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Formulating a logistic growth model to forecase population counts across counties in Liberia from 2021 to 2031

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Abstract
In this paper, the logistic growth model was formulated to forecast county-level population counts across Liberia from 2021 to 2031, addressing the limitations of traditional deterministic models currently used in national planning. For Liberia, most demographic projections have relied on constant growth rates under medium fertility scenarios, which fail to account for time-dependent population dynamics or resource constraints. The official projections from the Liberia Institute of Statistics and Geo-Information Services (LISGIS) were used for the period 2008 to 2020. This research identifies a nearly fixed annual growth rate across counties, highlighting the deterministic nature of current methods. A proposed logistic modeling framework integrates time-dependent growth rates and evolving carrying capacities to reflect more realistic demographic changes. The model is calibrated for each of Liberia's 15 counties and demonstrates that as populations approach their environmental or infrastructural limits, growth rates decline consistently with logistic dynamics. Forecasts reveal that Liberia's population will increase from approximately 4.71 million in 2021 to 6.38 million by 2031, with growth patterns varying by county. This refined approach provides a more adaptive and evidence-based forecasting tool for long-term planning in sectors such as healthcare, housing, and education, and encourages a shift toward dynamic, data-responsive population modeling in Liberia.

Keywords: Logistic growth model, population forecasting, time-dependent growth rate, carrying capacities, county-level projection, Liberia

1. Introduction

Accurate population forecasting is essential for effective national planning, particularly in ensuring equitable resource allocation and promoting sustainable development across all counties in Liberia. In its 2023 publication, the Liberia Institute of Statistics and Geo-Information Services (LISGIS) reveals the results of the national population housing census (NPHC), which utilized the medium fertility scenario to project population counts from 2008 to 2020. These projections suggest that the annual growth rate for each county from 2009 to 2020 remains nearly constant over time. Notable, most traditional demographic studies in Liberia continue to rely on methodologies that reflect a deterministic modeling assumption, as opposed to more adaptive dynamic or stochastic frameworks. For instance, population forecasting has historically relied on deterministic approaches used in Liberia, which assume fixed parameters and do not account for uncertainty or variability in demographic behavior. One of the most frequently applied methods is the exponential growth model, which assumes a constant growth rate and projects population using the formula as:

$$N(t) = N(0)e^{rt}.$$

Due to its mathematical simplicity, the linear growth method is widely used for implementation (LISGIS 2023) [2]. This approach assumes a steady annual increase in population, making it suitable for short-term projections where fluctuations in fertility or migration rates are minimal (Smath, *et. al.*, 2013) [4].

However, more sophisticated methods such as the cohort-component method for projecting population (CCMPP) are considered the gold standard in demographic forecasting globally but remain underutilized in Liberia due to limited access to detailed age and sex-disaggregated data, (United Nations Department of Economic and Social Affairs (UN DESA), 2017) [8]. According to UN DESA (2017), CCMPP is not a complete projection method on its own, as it requires the components of change to be projected in advance. Rather, it is an application of matrix algebra that enables demographers to calculate the effects of the assumed future patterns of fertility, mortality, and international migration on a population at some given point in the future (Preston, *et. al*., 2001; Whelpton, 1936) [3, 11]. The medium fertility scenario, modeled after the United Nations framework has been prominently employed by the Liberia Institute of Statistics and Geo-Information Services (LISGIS) in national reports, including the National Population Housing Census (NPHC). This method assumes a moderate fertility trajectory and typically applies a fixed growth rate assumption extrapolated from past census intervals to estimate population counts at the county level, thereby reinforcing a deterministic approach to county-level population projection (LISGIS 2023) [2].

Additionally, standard tools are built into Microsoft Excel, such as interpolation and extrapolation for sectoral ministries and county planning offices for estimating annual population counts. This approach is called the interpolation and extrapolation spreadsheet model, while the deterministic methods that provide a baseline for planning lack the flexibility to respond to abrupt changes in migration, fertility, or health outcomes. As such, there is a growing advocacy within the research community in Liberia to adopt more dynamic or stochastic models, such as logistic growth models, Bayesian inference, or Monte Carlo simulations, to better capture Liberia's changing demographic realities and improve the accuracy of long-term population forecasts (Verhulst 1838, Tsoularis and Wallace 2002, Allen 2003) [9, 5, 1].

In Liberia, given the limitations of traditional deterministic approaches and the scarcity of detailed demographic data, this study proposes a logistic growth modeling framework to forecast population trends across counties in Liberia from 2021 to 2031. By incorporating time-dependent growth rates and dynamic carrying capacities, the logistic growth model aims to better reflect changing demographic patterns and resource constraints. This approach not only provides a more nuanced understanding of population expansion but also serves as a valuable tool for long-term policy planning and development. Ultimately, it seeks to enhance the accuracy and relevance of population forecasts in Liberia's evolving socioeconomic context.

2. Materials and Methods

2.1 Study Design

The logistic growth function was adopted for the use of a quantitative modeling approach to model and forecast county-level population dynamics in Liberia and further investigates whether the population growth rates used in Liberia's official county-level population projections remain constant or vary over time from the national population housing census (NPHC) (2008). The analysis of this study focuses on medium fertility scenario projections published by the Liberia Institute of Statistics and Geo-Information Services (LISGIS) for the years 2008 to 2020 across all 15 counties. It compares year-over-year growth rates and fits a logistic curve to each county's population series. The study aims to determine if the underlying growth assumptions reflect changing demographic patterns or adhere to static growth estimates.

2.2 Data Source

This study utilizes a secondary dataset obtained from the Liberia Socioeconomic Statistical Bulletin: projected population by county, 2008-2020 (medium fertility scenario) published by LISGIS (2023). The projections are based on medium fertility assumptions and provide a basis for analysing temporal growth trends [2].

2.3 Logistic Growth Equation

Let i denote each county in Liberia. Using a logistic differential equation framework, this study derives a logistic population model tailored to Liberia's demographic structure. The logistic growth equation in its general form is given by:

$$\frac{dN_i(t)}{dt} = r_i(t)N_i(t), \tag{1}$$

Where the time-dependent growth rate $r_i(t)$ is defined as:

$$(t) = r_{i0}\left(1 - \frac{N_i(t)}{K_i^t}\right), \tag{2}$$

Substitute (2) into (1) and obtained this:

$$\frac{dN_i(t)}{dt} = N_i(t)r_{i0}\left(1 - \frac{N_i(t)}{K_i^t}\right).$$

In this formulation, $N_i(t)$ represents the population at time t for each county i, $r_i(t)$ is the instantaneous population growth rate at time t specific to each county, K_i is the carrying capacity and r_{io} is the initial population growth rate specific to each county. The equilibrium solution of the logistic growth model occurs when the rate of change of the population is zero, i.e. $\frac{dN_i(t)}{dt} = 0$, which yields two conditions: 1) $N_i(t) = 0$ and 2) $1 - \frac{N_i(t)}{K_i} = 0$, implying that $N_i(t) = K_i$ (population reaches carrying capacity). Solving the logistic differential equation for the initial growth rate r_{i0}, the study obtained:

$$r_{i0} = \frac{1}{N_i(t)\left(1 - \frac{N_i(t)}{K_i^t}\right)}\frac{dN_i(t)}{dt},$$

To integrate both sides, the study applied separation of variables:

$$\int \frac{1}{N_i(t)\left(1 - \frac{N_i(t)}{K_i^t}\right)} dN_i(t) = \int r_{i0} dt.$$

Using partial fraction decomposition, the left side becomes:

$$\frac{1}{N_i(t)\left(1 - \frac{N_i(t)}{K_i^t}\right)} = \frac{1}{K_i}\left(\frac{1}{N_i(t)} + \frac{1}{K_i - N - i\ (t)}\right),$$

Integrating both sides, yields:

$$\frac{1}{K_i^t}(\ln|N_i(t)| - \ln|K_i - N_i(t)) = K_i r_{i0} t + C.$$

Solving for r_{i0}, the equation becomes:

$$r_{i0} = \frac{1}{K_i^{t(2)}t}\left(\ln\left(\frac{N_i(t)}{K_i - N_i(t)}\right) - K_i C\right).$$

This study assumes that the initial growth rate is adjusted by taking the absolute value and scaling by a factor by 1000, such that

$$r_{i0} = |r_{i0}| \times 1000$$

This adjustment ensures that the growth rate remains positive and is proximately scaled for interpretation in the model. Applying rule of logarithmic, the solution simplifies to the logistic equation in exponential form:

$$\frac{N_i(t)}{K_i^t - N_i(t)} = Ce^{K_i r_{i0} t}. \quad (3)$$

To determine C, the researcher substitute $N_0^i(t)$ (initial population) yielding as:

$$C = \frac{N_0^i(t)}{K_i^t - N_0^i(t)}. \quad (4)$$

Substituting Equation (4) into Equation (3) and simplifying gives the study explicit logistic solution:

$$N_i(t) = \frac{N_0^i(t) K_i^t e^{K_i r_{i0} t}}{K_i^t - N_0^i(t) + N_0^i(t) e^{K_i^t r_{i0} t}}. \quad (5)$$

Finally, by multiplying the numerator and denominator by $\frac{1}{N_i(t)} e^{K_i^t r_{i0} t}$, the expression simplifies to the classic logistic population forecasting model used in this study.

$$N_i(t) = \frac{K_i^t}{1 + \frac{K_i^t - N_0^i(t)}{N_0^i(t)} e^{K_i^t r_{i0} t}}.$$

2.4 Carrying Capacity K_i^t

Tsoularis and Wallace (2002) [5] illustrated that the logistic growth model parameter K_i denotes the carrying capacity, representing the maximum sustainable population for a given county. Population growth is constrained by available resources and infrastructure, where carrying capacity serves as a theoretical upper limit. However, this capacity is not necessarily fixed and may evolve over time in response to various socioeconomic and environmental factors. For example, infrastructure expansion, education systems, urban development, and improvement of health care can increase a county's capacity to support a larger population. Conversely, environmental degradation or economic stagnation can reduce the effective carrying capacity. Thus, this study is applying logistic models to long-term population forecasting (2021 to 2031). It is often necessary to account for potential changes in K_i rather than treating it as a static value (United Nations 2022) [7].

To estimate carrying capacity K_i, the study adopts a compound growth rate g_i approach using projection data from 2008 to 2020. Specifically, the projected carrying capacity is modeled as:

$$K_i^t = K_i^{2020} \times (1 + g_i)^{t-2020},$$

Where the compound growth rate is computed as:

$$g_i = \frac{K_i^{2020} - K_i^{2008}}{K_i^{2008} \times 10}.$$

This process allows the model to reflect incremental changes in carrying capacity over time, consistent with the realities of demographic and infrastructure development.

3. Results

Table 1 reveals the results of the study's assessment of the underlying growth assumptions applied in forecasting population counts based on the 2008 National Population Housing Census (NPHC), utilizing the medium fertility scenario projections published by the Liberia Institute of Statistics and Geo-Information Services (LISGIS) for the period 2008 to 2020. The analysis reveals that the annual growth rate for each county from 2009 to 2020 remains nearly constant over time. This finding is strongly supported by three key indicators: i) a very low standard deviation of approximately 0.00112, ii) a consistent mean growth rate of around 0.022375 (i.e., ~2.24% per year), and iii) a coefficient of variation (CV) of only approximately 5%, indicating minimal relative fluctuation in the rate.

These findings indicate that LISGIS projection data applied a nearly fixed annual growth rate across all counties, which aligns with a deterministic modeling assumption rather than a dynamic or stochastic framework. In response, this study develops a logistic population forecast model designed to capture changing demographic patterns by allowing variability in the growth rate over time.

Table 1: Summary of annual growth rate variability by County (2008-2020)

County	Mean Growth Rate	SD	Min	Max	CV (%)
Bomi	0.022375	0.00112	0.02056	0.02427	5.00
Bong	0.022375	0.00112	0.02056	0.02427	5.01
Cape Mount	0.022375	0.00112	0.02055	0.02428	5.01
Gbarpolu	0.022375	0.00112	0.02056	0.02427	5.00
Grand Bassa	0.022376	0.00112	0.02056	0.02427	5.00
Grand Gedeh	0.022375	0.00112	0.02055	0.02427	5.00
Grand Kru	0.022375	0.00112	0.02055	0.02427	5.00
Lofa	0.022375	0.00112	0.02056	0.02427	5.00
Margibi	0.022376	0.00112	0.02056	0.02428	5.01
Maryland	0.022375	0.00112	0.02055	0.02427	5.00
Montserrado	0.022375	0.00112	0.02056	0.02427	5.00
Nimba	0.022375	0.00112	0.02056	0.02427	5.00
River Cess	0.022376	0.00112	0.02056	0.02427	5.00
River Gee	0.022374	0.00112	0.02055	0.02427	5.00
Sinoe	0.022375	0.00112	0.02056	0.02427	5.00

Table 2 presents a noticeable decline in the logistic growth rate over time. This trend can be attributed to resource limitations, that emerge as the population size approaches the carrying capacity, K_i^t. The carrying capacity is the maximum number of individuals that the environment can sustain. As the population increases and approaches K_i^t or $N_i(t) = K_i^t$, the resources become scarce, and competition intensifies. Consequently, this results in a gradual slowdown of the population growth rate.

Table 2: Projected logistic growth rate by county in% (2021 to 2031)

County	2021	2022	2023	2024	2025	2026	2027	2028	2029	2030	2031
Bomi	5.56	4.83	4.42	3.71	3.27	2.9	2.57	2.3	2.05	1.84	1.65
Bong	1.4	1.22	1.06	0.93	0.82	0.73	0.65	0.58	0.52	0.46	0.42
Gbarpolu	5.61	4.87	4.26	3.73	3.3	2.92	2.6	2.32	2.1	1.86	1.6
Grand Bassa	2.11	1.83	1.6	1.41	1.25	1.1	0.98	0.87	0.78	0.7	0.63
Cape Mount	3.68	3.2	2.79	2.45	2.16	1.92	1.7	1.52	1.36	1.22	1.1
Grand Gedeh	3.73	3.24	2.83	2.49	2.2	1.95	1.73	1.54	1.38	1.24	1.1
Grand Kru	8.1	7.0	6.13	5.38	4.75	4.21	3.74	3.33	2.98	2.67	2.4
Lofa	1.69	1.47	1.28	1.13	1.0	0.88	0.78	0.7	0.62	0.56	0.5
Margibi	2.23	1.93	1.69	1.49	1.31	1.16	1.0	0.91	0.82	0.74	0.66
Maryland	3.44	2.99	2.61	2.29	2.0	1.79	1.59	1.42	1.27	1.14	1.0
Montserrado	0.42	0.36	0.32	0.28	0.25	0.22	0.19	0.17	0.15	0.14	0.12
Nimba	0.1	0.88	0.77	0.67	0.6	0.53	0.47	0.42	0.37	0.33	0.3
Rivercess	6.54	5.68	4.96	4.36	3.85	3.41	3.0	2.7	2.41	2.16	1.94
Sinoe	4.57	3.97	3.47	3.0	2.69	2.38	2.12	1.89	1.69	1.51	1.36
River Gee	7.0	6.1	5.31	4.67	4.12	3.65	3.24	2.89	2.59	2.32	2.1

Table 3 presents the forecasted population trends across Liberia's fifteen counties from 2021 to 2031, based on a logistic forecast population model. The projections indicate a steady and consistent increase in population across all counties, aligning with the moderate growth assumptions previously derived from the logistic population framework.

Montserrado County, home to the national capital city, Monrovia, remains the most populous region throughout the forecast period, with the population increasing from approximately 1.51 million in 2021 to over 2.05 million by 2031. Similarly, Nimba and Bong counties also exhibit significant population increases, reaching 847,594 and 611,775, respectively, by 2031. These trends reflect the concentration of socioeconomic activities, migration, and possibly higher fertility rates in these counties. Other counties such as Grand Kru, River Gee, and Gbarpolu, maintain relatively smaller populations; still, their annual increases are consistent, indicating proportional growth based on their respective carrying capacities and initial conditions.

At the national level, Liberia's total population is projected to grow from approximately 4.71 million in 2021 to 6.38 million by 2031. This reveals a total increase of nearly 1.67 million people over a decade, or an average annual increase of about 167,900 individuals. This sustained rise is underpinned by modest but stable annual growth rates and assumes that the logistic constraints such as infrastructure, healthcare, and environmental factors, are continuing to evolve gradually without sudden shocks.

These forecasts importantly reflect the time-dependent nature of logistic growth, where population expansion slows as counties approach their carrying capacities. This declaration becomes especially apparent in the later years of the projection period, with noticeable flattening of population growth rates, particularly in densely populated counties such as Montserrado and Nimba. The projections thus provide critical insights for long-term planning in areas such as education, housing, healthcare and employment.

Table 3: Liberia forecasted population counts across counties from 2021 to 2031

County	2021	2022	2023	2024	2025	2026	2027	2028	2029	2030	2031
Bomi	113937	117447	121064	124793	128637	132599	136683	140893	145232	149706	154317
Bong	451694	465606	479947	494730	509968	525676	541867	558557	575761	593495	611775
Gbarpolu	112948	116427	120013	123709	127520	131447	135496	139669	143971	148406	152977
Grand Bassa	300280	309528	319062	328890	339020	349462	360226	371321	382758	394548	406700
Cape Mount	172122	177424	182888	188521	194328	200314	206483	212843	219399	226157	233122
Grand Gedeh	169659	174885	180272	185824	191547	197447	203529	209797	216259	222920	229786
Grand Kru	78442	80858	83348	85915	88561	91289	94101	96999	99987	103066	106241
Lofa	375006	386557	398463	410736	423387	436427	449870	463726	478009	492732	507908
Margibi	284337	293095	302123	311428	321021	330908	341101	351607	362437	373600	385107
Maryland	184126	189797	195643	201669	207880	214283	220883	227687	234699	241928	249380
Montserrado	1514638	1561290	1609379	1658949	1710046	1762716	1817009	1872974	1930663	1990129	2051427
Nimba	625806	645082	664951	685432	706544	728306	750738	773862	797697	822267	847594
Rivercess	96858	99842	102917	106087	109355	112723	116195	119774	123463	127266	131186
Sinoe	138687	142959	147362	151901	156579	161402	166374	171498	176780	182225	187838
River Gee	90464	93250	96122	99083	102134	105280	108523	111865	115310	118862	122523
Liberia	4709004	4854047	5003554	5157667	5316527	5480279	5649078	5823072	6002425	6187307	6377881

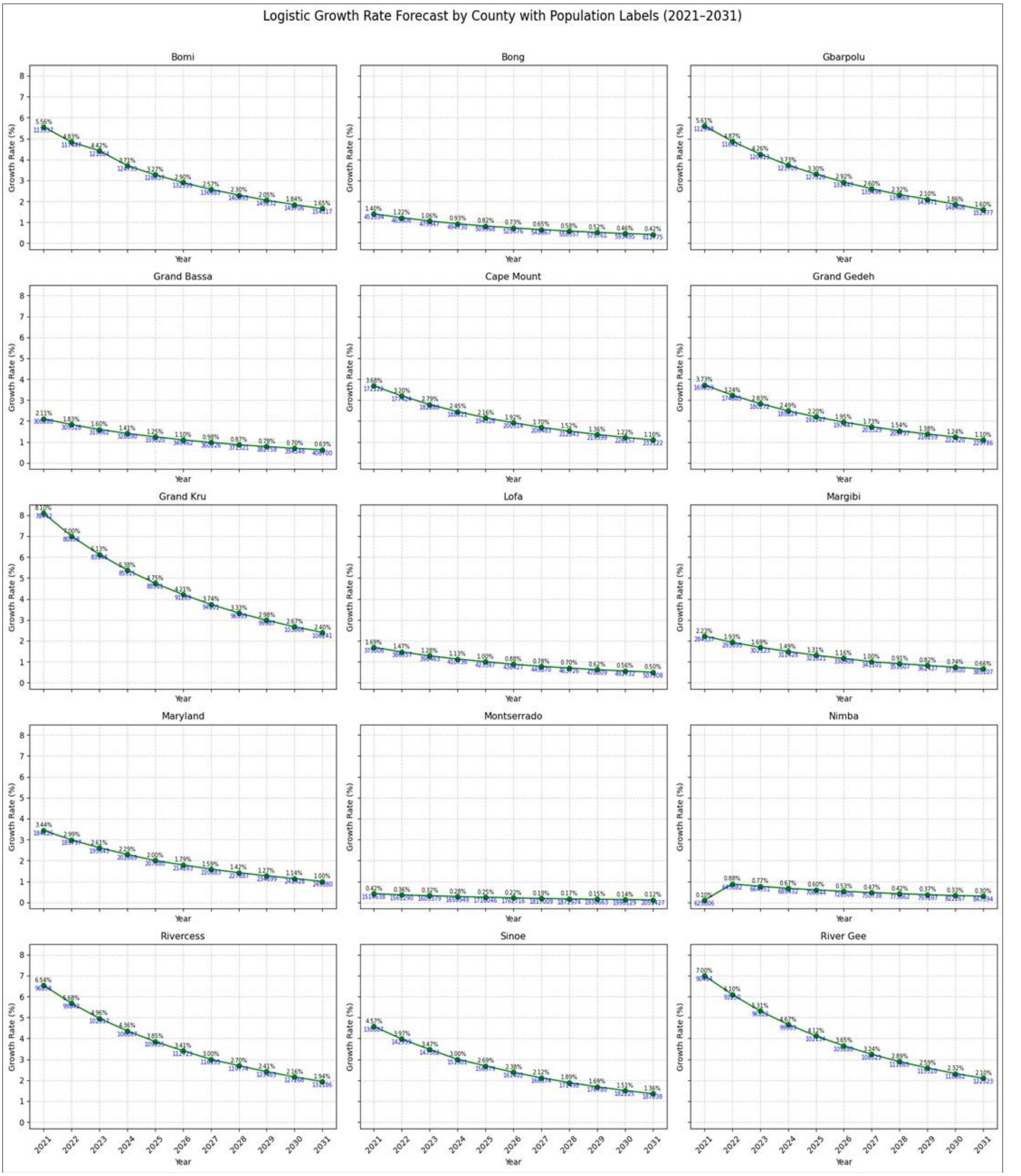

Fig 1: Logistic growth rate forecast across counties with population labels (2021-2031

4. Discussions

The proposed logistic growth model produced significant results, where the performance of the model accounts for time-dependent changes in growth rates and carrying capacities. The study provides a more realistic basis for forecasting population dynamics across counties. This allows policymakers to better anticipate demands for essential services, such as healthcare, education, housing, and employment. Additionally, the model offers a framework that can be adapted and updated as new demographic data becomes available, thereby supporting evidence-based decision-making and promoting sustainable development. Moreover, it encourages a shift away from static, deterministic assumptions toward more dynamic modeling practices in demographic research in Liberia.

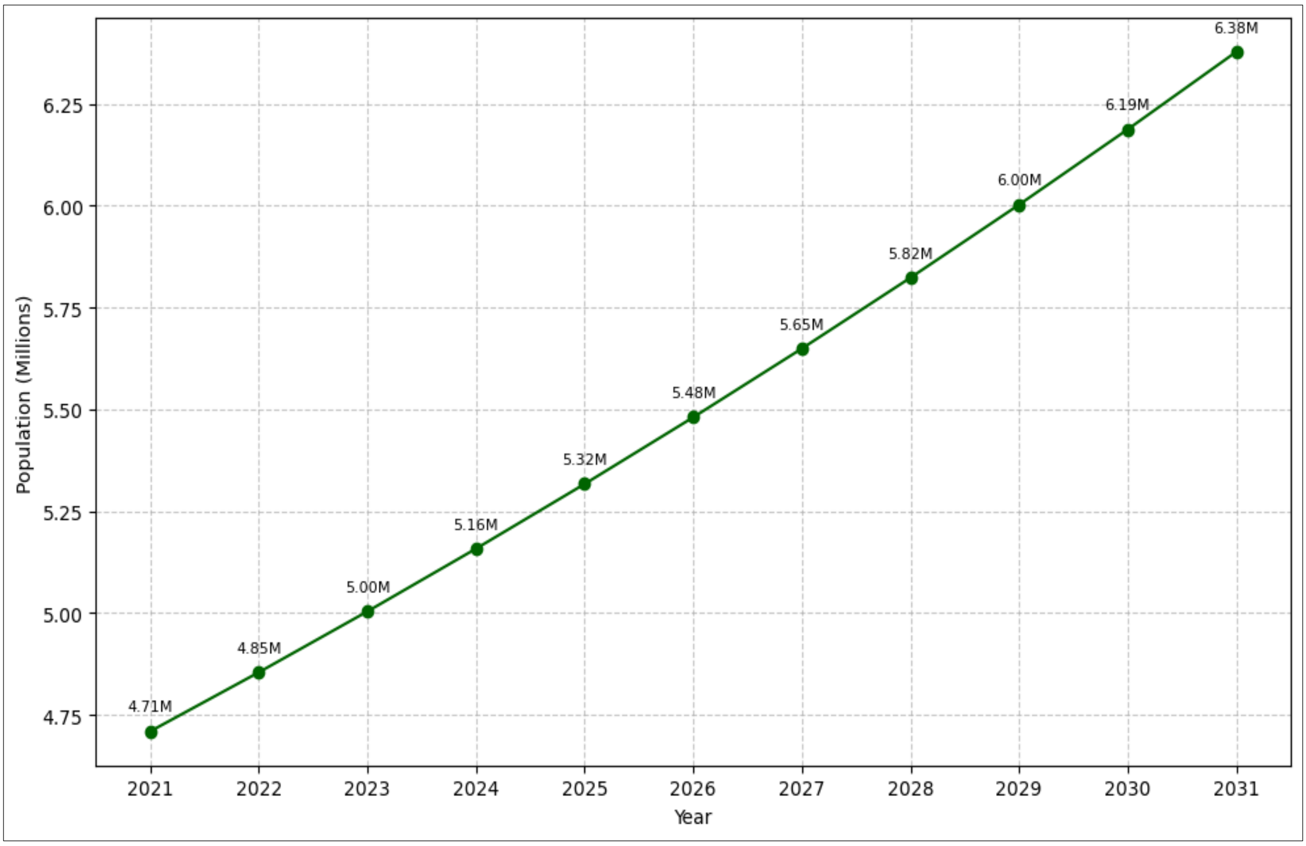

Fig 2: Forecasted Population of Liberia (2021-2031)

5. Conclusion

Existing practices of demographic forecasting in Liberia have been enhanced by applying a logistic growth model in this study. Unlike conventional deterministic approaches that rely on fixed growth rates, the proposed model accounts for the dynamic nature of population growth by incorporating both time-dependent growth rates and flexible carrying capacities. The analysis demonstrates that population growth in Liberia is not uniform and that growth rates decline over time as counties approach their respective carrying capacities because of the logistic growth behaviour. This finding not only aligns with biological and environmental growth principles but also mirrors the infrastructural and economic constraints present in many counties.

Along with supporting growth principles, these projections offer valuable insights for policymakers, suggesting where population pressures may intensify and where strategic investments in infrastructure, health, and education are most needed. Ultimately, the study reinforces the effectiveness of logistic models in long-term population forecasting while advocating for their inclusion in Liberia's planning and national statistical frameworks to improve Liberia's development initiatives.

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