International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452
Maths 2025, 10(6): 128-133

https://www.mathsjournal.com
Received: 25-04-2025
Accepted: 26-05-2025

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Construction of control charts for moving range with six sigma under interquartile range

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DOI: https://www.doi.org/10.22271/maths.2025.v10.i6b.2066

Abstract
A company's ability to satisfy customer demands is crucial in today's fiercely competitive market as attaining sustained growth depends heavily on the caliber of its goods and services. Any industry may succeed in business as long as it offers high-quality goods and services that maximize client pleasure. One statistical method used to assess whether or not the process is under control is a control chart W.A. Shewhart created it in 1931. There are several six-sigma-based control charts that can be utilized when the normalcy assumption is broken. Both normal and exponential distributions can use this chart, which estimates the moving range using the interquartile range (IQR).

Keywords: Moving range, six sigma and process capability analysis

1. Introduction

Process is the fundamental source of all quality issues. One that is principally to blame for the quality issue is known as the root cause. The process/product quality should approach perfection if the underlying reason is removed. Thus, flawless processes produce flawless results. In order for the process's output to consistently satisfy its clients, it is crucial that it be flawless both in the short and long term. A quality control methodology called statistical process control (SPC) uses statistical methods to track and guarantee a process's stability. Two categories can be used to classify process variation:

- Common (or chance) variation is brought about by arbitrary, unidentified elements that are part of the process itself.
- Assignable (or special) variation: this type of variation may be fixed or avoided and is brought about by particular, identifiable sources.

2. Concept of Six Sigma

Six Sigma is a collection of procedures and initiatives that were first created by the multinational American telecommunications corporation Motorola. The Motorola people acknowledged that their high-quality goods almost never failed during usage. The Six Sigma approach, which aims to systematically improve processes by removing faults, was developed as a result of this realization. Six Sigma is the ability of highly skilled procedures to produce results that meet standards. In particular, systems driven by Six Sigma quality produce less than 3.4 defects for every million chances. Radhakrishnan (2009) [6] developed a single sample plan oriented towards the quality level of Six Sigma, using weighted Poissons, and intervened with random effectsPoisson distribution as the basis line. By expanding this, created a break error control diagram with a modifiable sample size that is particularly suitable for companies used in the product and services industry. Uma and Balamurugan (2021) [8] then organized a powerful control strategy based on the six Sigma concepts for a calm situation. Sivakumaran and Kanneswari (2024). Development of six sigma based control chart for standard deviation under robust measure Sivakumaran and Kanneswari (2024) Control chart based on Six Sigma for Range with Robust Scale Estimator.

3. Process Capability (C_P)

Process capabilities are estimated using process capability analysis. Regular process parameter estimate is necessary for a quality improvement initiative. The process will function as efficiently as feasible if these variables are monitored. Corrective action must be made if the process is not naturally competent. Examples include determining whether investing in better materials or modern equipment would reduce the process spread and comparing the process mean with the desired specification. Finding out what the client wants is another way to assess the specifications' adaptability with LSL denoting the lower specification limit and USL the upper specification limit.

$$C_p = \frac{\text{USL-LSL}}{6\sigma}$$

4. Interquartile Range (IQR)

The interquartile range (IQR) is a measure of variability calculated from the quartiles of a dataset. When data is rank-ordered, it can be divided into four equal parts using three quartiles: Q1 (first quartile), Q2 (second quartile, or median), and Q3 (third quartile), (Adekeye, 2012) [2].

- Q1 represents the median of the first half of the data.
- Q2 is the overall median of the dataset.
- Q3 is the m*edia*n of the second half of the data.

The IQR is computed as the difference between Q3 and Q1 (IQR = Q3-Q1), providing a robust measure of statistical dispersion.

5. Methods and Materials

Process monitoring uses a sample size of n=1, meaning each individual unit is treated as a sample (Montgomery, 2008) [3].

This approach is useful in the following situations:

- **Automated Inspection:** Every unit is checked, so logical subgrouping isn't needed.
- **Slow Data Collection:** Waiting for larger samples (*n* > 1) is impractical.
- **High Variability:** Subgrouping is difficult due to inconsistent data (common in both manufacturing and non-manufacturing processes).
- **Chemical Processes:** Repeated measurements vary only due to lab or testing errors.
- **Semiconductor Manufacturing:** A single wafer is measured at multiple points (e.g., oxide thickness).
- **Low-Variation Processes:** Some parameters (e.g., paper coating thickness) show minimal variation, making subgrouping unnecessary.

Control Chart for Individual Units

When using individuals control charts (I-charts), process variability is estimated using the moving range (MR) the absolute difference between two consecutive measurements.

$$MR_i = \left|X_i - X_{i-1}\right|, \; for \; i = 1 \; to \; N.$$

It is also possible to create a moving r*a*nge control chart.

Montgomery (2008) [3] constructed the three-sigma probability level under normal distribution $p(z \le z_{3\sigma}) = 1-\alpha_1, \alpha_1 = 0.0027$ for constructing a control chart based on 3σ.

$$UCL^{3\sigma}_{\bar{X}-MR} = \bar{\bar{X}}_{MR} + \left(\frac{3\bar{MR}}{d_2}\right)$$

$$CL^{3\sigma}_{\bar{X}-MR} = \bar{\bar{X}}_{MR}$$

$$LCL^{3\sigma}_{\bar{X}-MR} = \bar{\bar{X}}_{MR} - \left(\frac{3\bar{MR}}{d_2}\right)$$

The standard deviation $\left(\sigma^{3\sigma-IQR}_{N.\bar{X}-MR}\right)$ for creating three-sigma control limits using IQR for shifting range under a normal distribution in the manner described below:

$$UCL^{3\sigma.IQR}_{N.\bar{X}-MR} = \bar{\bar{X}}_{MR} + 3\left(\frac{\sigma^{3\sigma-IQR}_{N.\bar{X}-MR}}{\sqrt{n}}\right)$$

$$CL^{3\sigma.IQR}_{N.\bar{X}-MR} = \bar{\bar{X}}_{MR}$$

$$LCL^{3\sigma.IQR}_{N.\bar{X}-MR} = \bar{\bar{X}}_{MR} - 3\left(\frac{\sigma^{3\sigma-IQR}_{N.\bar{X}-MR}}{\sqrt{n}}\right)$$

The standard deviation $\left(\sigma^{3\sigma-IQR}_{E.\bar{X}-MR}\right)$ for building three-sigma control limits using IQR for shifting range under an exponential distribution in the manner described below:

$$UCL^{3\sigma.IQR}_{E.\bar{X}-MR} = \bar{\bar{X}}_{MR} + 3\left(\frac{\sigma^{3\sigma-IQR}_{E.\bar{X}-MR}}{\sqrt{n}}\right)$$

$$CL^{3\sigma.IQR}_{E.\bar{X}-MR} = \bar{\bar{X}}_{MR}$$

$$LCL^{3\sigma.IQR}_{E.\bar{X}-MR} = \bar{\bar{X}}_{MR} - 3\left(\frac{\sigma^{3\sigma-IQR}_{E.\bar{X}-MR}}{\sqrt{n}}\right)$$

In *t*his control *c*hart, the center line represents the me*a*n moving range, with control limits derived from the Downton estimator, which provides a resistant average of subgroup values. $\left(Dn^{3\sigma}_{\bar{X}-MR}\right)$

The technique makes the chart more resilient to outliers in normally distributed data by calculating control limits using subgroup-based standard deviation and averaging moving ranges to determine the process mean.

$$UCL^{3\sigma}_{\bar{X}-MR} = \bar{\bar{X}}_{MR} + 3\left(\frac{Dn^{3\sigma}_{\bar{X}-MR}}{\sqrt{n}}\right)$$

$$CL^{3\sigma}_{\bar{X}-MR} = \bar{\bar{X}}_{MR}$$

$$LCL^{3\sigma}_{\bar{X}-MR} = \bar{\bar{X}}_{MR} - 3\left(\frac{Dn^{3\sigma}_{\bar{X}-MR}}{\sqrt{n}}\right)$$

Proposed the six-sigma probability level under normal distribution $p(z \le z_{6\sigma}) = 1-\alpha_1, \alpha_1 = 3.4\times10^{-6}$ for constructing a moving range control chart based on 6σ.

$$UCL_{\bar{X}-MR}^{6\sigma} = \bar{\bar{X}}_{MR} + \phi_{6\sigma}\left(\frac{\sigma_{MR}^{6\sigma}}{\sqrt{n}}\right)$$

$$CL_{\bar{X}-MR}^{6\sigma} = \bar{\bar{X}}_{MR}$$

$$LCL_{\bar{X}-MR}^{6\sigma} = \bar{\bar{X}}_{MR} - \phi_{6\sigma}\left(\frac{\sigma_{MR}^{6\sigma}}{\sqrt{n}}\right)$$

$\left(D_{n.MR}^{6\sigma}\right)$ By calculating the average of the subgroup Downton estimator, the six-sigma moving range control chart establishes the center line and control boundaries based on the process standard deviation.

$$UCL_{Dn.MR}^{6\sigma} = \bar{\bar{X}}_{MR} + \phi_{6\sigma}\left(\frac{D_{n.MR}^{6\sigma}}{\sqrt{n}}\right)$$

$$CL_{Dn.MR}^{6\sigma} = \bar{\bar{X}}_{MR}$$

$$LCL_{Dn.MR}^{6\sigma} = \bar{\bar{X}}_{MR} - \phi_{6\sigma}\left(\frac{D_{n.MR}^{6\sigma}}{\sqrt{n}}\right)$$

The interquartile range (IQR) ensures resilience for normally distributed data, and the suggested standard deviation $\left(\tau_{N.6\sigma}^{MR-IQR}\right)$ is used to generate six sigma control limits, which establish tolerance levels (TL) and process capability.

$$UCL_{N.MR}^{6\sigma.IQR} = \bar{\bar{X}}_{MR} + \left(\frac{\phi_{6\sigma}\tau_{N.6\sigma}^{MR-IQR}}{\sqrt{n}}\right)$$

$$CL_{N.MR}^{6\sigma.IQR} = \bar{\bar{X}}_{MR}$$

$$LCL_{N.MR}^{6\sigma.IQR} = \bar{\bar{X}}_{MR} - \left(\frac{\phi_{6\sigma}\tau_{N.6\sigma}^{MR-IQR}}{\sqrt{n}}\right)$$

To determine Tolerance Levels (TL) and process capabilities, this method uses the proposed process standard deviation $\left(\tau_{E.6\sigma}^{MR-IQR}\right)$ to establish Six Sigma moving range control limits. Instead of relying on the standard deviation (σ) from control charts, it calculates moving range values using the process standard deviation, with the interquartile range (IQR) applied for exponential distributions.

$$UCL_{E.MR}^{6\sigma.IQR} = \bar{\bar{X}}_{MR} + \left(\frac{\phi_{6\sigma}\tau_{E.6\sigma}^{MR-IQR}}{\sqrt{n}}\right)$$

$$CL_{E.MR}^{6\sigma.IQR} = \bar{\bar{X}}_{MR}$$

$$LCL_{E.MR}^{6\sigma.IQR} = \bar{\bar{X}}_{MR} - \left(\frac{\phi_{6\sigma}\tau_{E.6\sigma}^{MR-IQR}}{\sqrt{n}}\right)$$

6. Illustration

Here, the example given by Richard S Leavenworth and Eugene L Grant (1952) [1] is taken into consideration. Based on their measurements in inches, the following information is gathered from the production of airplane parts in a terminal block.

Table 1: Data measured from aircraft parts in terminal block based on their dimensions

Group No.	I	II	III	IV	V	Mean	Moving range	SD	IQR_{Norm}	IQR_{Exp}	Downton
1	77	80	78	72	78	77.00	-	3.00	0.74	0.91	3.01
2	76	79	73	74	73	75.00	2.00	2.55	2.22	2.73	2.66
3	76	77	72	76	74	75.00	0.00	2.00	1.48	1.82	2.13
4	74	78	75	77	77	76.20	1.20	1.64	1.48	1.82	1.77
5	80	73	75	76	74	75.60	0.60	2.70	1.48	1.82	2.84
6	78	81	79	76	76	78.00	2.40	2.12	2.22	2.73	2.30
7	75	77	75	76	77	76.00	2.00	1.00	1.48	1.82	1.06
8	79	75	78	77	76	77.00	1.00	1.58	1.48	1.82	1.77
9	76	75	74	75	75	75.00	2.00	0.71	0.00	0.00	0.71
10	71	73	71	70	73	71.60	3.40	1.34	1.48	1.82	1.42
11	72	73	75	74	75	73.80	2.20	1.30	1.48	1.82	1.42
12	75	73	76	73	73	74.00	0.20	1.41	1.48	1.82	1.42
13	75	76	78	79	77	77.00	3.00	1.58	1.48	1.82	1.77
14	77	77	78	77	76	77.00	0.00	0.71	0.00	0.00	0.71
15	77	76	77	77	77	76.80	0.20	0.45	0.00	0.00	0.35
Average						75.67	1.44	1.61	1.24	1.52	1.69

Source: Richard S. Leavenworth and Eugene L Grant (1952) [1]

(I) The following formula is used to determine the moving range chart's 3σ control limits

$$UCL_{\bar{X}-MR}^{3\sigma} = \bar{\bar{X}}_{MR} + \left(\frac{3\bar{M}R}{d_2}\right) = 75.67 + \left(\frac{3 \times 1.44}{2.326}\right) = 77.53$$

$$CL_{\bar{X}-MR}^{3\sigma} = \bar{\bar{X}}_{MR} = 75.67$$

$$LCL_{\bar{X}-MR}^{3\sigma} = \bar{\bar{X}}_{MR} - \left(\frac{3\bar{M}R}{d_2}\right) = 75.67 - \left(\frac{3 \times 1.44}{2.326}\right) = 73.81$$

(II) The moving range control chart uses these 3σ limits, calculated with IQR for normal distributions

$$UCL_{N.\bar{X}-MR}^{3\sigma.IQR} = \bar{\bar{X}}_{MR} + 3\left(\frac{\sigma_{N.\bar{X}-MR}^{3\sigma-IQR}}{\sqrt{n}}\right) = 75.67 + 3\left(\frac{1.24}{\sqrt{5}}\right) = 77.32$$

$$CL_{N.\bar{X}-MR}^{3\sigma.IQR} = \bar{\bar{X}}_{MR} = 75.67$$

$$LCL_{N.\bar{X}-MR}^{3\sigma.IQR} = \bar{\bar{X}}_{MR} - 3\left(\frac{\sigma_{N.\bar{X}-MR}^{3\sigma-IQR}}{\sqrt{n}}\right) = 75.67 - 3\left(\frac{1.24}{\sqrt{5}}\right) = 74.01$$

(III) For exponentially distributed data, the control limits using IQR at 3σ are

$$UCL^{3\sigma.IQR}_{E.\bar{X}-MR} = \bar{\bar{X}}_{MR} + 3\left(\frac{\sigma^{3\sigma-IQR}_{E.\bar{X}-MR}}{\sqrt{n}}\right) = 75.67 + 3\left(\frac{1.52}{\sqrt{5}}\right) = 77.70$$

$$CL^{3\sigma.IQR}_{E.\bar{X}-MR} = \bar{\bar{X}}_{MR} = 75.67$$

$$LCL^{3\sigma.IQR}_{E.\bar{X}-MR} = \bar{\bar{X}}_{MR} - 3\left(\frac{\sigma^{3\sigma-IQR}_{E.\bar{X}-MR}}{\sqrt{n}}\right) = 75.67 - 3\left(\frac{1.52}{\sqrt{5}}\right) = 73.63$$

(IV) Based on Downton's statistic, the moving range control chart's 3-sigma limits are

$$UCL^{3\sigma}_{\bar{X}-MR} = \bar{\bar{X}}_{MR} + 3\left(\frac{Dn^{3\sigma}_{\bar{X}-MR}}{\sqrt{n}}\right) = 75.67 + 3\left(\frac{1.69}{\sqrt{5}}\right) = 77.93$$

$$CL^{3\sigma}_{\bar{X}-MR} = \bar{\bar{X}}_{MR} = 75.67$$

$$LCL^{3\sigma}_{\bar{X}-MR} = \bar{\bar{X}}_{MR} - 3\left(\frac{Dn^{3\sigma}_{\bar{X}-MR}}{\sqrt{n}}\right) = 75.67 - 3\left(\frac{1.69}{\sqrt{5}}\right) = 73.40$$

(V) The six-sigma based control limits for the moving range chart are

$$UCL^{6\sigma}_{\bar{X}-MR} = \bar{\bar{X}}_{MR} + \phi_{6\sigma}\left(\frac{\sigma^{6\sigma}_{MR}}{\sqrt{n}}\right) = 75.67 + 4.831\left(\frac{0.21}{\sqrt{5}}\right) = 76.13$$

$$CL^{6\sigma}_{\bar{X}-MR} = \bar{\bar{X}}_{MR} = 75.67$$

$$LCL^{6\sigma}_{\bar{X}-MR} = \bar{\bar{X}}_{MR} - \phi_{6\sigma}\left(\frac{\sigma^{6\sigma}_{MR}}{\sqrt{n}}\right) = 75.67 - 4.831\left(\frac{0.21}{\sqrt{5}}\right) = 75.21$$

(VI) Six-sigma moving range limits (Downton method)

$$UCL^{6\sigma}_{Dn.MR} = \bar{\bar{X}}_{MR} + \phi_{6\sigma}\left(\frac{D^{6\sigma}_{n.MR}}{\sqrt{n}}\right) = 75.67 + 4.831\left(\frac{0.22}{\sqrt{5}}\right) = 76.15$$

$$CL^{6\sigma}_{Dn.MR} = \bar{\bar{X}}_{MR} = 75.67$$

$$LCL^{6\sigma}_{Dn.MR} = \bar{\bar{X}}_{MR} - \phi_{6\sigma}\left(\frac{D^{6\sigma}_{n.MR}}{\sqrt{n}}\right) = 75.67 - 4.831\left(\frac{0.22}{\sqrt{5}}\right) = 75.19$$

(VII) Six-sigma moving range limits (Downton method)

$$UCL^{6\sigma.IQR}_{N.MR} = \bar{\bar{X}}_{MR} + \left(\frac{\phi_{6\sigma}\tau^{MR-IQR}_{N.6\sigma}}{\sqrt{n}}\right) = 75.67 + \left(\frac{4.831\times 0.19}{\sqrt{5}}\right) = 76.07$$

$$CL^{6\sigma.IQR}_{N.MR} = \bar{\bar{X}}_{MR} = 75.67$$

$$LCL^{6\sigma.IQR}_{N.MR} = \bar{\bar{X}}_{MR} - \left(\frac{\phi_{6\sigma}\tau^{MR-IQR}_{N.6\sigma}}{\sqrt{n}}\right) = 75.67 - \left(\frac{4.831\times 0.19}{\sqrt{5}}\right) = 75.27$$

(VIII) Six Sigma moving range control limits using IQR (exponential data)

$$UCL^{6\sigma.IQR}_{E.MR} = \bar{\bar{X}}_{MR} + \left(\frac{\phi_{6\sigma}\tau^{MR-IQR}_{E.6\sigma}}{\sqrt{n}}\right) = 75.67 + \left(\frac{4.831\times 0.23}{\sqrt{5}}\right) = 76.16$$

$$CL^{6\sigma.IQR}_{E.MR} = \bar{\bar{X}}_{MR} = 75.67$$

$$LCL^{6\sigma.IQR}_{E.MR} = \bar{\bar{X}}_{MR} - \left(\frac{\phi_{6\sigma}\tau^{MR-IQR}_{E.6\sigma}}{\sqrt{n}}\right) = 75.67 - \left(\frac{4.831\times 0.23}{\sqrt{5}}\right) = 75.18$$

Table 2: An overview of the movable range's three- and six-sigma control charts

S. No	Control chart	UCL	LCL	CLI
1	The 3σ Shewhart	77.53	73.81	3.72
2	The 3σ with IQR under Exponential distribution	77.70	73.63	4.07
3	The 3σ with IQR under Normal distribution	77.32	74.01	3.32
4	The 3σ using Downton	77.93	73.40	4.53
5	The6σ using Downton	76.15	75.19	0.96
6	The6σ	76.13	75.21	0.92
7	The 6σ methodology using interquartile range (IQR) for exponential distributions	76.16	75.18	0.98
8	The6σ methodology using interquartile range (IQR) for Normal distribution	76.07	75.27	0.80

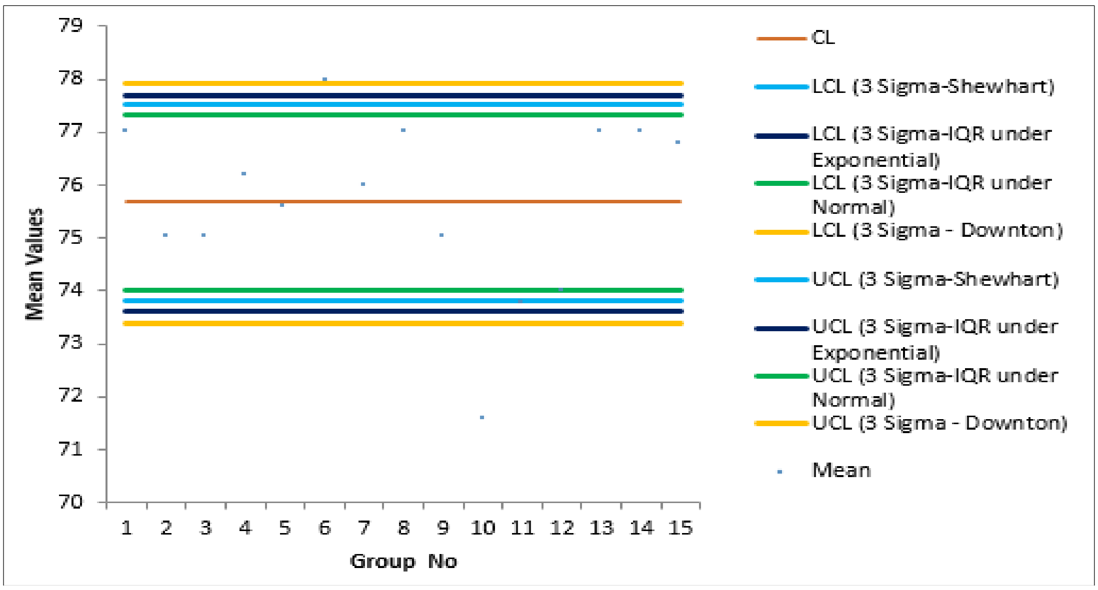

Fig 1: Three-sigma control charts for moving range

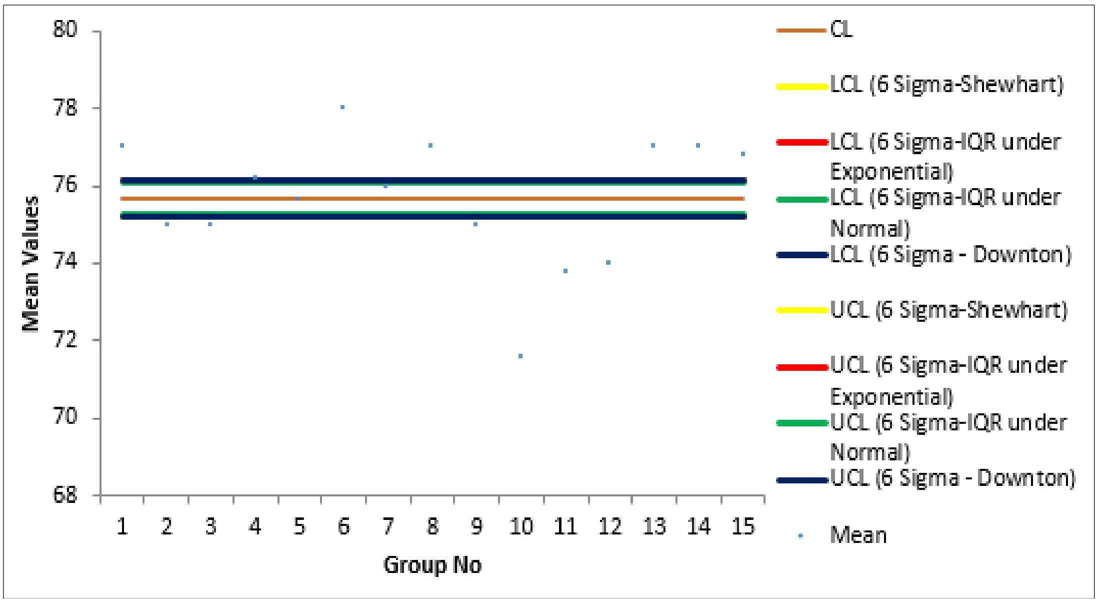

Fig 2: Six-sigma control charts for moving range

The results indicate an out-of-control process when using Six Sigma moving range control charts with IQR (normal distribution). Only two sample points fell within the control limits, yielding a control limit interval (CLI) of 0.80 for n=5 (Table 1, Figures 1-2). Comparative analysis shows the Six Sigma IQR-based chart outperforms traditional 3σ and other control charts in detecting shifts, as measured by average run length (Table 3, Figures 3-4).

Table 3: ARL of three-sigma and six-sigma control charts for moving range

S. No	Multiple of σ and Control chart	0.5	0.6	0.7	0.8	0.9	1.0
1	The 3σShewhart	57.228	39.578	27.883	20.040	14.700	11.004
2	3σ with IQR under Exponential distribution	81.347	56.322	39.639	28.405	20.734	15.418
3	3σwith IQR under Normal distribution	52.358	34.279	23.065	15.957	11.349	8.294
4	Three-sigma (Downton)	99.389	70.893	51.155	37.426	27.787	20.941
5	The Six-*sig*ma (Downton)	1.723	1.125	1.013	1.001	1.000	1.000
6	The Six-sigma	1.504	1.073	1.005	1.000	1.000	1.000
7	*Six* Sigm*a* methodology using interquartile range (IQR) for exponential distributions	1.856	1.166	1.021	1.000	1.000	1.000
8	Six Sigma methodology using interquartile range (IQR) for Normal distribution	1.136	1.009	1.000	1.000	1.000	1.000

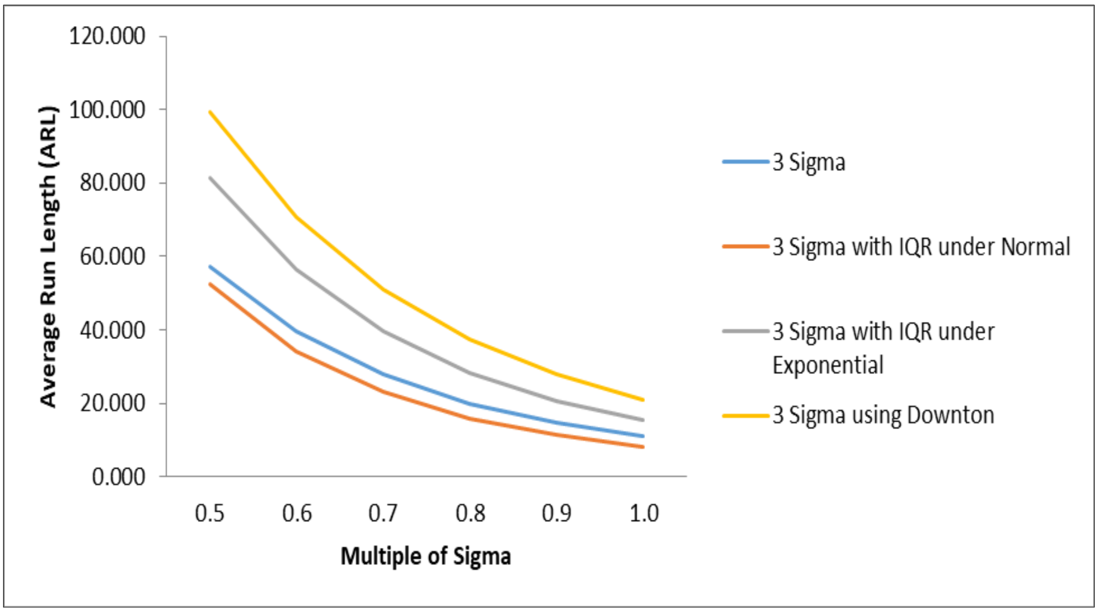

Fig 3: ARL for moving range three-sigma control charts

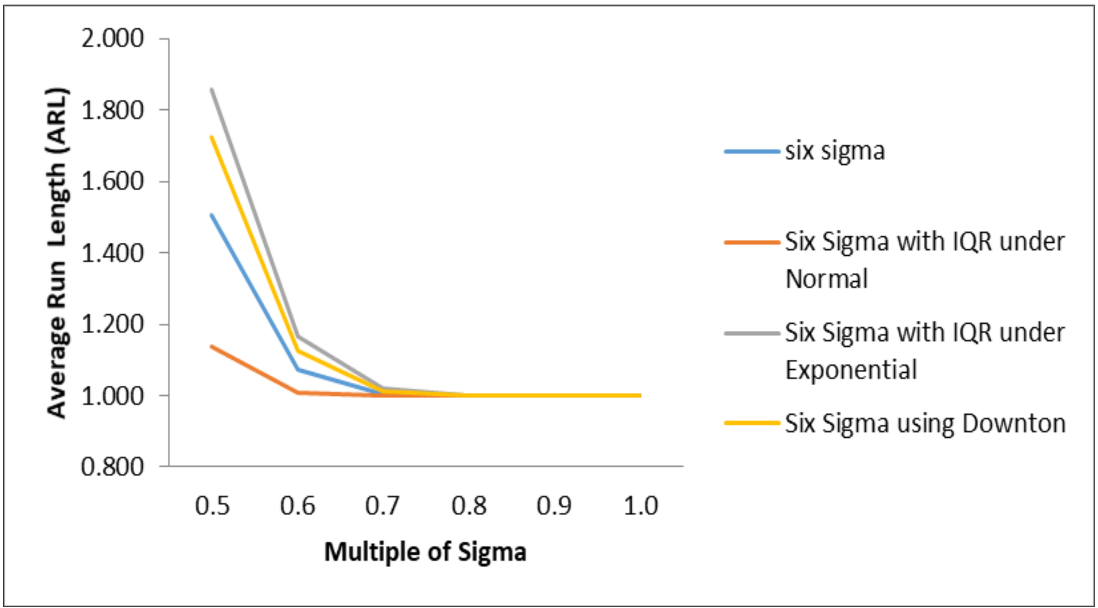

Fig 4: ARL for moving range six-sigma control charts

8. Conclusion

The results show that the process remains out of statistical control when using Shewhart 3-sigma, IQR-based, and 6-sigma IQR control limit for both normal and exponential distributions of the Moving Range. To ensure compliance, improvements in product quality are necessary.

9. References

1. Eugene L, Leavenworth R. Statistical Quality Control. Tata McGraw-Hill Publishing Company Limited, 1952.
2. Adekeye KS. Modified simple robust control chart based on median absolute deviation. Transnat J Sci Technol. 2012;2(4):1-10.
3. Montgomery DC. Introduction to statistical Quality Control. 4th ed. Hoboken: John Wiley & Sons, 2008.
4. Abu-Shawiesh MO. A Simple Robust Control Chart Based on MAD. J Math Stat. 2008;4(2):102-107.
5. Radhakrishnan R. Construction of Six sigma-based sampling plans [Thesis]. Coimbatore: Bharathiar Univ, 2009.
6. Radhakrishnan R, Sivakumaran PK. Construction of double sampling plans through six sigma quality levels. In: Proceedings of the IEEE Second International Joint Conference on Computational Sciences and Optimization, Sanya, Hainan, China, 2009 Apr 24-26, p. 1027-1030.
7. Shewhart WA. Economic Control of Quality of Manufactured Product. New York: Van Nostrand, 1931.
8. Uma S, Balamurugan P. Six Sigma Based Robust Control Chart Under Moderateness. Stoch Modelling Appl. 2021;25(2):207-215.
9. Sivakumaran PK, Kanneswari T. Control chart based on Six Sigma for Range with Robust Scale Estimator. J Comput Anal Appl. 2024;33(8):72-81.
10. Sivakumaran PK, Kanneswari T. Development of six sigma based control chart for standard deviation under robust measure. Int J Stat Appl Math. 2024;9(6):235-242.