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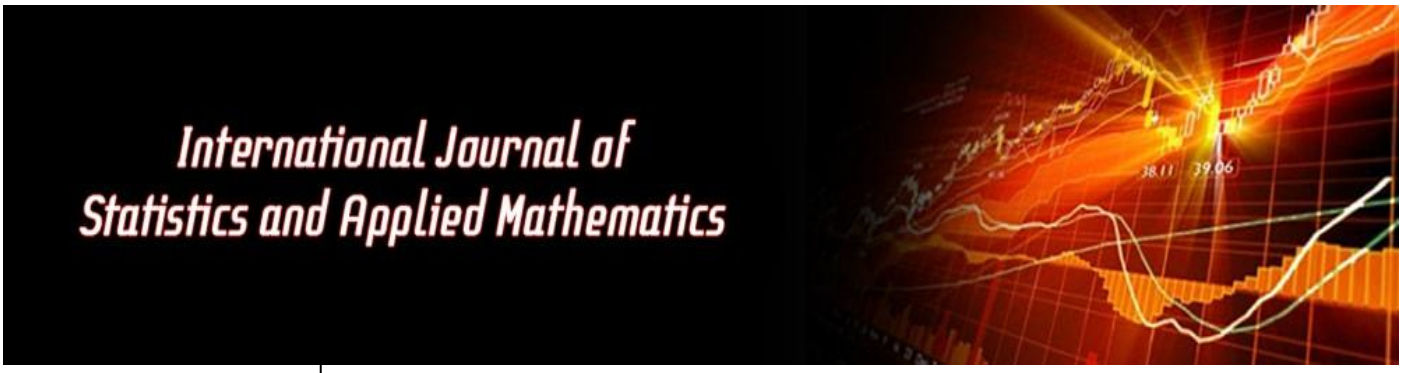

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The inhomogeneity effects on love waves in heterogeneous transversely isotropic layered media

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Abstract
This paper is concerned with to investigate the effects of inhomogeneities on Love waves in a laterally and vertically inhomogeneous transversely isotropic elastic media composed of a half-space under a layer of finite thickness with stress-free upper surface. The elastic parameters and density are assumed to vary exponentially. This is a two-dimensional problem. To analyse the effects of inhomogeneities numerical computations are performed to obtain the dimensionless phase velocity and dimensionless group velocity. Numerical calculations are done in two particular cases. All the numerical results are plotted graphically and discussed. It is observed that inhomogeneities have significant effect on Love wave propagation in a layered media.

Keywords: Inhomogeneities, love waves, transversely isotropic, elastic media, numerical computations

1. Introduction
Reasonable grounds are available which ensure the earth structure is not homogeneous and isotropic throughout. Various types of inhomogeneity and anisotropy have been incorporated in research literature. The variations in elastic parameters effect the propagation of surface waves. Bhattacharya (1970) [2], Chatterjee (1972) [5], Negi and Singh (1973) [7] have derived the frequency equation for Love waves in a laterally inhomogeneous layer lying over a homogeneous half-space. The various authors and researchers, Chattaraj *et al*. (2013) [4], Kundu *et al*. (2016) [8], Manna *et al*. (2015) [9] and S. Bhatt S. Manna (2024) [3] have worked on the propagation of surface waves taking different modal of elastic layered media.
Anita Rana (1997) [1] has derived the frequency equation and group velocity expressions by taking heterogeneous and transversely isotropic layered media having a half-space under layer and with stress-free upper surface. Using the material values of parameters [Umesh, 1977] computation calculations are performed on the expressions so obtained to analyse the effect of inhomogeneities on Love waves. Numerical calculation is done in two particular cases. All the results are discussed and plotted graphically.

2. Formulation of the problem
A layered media consisting of a layer of finite thickness *H* having stress-free upper surface lying over a half-space in a laterally and vertically inhomogeneous transversely isotropic elastic media is considered. The Cartesian co-ordinate system is considered such that the *x*-axis is in the duration of propagation of Love wave and *z*-axis is vertically downwards. The displacement is only along *y*-axis. The problem is two-dimensional in *xz*-plane and all quantities are independent of *y*-coordinates i.e. $\frac{\partial}{\partial y}=0$. The rigidities and densities of the layer and half-space are assumed as:

For layer

$$[N, L, \rho]=[N_0, L_0, \rho_0]p(x)r(z)$$

where N_0, L_0, ρ_0 are constants.

For half-space

$$[N, L, \rho] = [N_{01}, L_{01}, \rho_{01}]\, p(x) r(z)$$

where $N_{01}, L_{01}, \rho_{01}$ are constants.

N, L are directional rigidities and ρ is the density of the medium.
Geometry of the problem is shown in Fig. 1.

3. Equation of Motion and Its Solution

Describing *SH*-type of motion for which $u_1 = u_3 = 0, u_2 = v$, the equation of motion becomes

$$\frac{\partial}{\partial x}\left(N \frac{\partial v}{\partial x} \right) + \frac{\partial}{\partial z}\left(L \frac{\partial v}{\partial z} \right) = \rho \frac{\partial^2 v}{\partial t^2}, \tag{1}$$

Considering time harmonic solution, the solution of (1) gives the displacement

$$v(x, z, t) = \frac{1}{\left[L_0 p(x) r(z) \right]^{\frac{1}{2}}} \left[A e^{S_1 z} + B e^{-S_1 z} \right] e^{ik(x - ct)}, \tag{2}$$

and stress component

$$\tau_{yz} = L e_{yz},$$

$$= L \frac{\partial v}{\partial z},$$

$$= \left[\left\{ L_0 p(x) r(z) \right\}^{\frac{1}{2}} S_1 \left(A e^{S_1 z} - B e^{-S_1 z} \right) - \frac{1}{2} \frac{L_0 p(x) r'(z)}{\left[L_0 p(x) r(z) \right]^{\frac{1}{2}}} \left(A e^{S_1 z} + B e^{-S_1 z} \right) \right] e^{ik(x - ct)}, \tag{3}$$

Where

$$S_1 = \left(\frac{N_0}{L_0} \right)^{\frac{1}{2}} \left(\varepsilon^2 + k^2 + \frac{L_0}{N_0} b^2 - \frac{\omega^2}{\beta_{110}^2} \right)^{\frac{1}{2}}, \text{ if } \left(\varepsilon^2 + k^2 + \frac{L_0}{N_0} b^2 \right)^{\frac{1}{2}},$$

$$-i \left(\frac{N_0}{L_0} \right)^{\frac{1}{2}} \left(\frac{\omega^2}{\beta_{110}^2} - \varepsilon^2 - k^2 - \frac{L_0}{N_0} b^2 \right)^{\frac{1}{2}}, \text{ if } \frac{\omega}{\beta_{110}} > \left(\varepsilon^2 + k^2 + \frac{L_0}{N_0} b^2 \right)^{\frac{1}{2}}. \tag{4}$$

$$\omega = ck, \beta_{11} = \left(\frac{N}{\rho} \right)^{\frac{1}{2}}, \beta_1 = \left(\frac{L}{\rho} \right)^{\frac{1}{2}}.$$

In half-space: The displacement component v_1 and stress component $\left(\tau_{yz} \right)$ are

$$V_1(x, z, t) = \frac{1}{\left[L_{01} p(x) \ell(z) \right]^{\frac{1}{2}}} A_1 e^{-S_2 z} e^{ik(x - ct)}, \tag{5}$$

$$\left(\tau_{yz}\right)_1 = L_1 \frac{\partial v_1}{\partial z} = -\left[\left\{L_{01} p(x) \ell(z)\right\}^{\frac{1}{2}} S_2 + \frac{1}{2} \frac{L_{01} p(x) \ell'(z)}{\left[L_{01} p(x) \ell(z)\right]^{\frac{1}{2}}}\right] A_1 e^{-S_2 z} e^{ik(x-ct)}, \tag{6}$$

where

$$S_2 = \left(\frac{N_{01}}{L_{01}}\right)^{\frac{1}{2}} \left[\varepsilon^2 + k^2 + \frac{L_{01}}{N_{01}} a^2 - \frac{\omega^2}{\beta_2^2}\right], \tag{7}$$

and

$$\beta_2 = \left(\frac{N_{01}}{\rho_{01}}\right)^{\frac{1}{2}}.$$

Using the appropriate boundary conditions and assuming

$$r(z) = e^{d_1 z},$$

$$\ell(z) = e^{d_2 z},$$

$$p(x) = e^{\delta x}, \tag{8}$$

where d_1, d_2, δ are constants. The dispersion equation

$$\tan(SH) = \frac{(SH)\frac{L_{01}}{L_0}\left(S_2 H + \frac{1}{2} d_2 H\right)}{\frac{1}{2}(d_1 H)\left\{\frac{1}{2} d_1 H - \frac{L_{01}}{L_0}\left(S_2 H + \frac{1}{2} d_2 H\right)\right\} + (SH)^2}, \tag{9}$$

where

$$SH = \left(\frac{N_0}{L_0}\right)^{\frac{1}{2}} \left\{kH\left[\frac{c^2}{\beta_{110}^2} - \frac{(\delta H)^2}{4(kH)^2} - 1 - \frac{L_0}{N_0} \frac{(d_1 H)^2}{4(kH)^2}\right]^{\frac{1}{2}}\right\}, \tag{10}$$

and

$$S_2 H = \left(\frac{N_{01}}{L_{01}}\right)^{\frac{1}{2}} \left\{kH\left[1 + \frac{L_{01}}{N_{01}} \frac{(d_2 H)^2}{4(kH)^2} + \frac{(\delta H)^2}{4(kH)^2} - \frac{c^2}{\beta_{110}^2} \frac{\beta_{110}^2}{\beta_2^2}\right]^{\frac{1}{2}}\right\}, \tag{11}$$

Group velocity is

$$\frac{U}{\beta_{110}}=\frac{\begin{array}{l}\dfrac{N_0}{L_{01}}\dfrac{1}{SH}E^2\sec^2(SH)-\dfrac{N_0}{L_0}\dfrac{1}{SH}\left(S_2H+\dfrac{1}{2}d_2H\right)\left\{E-2(SH)^2\right\}\\ +\dfrac{SH}{S_2H}\left[\dfrac{N_{01}}{L_{01}}E+\dfrac{1}{2}\dfrac{N_{01}}{L_0}d_1H\left(S_2H+\dfrac{1}{2}d_2H\right)\right]\end{array}}{\begin{array}{l}E^2\sec^2(SH)\dfrac{N_0}{L_{01}}\dfrac{1}{SH}\dfrac{c}{\beta_{110}}-\dfrac{N_0}{L_0}\dfrac{1}{SH}\dfrac{c}{\beta_{110}}\left(S_2H+\dfrac{1}{2}d_2H\right)\left\{E-2(SH)^2\right\}\\ +\dfrac{SH}{S_2H}\dfrac{c}{\beta_{110}}\left(\dfrac{\beta_{110}}{\beta_2}\right)^2\left[\dfrac{N_{01}}{L_{01}}E+\dfrac{1}{2}\dfrac{N_{01}}{L_0}d_1H\left(S_2H+\dfrac{1}{2}d_2H\right)\right]\end{array}} \tag{12}$$

where

$$E=\frac{1}{2}(d_1H)D+(SH)^2, \tag{13}$$

and

$$D=\frac{1}{2}d_1H-\frac{L_{01}}{L_0}\left(S_2H+\frac{1}{2}d_2H\right). \tag{14}$$

$$\beta_{110}=\left(\frac{N_0}{\rho_0}\right)^{\frac{1}{2}},\ \beta_2=\left(\frac{N_{01}}{\rho_{01}}\right)^{\frac{1}{2}}.$$

4. Numerical Calculations and Discussion

The values selected for non-dimensional parameters are [Umesh, 1977].

$\frac{N_0}{L_0}=0.68$, $\frac{N_{01}}{L_{01}}$=0.54, $\frac{L_{01}}{L_0}$ = 1.845,

$\frac{N_{01}}{L_{01}}$ = 0.997, $\frac{N_{01}}{L_{01}}$ = 0.366, $\frac{\beta_{110}^2}{\beta_2^2}$ = 0.2

δH =0, 1, 1.8; d_1 H=0.035, 0.3; d_2H = 0.175, 0.35.

The parameter δH defines the lateral inhomogeneity. The parameter d_1H may define the effect of vertical inhomogeneity of the layer while d_2H the vertical inhomogeneity of the half-space for fixed H.

The dimensionless phase velocity $\frac{c}{\beta_{110}}$ and group velocity $\frac{U}{\beta_{110}}$ are calculated from (9) and (12) for taking different values of δH keeping d_1H and d_2H fixed. The computed results are plotted graphically in Fig. (A-1, A-2) to show the variations of phase velocity and group velocity against wave number. It is observed that phase velocity increases with the increase of δH, but group velocity decreases. The phase velocity and group velocity decrease when wave number increases.

To study the effect of vertical inhomogeneity different values are taken for *d, H* keeping δH and d_2H fixed. The results are presented graphically in Fig. (B-1, B-2). Increase in vertical inhomogeneity increases the phase velocity but decreases group velocity. Phase velocity decreases with increase of wave number.

By keeping δH and d_1H fixed and taking different values of d_2H the results are plotted in Fig. (C-1, C-2). In this case phase velocity increases and group velocity is decreased.

Particular

Case (i), when only half-space is isotropic we take $\frac{N_{01}}{L_{01}}=1$. The results obtained giving variation in one inhomogeneity and keeping the other two fixed. The results are shown in Fig. (F-1, F-2), (T-1, T-2) and (B-1, B-2).

In this model same phenomenon is observed.

Case (ii), when both half-space and layer are isotropic. In this case we put $\frac{N_0}{L_0} = \frac{N_{01}}{L_{01}} = 1.$ The numerical results so estimated are shown graphically in Fig. (D-1, D-2), (G-1, G-2) and (K-1, K-2). Increase in vertical inhomogeneity is the half-space increases the phase velocity and group velocity.

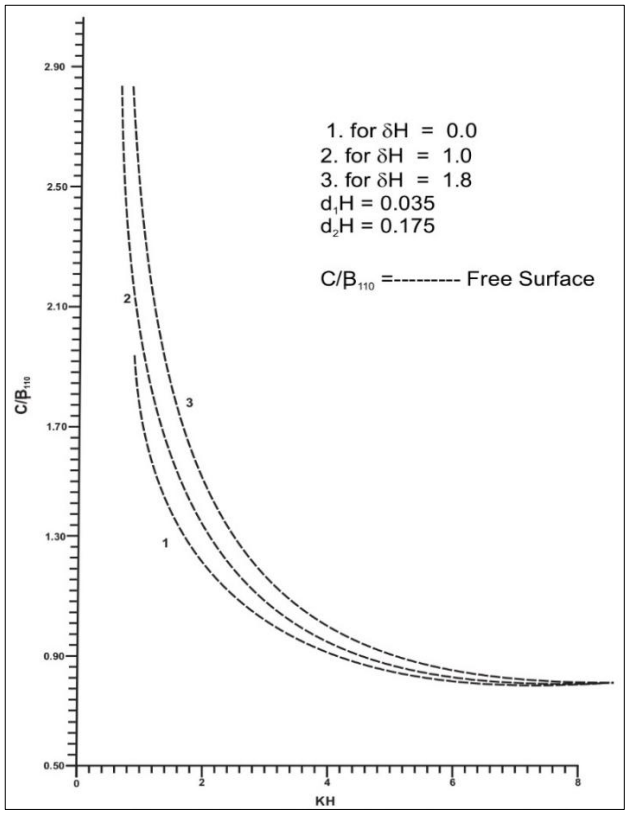

Fig A-1: Variations of phase velocity with wave number for transversely isotropic halfspace under a transversely isotropic layer with free surface

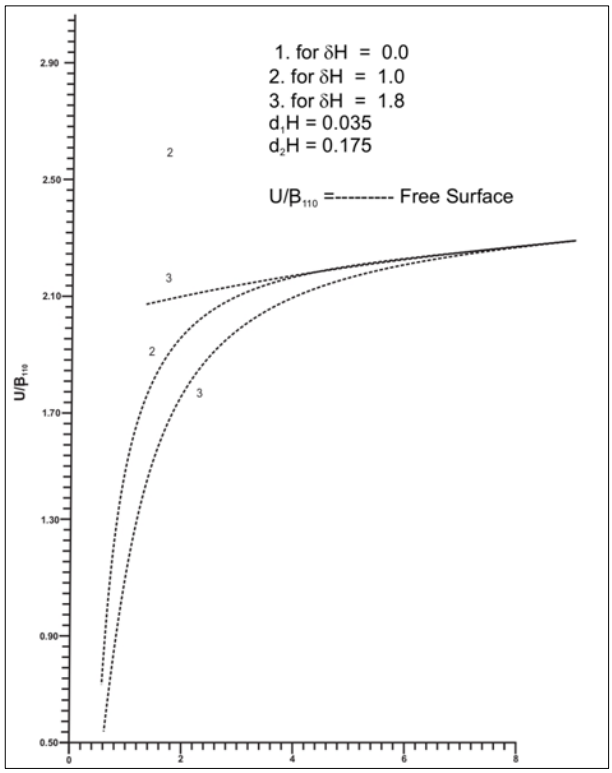

Fig A-2: Variations of group velocity with wave number for transversely isotropic halfspace under a transversely isotropic layer with free surface

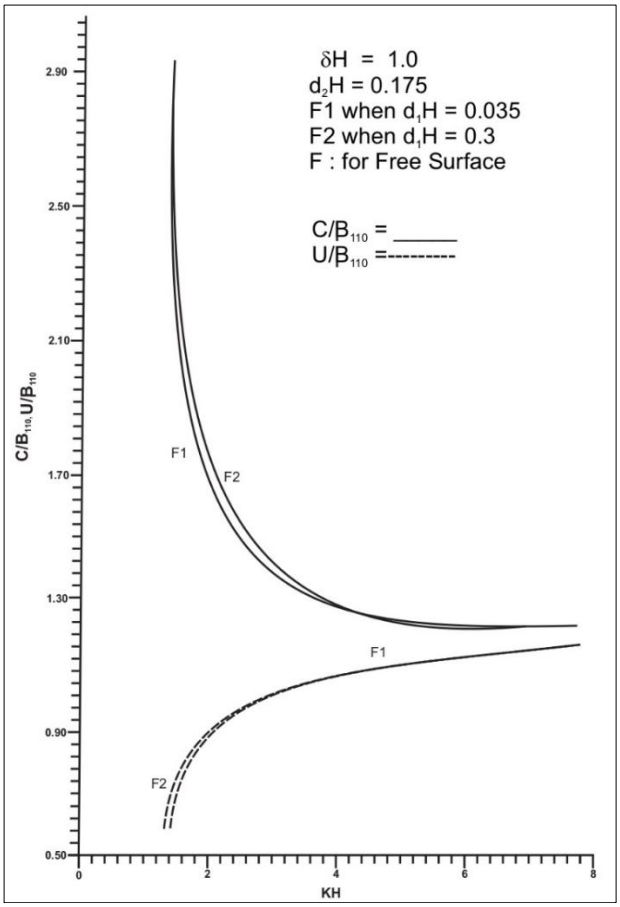

Fig. C1, C2: Variations of phase and group velocity with wave number for a transversely isotropic halfspace under transversely isotropic layer with free surface

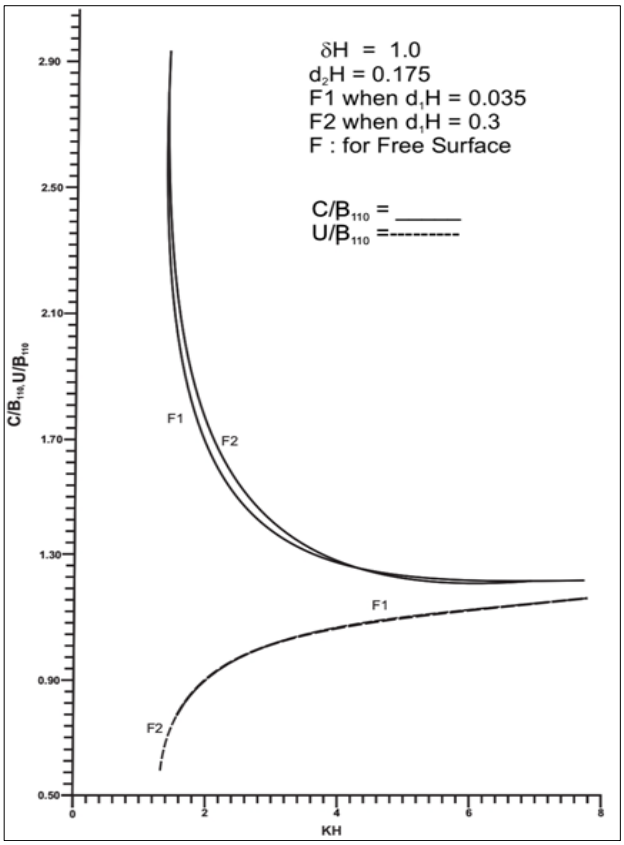

Fig B1, B2: Variations of phase and group velocity with wave number for a transversely isotropic halfspace under transversely isotropic layer with free surface

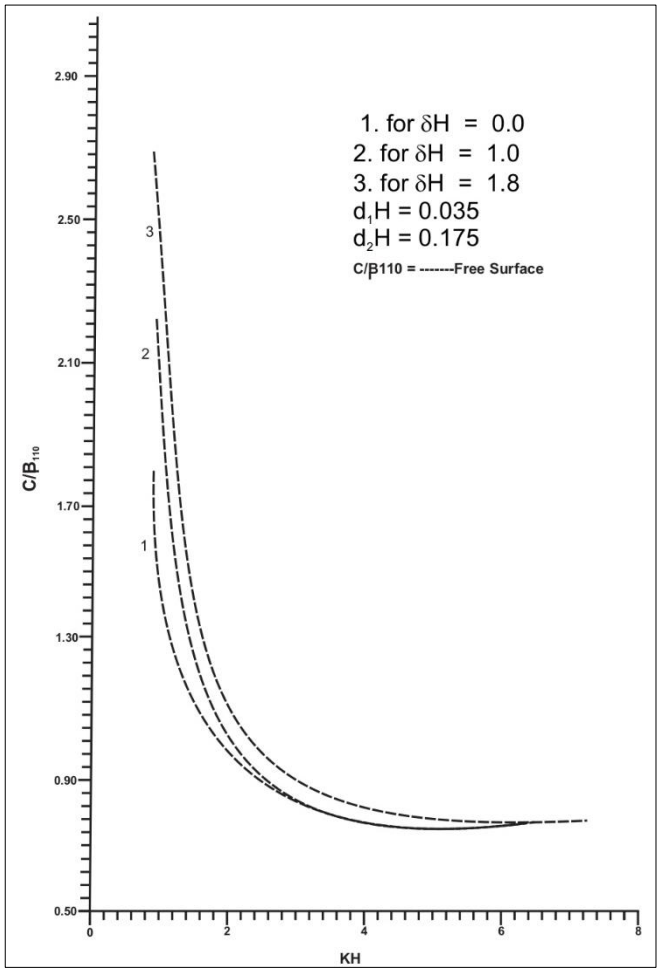

Fig D1: Variations of phase velocity with wave number for an isotropic halfspace under an isotropic layer with free surface

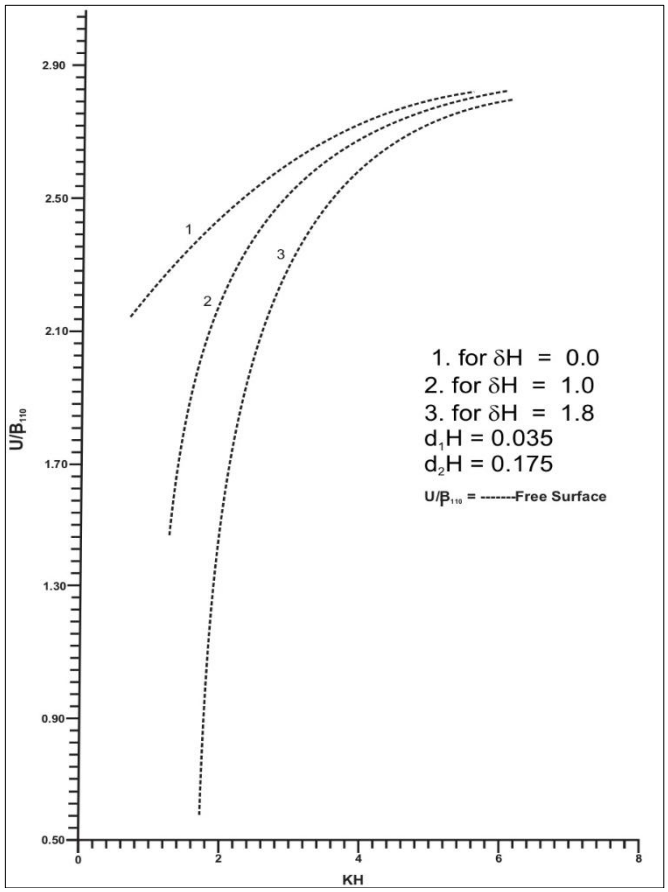

Fig D2: Variations of phase velocity with wave number for an isotropic halfspace under an isotropic layer with free surface

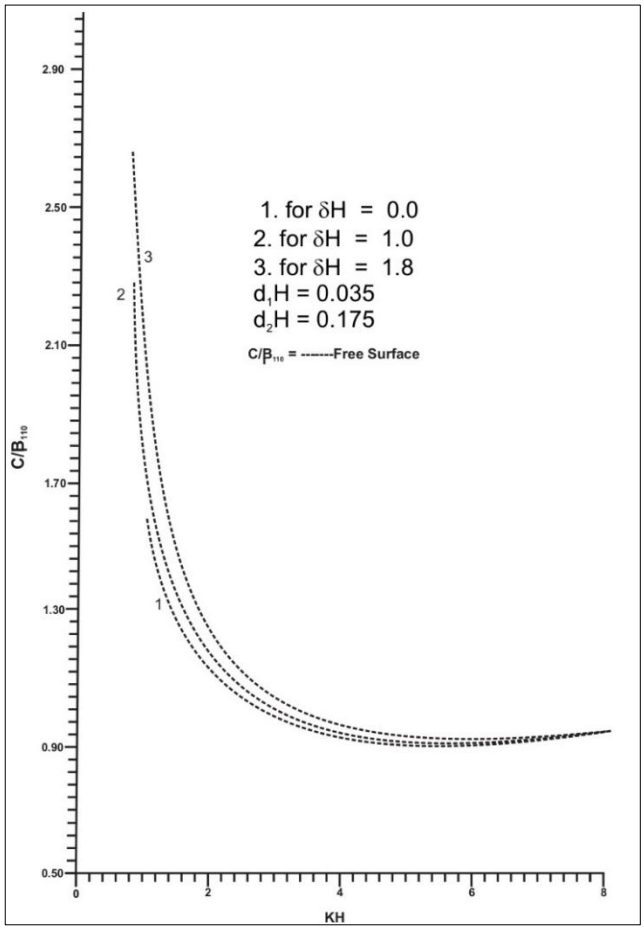

Fig F1: Variations of phase velocity with wave number for an isotropic halfspace under an isotropic layer with free surface

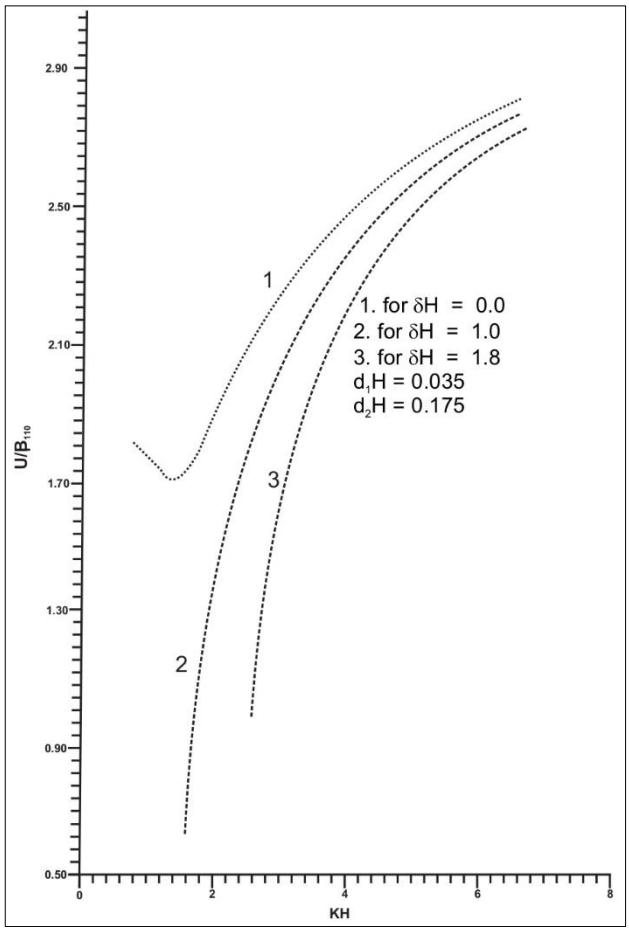

Fig F2: Variations of phase velocity with wave number of an isotropic halfspace under an isotropic layer with free surface

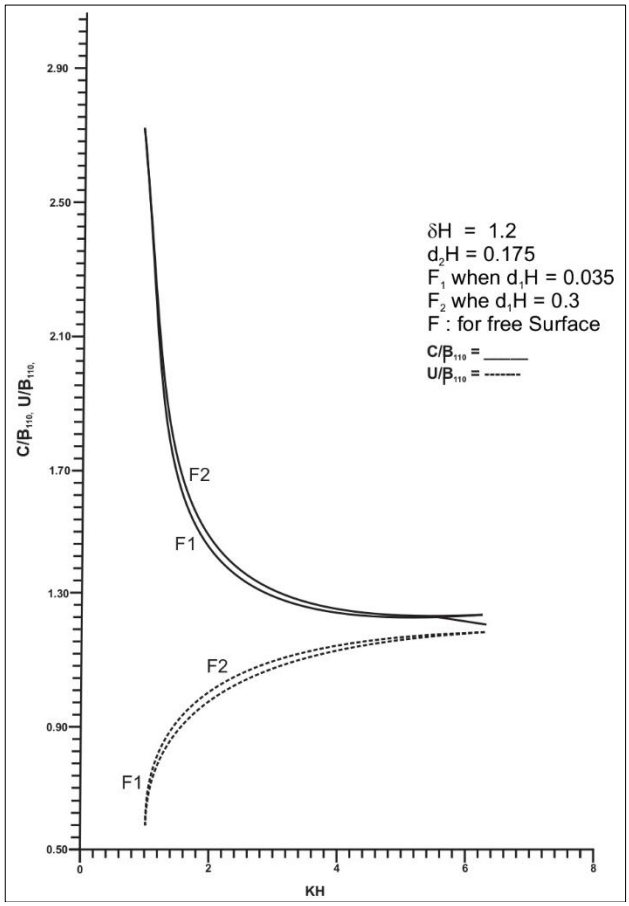

Fig G1, G2: Variations of phase velocity with wave number for an isotropic halfspace under an isotropic layer with free surface

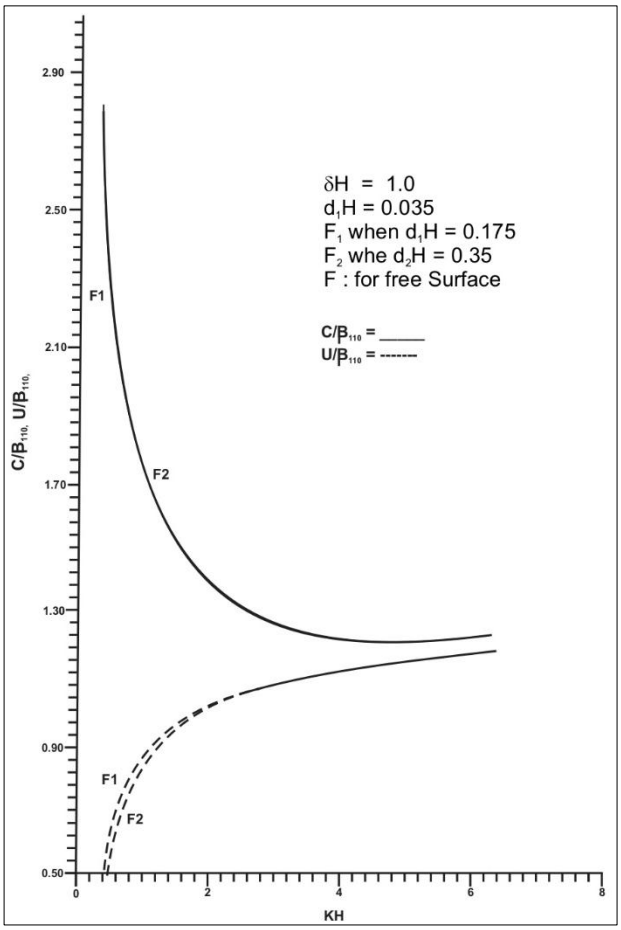

Fig K-1, K-2: Variations of phase and group velocity with wave number for an isotropic halfspace under an isotropic layer with free surface

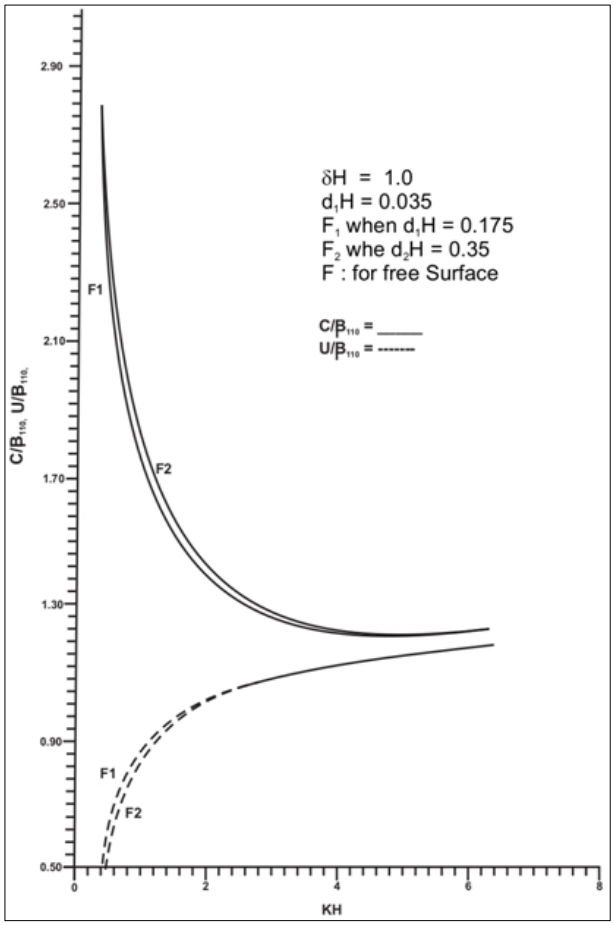

Fig T-1, T-2: Variations of phase and group velocity with wave number for an isotropic halfspace under a transversely isotropic layer with free surface

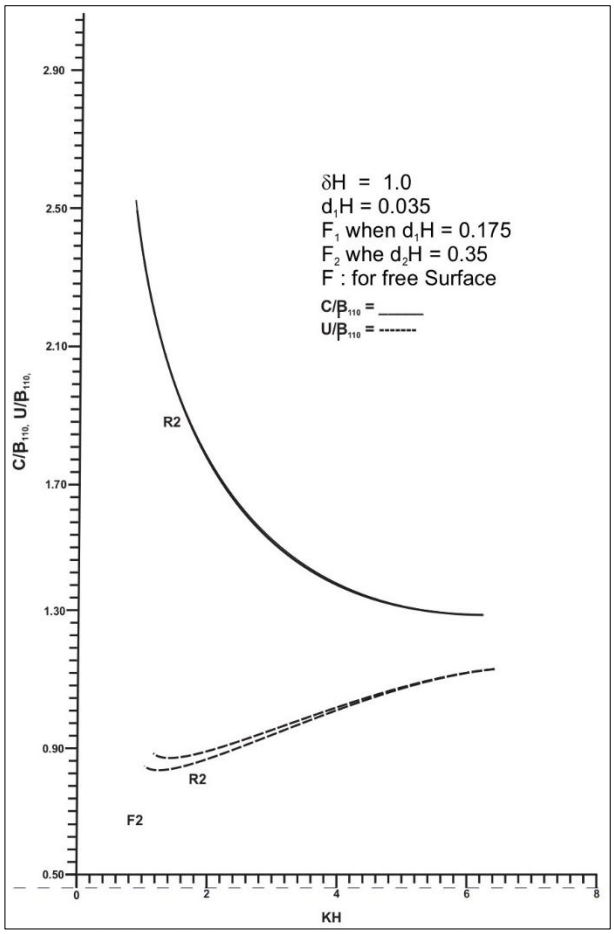

Fig Q-1, Q-2: Variations of phase and group velocity with wave number for an isotropic halfspace under a transversely isotropic layer with free surface

Conclusion

The effect of lateral and vertical inhomogeneities of the layer and of half-space is considered in this work. It is observed that lateral inhomogeneity effects strongly than vertical inhomogeneity on group velocity and phase velocity. Phase velocity increases but group velocity decrease with the increase of lateral inhomogeneity. Increase in vertical inhomogeneity in the layer and half-space increases the phase velocity but decreases the group velocity.

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