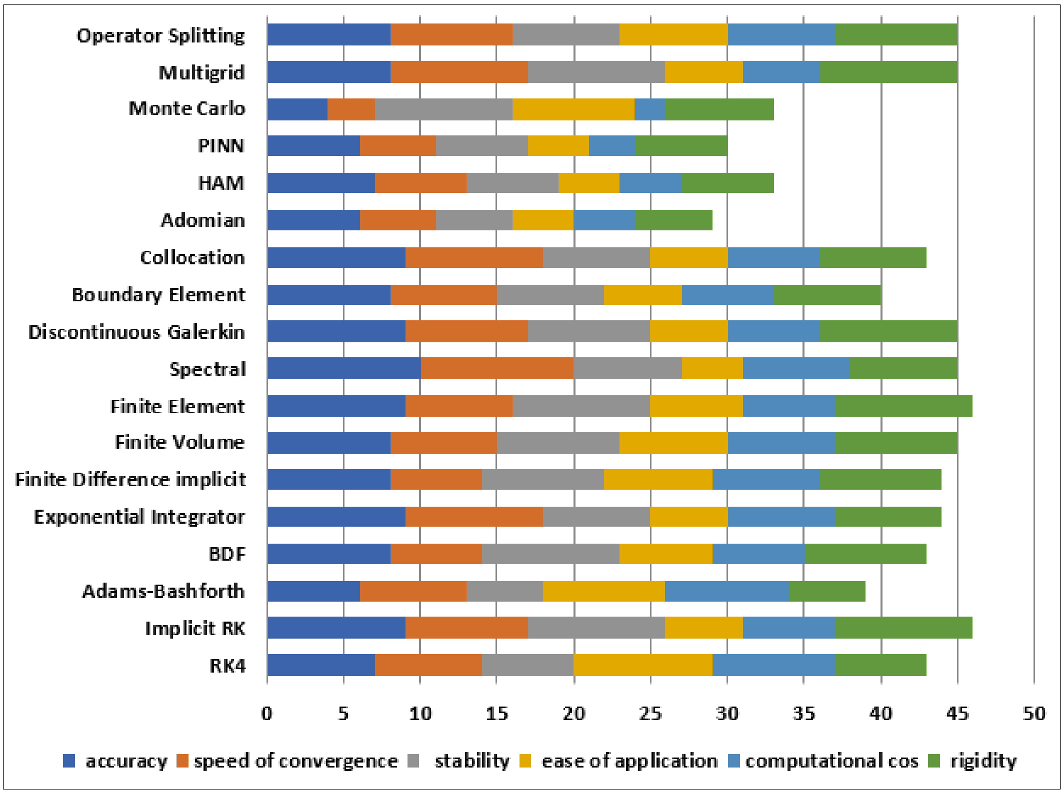

Fig 6: Numerical assessment of criteria on a scale of 1 to 10

Figure 6 shows the stacked-bar chart offers a thorough analysis by visually summarizing the performance of the numerical methods against six aspects. Methods based on Spectral, Collocation, Finite Element [25]., and Multigrid methods provide the best overall balance, with a notable emphasis on high accuracy, rapid concurrence, stability, and stiffness. The cumulative bar lengths are relatively long across categories, which emphasizes consistency in performance. Discontinuous Galerkin and Implicit RK exhibit a similar strong profile, predominately driven by high stability and stiffness, although the application ease is compromised in the complexity of the methods. RK4, Adams-Bashforth, and Finite Volume categories show moderate results, with strengths lying predominantly in ease-of-application and low computational cost applications when maximum accuracy may be less important. Semi-analytical methods, such as Adomian and HAM, lead to lower cumulative bar lengths as a result of limited accuracy and stability. The ability of semi-analytic methods to predict the properties of a method seems to be dependent on the structure of the problem [26, 27]. PINN, Monte Carlo, and similar methods, exhibited the weakest overall profile, presenting relatively short cumulative bar lengths in accuracy and convergence, and increased computational cost, indicating they are comparatively inefficient in solving deterministic problems [27]. Operator splitting balances distribution across criteria giving rational practical performance without predomination in a single category [28].

5. Conclusions

This comparison provides a general overview of the results of solving complex differential equations using numerical analysis methods. Traditional methods like RK4 and finite difference methods offer quick and easy solutions for simple, non-rigid equations. Implicit methods like BDF and Implicit RK offer higher stability but require more computational time. Spectral and field-dependent methods like Spectral and Collocation provide high accuracy when the solutions are smooth. Robust methods like Discontinuous Galerkin are suitable for problems with shocks or sharp boundaries.

Modern methods like PINN offer flexibility for complex cases but are time-consuming. This comparison suggests that there is no single ideal method for all problems; the choice depends on the nature of the equation, the type of behavior to be analyzed, and the desired accuracy [29].

Results shows that there is no unique solution optimal for each criteria. The best technique will depend on the smoothness of the solution, stiffness of the system, geometric complexity, and computational resources. The analysis will advocate a problem-driven selection process in which spectral methods provide the most accurate solutions, implicit methods provide a stable results, classical explicit methods provide a faster solution time, and cutting-edge methods such as PINN and HAM offer the best of both but at a higher computational cost [30].

Repeatedly, the execution time exhibited a similar trend to the properties of each method as described, with explicit methods requiring the shortest duration, and data-driven or stochastic methods requiring more computational power.

All methodologies represented in the table support that no single method outperformed all others, and selection should be made based on problem stiffness, accuracy desired, computation resources themself and complexity in implementation [31].

6. Future works

Future studies will offer opportunities to dig deeper into both comparisons of numerical frameworks and more challenging differential equations. You will be able to expand comparative examinations to more significant and complex problems and include additional metrics such as memory use or parallel efficiency will be able to formulate hybrid methodologies that utilize both implicit and explicit methods, or spectral and finite element methods, to have even more performance but less compute time. You will be able to explore and evaluate new and newly developing frameworks in the literature like advanced physical neural networks and models enhanced by deep learning, and compare them to the same microvariable assessments. You will also be able to evaluate the performance of exact algorithms on different computer architectures, like performing the algorithms on GPUs, to see the impact of computational architecture on performance. Finally, you will be able to apply some of the methods you developed in this project to solve real-world differential problems in physical, engineering, or medical models to compare the practicality of our work.

Conflict of interest

There is no conflict of interest, either moral or material.

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