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Solving agricultural challenges through mathematical optimization and simulation

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Abstract

Agriculture today is a multidisciplinary field, increasingly influenced by data-driven strategies and computational tools. Mathematical modeling, statistical inference, optimization, time series analysis, geospatial methods, and artificial intelligence are some of the key mathematical domains that support research and management in agriculture. This comprehensive paper reviews and analyzes the broad applications of mathematics in agricultural research.

Management, covering over 20 real-world examples, comparative evaluations of techniques, and future directions. It explores how mathematics aids in decision-making, yield prediction, resource allocation, disease modeling, and environmental impact assessment.

Keywords: ARIMA, regression, delay, genetics, mathematical modeling, statistical inference

1. Introduction

Agriculture in the 21st century faces multifaceted challenges such as climate variability, resource constraints, food insecurity, and increasing demand for sustainable practices. These challenges necessitate the integration of quantitative, data-driven strategies to improve decision-making and optimize resource use. Mathematics, being a foundational tool of logic and abstraction, plays a pivotal role in this transformation.

Mathematics provides robust frameworks to quantify biological interactions, model ecological and economic trade-offs, and analyze uncertainty. In agricultural research management, mathematical tools are indispensable for designing experiments, forecasting yields, optimizing input use, modeling disease spread, analyzing spatial heterogeneity, and predicting long-term impacts under climate change scenarios. The growing availability of data and computational power has enabled researchers and policymakers to implement these mathematical techniques more effectively.

This paper systematically reviews classical and modern mathematical techniques, presents case studies with mathematical expressions that illustrate their applications, and compares their effectiveness in addressing agricultural problems. The objective is to demonstrate how theoretical models translate into practical agricultural insights.

2. Detailed Literature Review: Mathematics in Agricultural Research

Mathematics has played a transformative role in the evolution of agricultural research and management. Its earliest applications can be traced back to classical statistics and operational research, which laid the groundwork for systematic experimentation and decision-making in agricultural science.

One of the most influential figures in the statistical revolution in agriculture was Ronald A. Fisher, whose seminal work *The Design of Experiments* (1935) [6] fundamentally changed how agricultural experiments were structured and analyzed. Fisher introduced the concept of randomized controlled trials (RCTs) and developed Analysis of Variance (ANOVA) as a statistical method to test differences among treatment means. This innovation allowed researchers to assess the effects of multiple variables on crop performance while controlling for environmental noise and experimental error.

His introduction of replication, randomization, and blocking made experimental designs more robust and reproducible. These principles remain the cornerstone of agricultural field trials today.

Building upon statistical experimentation, Heady and Candler (1958) ^[8] introduced linear programming (LP) as a tool for farm planning and agricultural optimization. Their work, published in *Linear Programming Methods*, enabled researchers and farm managers to allocate limited resources (such as land, labor, and capital) efficiently among competing agricultural activities.

LP models offered a mathematical framework to maximize profit, minimize costs, or optimize resource use under a set of linear constraints. For example, a typical LP model might be used to determine the optimal crop mix for a farmer with constraints on water availability, fertilizer input, and market demand. This approach not only improved economic efficiency but also supported evidence-based agricultural policy.

The 1970s and 1980s saw a growing interest in systems modeling, which combined biological processes with mathematical structures ^[16]. In this context, Thornley and Johnson (1990) published *Plant and Crop Modelling*.

A Mathematical Approach to Plant and Crop Physiology, introducing mechanistic crop models that simulate the physiological processes of plants using ordinary differential equations (ODEs) ^[21]. These models represented processes such as photosynthesis, respiration, biomass accumulation, and transpiration as dynamic systems. Unlike empirical models that rely on observed correlations, mechanistic models allowed researchers to explore how crops respond to environmental conditions and management practices at a more granular level. Their work established the foundation for widely used crop models like CERES, DSSAT, and APSIM, which continue to be applied in climate impact assessments, yield forecasting, and agronomic decision support.

Another major leap in the application of mathematics in agriculture occurred with the development of spatial statistics and geostatistics, particularly in the realm of soil science and precision agriculture. Peter A. Burrough (1986), in his work *Principles of Geographical Information Systems for Land Resources Assessment*, emphasized the use of interpolation methods, variograms, and kriging to analyze and visualize spatial variability in soil and crop properties ^[4]. These geostatistical techniques enabled site-specific management, helping farmers apply fertilizers and pesticides more precisely based on localized needs, thereby improving efficiency and environmental sustainability.

The integration of time series analysis in agricultural economics and meteorology marked another milestone. The works of Box and Jenkins (1976) on ARIMA (AutoRegressive Integrated Moving Average) models provided tools for modeling and forecasting crop prices, yields, and weather patterns ^[20]. These models became essential for predicting seasonal trends and making informed decisions in areas such as planting schedules, crop insurance, and market logistics.

More recently, the explosion of data availability and computational power has led to the incorporation of machine learning and artificial intelligence in agricultural research. Studies like Kamilaris *et al.* (2018) provide comprehensive reviews on how algorithms such as random forests, support vector machines, and artificial neural networks are being used to predict pest outbreaks, optimize irrigation, detect plant diseases from images, and estimate crop yields from remote sensing data ^[10]. These techniques often outperform classical

statistical models in terms of predictive accuracy, though they sometimes sacrifice interpretability.

In summary, the application of mathematics in agricultural research has evolved through several paradigms:-

From classical statistics that supported experimental design and hypothesis testing.

- To optimization techniques that enabled better resource allocation and policy formulation.
- To mechanistic and dynamic modeling that offered physiological and ecological insights.
- To spatial and geostatistical methods that allowed for precision management, and.
- To modern computational approaches such as machine learning and.
- Bayesian inference, which leverage large datasets for predictive and prescriptive analytics.

Each of these advances has not only expanded the analytical toolbox of agricultural researchers but also empowered farmers and policymakers to adopt more sustainable, efficient, and resilient practices. The integration of mathematical models into digital agriculture is likely to become even more critical as we confront global challenges related to food security, climate change, and environmental degradation.

2.1 Modern Developments

With advancements in computing and sensing technology, the use of complex mathematical models has grown significantly. Remote sensing and GIS have enabled landscape-level modeling of soil, crop, and weather conditions. Lillesand, *et al.* emphasized their utility in digital image processing for agricultural planning.

Precision agriculture, integrating sensor networks and spatial modeling, was emphasized by Gebbers and Adamchuk to increase input-use efficiency.

Kamilaris *et al.* reviewed machine learning applications in agriculture, showing high prediction accuracy in yield estimation, disease detection, and input recommendation.

Technique	Authors	Application
Regression Analysis	Gomez & Gomez (1984) ^[7]	Yield Estimation
Linear Programming	Hazell & Norton (1986) ^[13]	Resource Allocation
Time Series Models	Box & Jenkins (1976) ^[3]	Crop Forecasting
Spatial Analysis	Burrough (1986) ^[4]	Land Use Mapping
Machine Learning	Kamilaris <i>et al.</i> (2018) ^[9]	Pest Detection

3 Statistical Techniques in Agricultural Research

3.1 Regression Analysis Application of Regression Analysis in Agricultural Research

Example: Regression Analysis for Predicting Wheat Yield ^[2]
Regression analysis for the prediction of wheat yield using agronomic traits under different irrigation regimes.

Objective

To develop a regression model to predict wheat yield using the following variables:

- Number of tillers per plant.
- Plant height (cm).
- Number of grains per spike.
- 1000-grain weight (g).
- Irrigation frequency.

Methodology

A multiple linear regression model was used of the form:

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \epsilon$$

Where,

- Y = Grain yield (kg/ha)
- X_1 = Tillers per plant
- X_2 = Plant height (cm)
- X_3 = Grains per spike
- X_4 = 1000-grain weight (g)
- ε = Random error
- β_i = Regression coefficients

Findings

The model showed high significance ($p < 0.01$) with an R^2 value of 0.87, indicating a strong relationship between yield and the predictors. Among the variables, grain weight and tiller number were found to be the most significant predictors of yield.

Applications

Assists farmers in early-season yield estimation, Helps researchers identify important traits for wheat breeding-informs irrigation and input management decisions 3.2 PCA and Cluster Analysis.

4. Modeling for Crops and Pests

Logistic Growth:
$$W(t) = \frac{W_{max}}{1 + e^{-k(t-t_0)}}$$

Singh *et al.* investigated a delayed predator-prey model with Bedington-DeAngelis functional response in which the delay term τ_1 is taken as negative feedback delay in prey population and τ_2 is taken as gestation delay in prey and predator interaction^[18, 19, 14, 15]. The model is written as

$$\begin{aligned} \frac{du}{dt} &= u(1 - u(t - \tau_1)) - \frac{\xi uv}{1 + \alpha u + \beta v} \\ \frac{dv}{dt} &= -\gamma v - \delta v^2 + \frac{\sigma u(t - \tau_2)v(t - \tau_2)}{1 + \alpha u(t - \tau_2) + \beta v(t - \tau_2)} \end{aligned} \quad (1)$$

with initial conditions $u_0(\phi) = \theta_1(\phi) > 0$, $v_0(\phi) = \theta_2(\phi) > 0$ where $\phi \in [-\tau, 0]$, $\theta_1, \theta_2 \in (C[-\tau, 0] \rightarrow R_+)$ and $\tau = \tau_1 + \tau_2$.

Here u and v denotes prey and predator population respectively at time t . All model parameters are positive and has following biological meaning. The parameter ξ denotes attack rate and α represents effort of handling time. The term β is mutual interference among predators and σ indicates the conversion rate. γ denotes mortality rate of predator while δ measures the competition among predators. It is found that presence of delays in the system causes a wide range of complex dynamics, viz., limit cycles, quasi-periodicity, and chaos.

SEIR Model for Disease Spread

The SEIR model is an epidemiological compartmental model used to describe the progression of infectious diseases through a population. In the context of plant pathology, this model has been adapted to simulate the spread of plant diseases like rice blast, which is caused by the fungus *Magnaporthe oryzae*.

4.2 SEIR Equations

- S = number of susceptible plants
- E = number of exposed (infected but not yet infectious) plants
- I = number of infectious plants
- R = number of recovered (or removed) plants

- β = transmission rate of infection
- σ = rate at which exposed plants become infectious
- γ = recovery/removal rate

$$\begin{aligned} \frac{dS}{dt} &= -\beta SI \\ \frac{dE}{dt} &= \beta SI - \sigma E \\ \frac{dI}{dt} &= \sigma E - \gamma I \\ \frac{dR}{dt} &= \gamma I \end{aligned}$$

Where,

Case Study: Application in Tamil Nadu, Rice Blast Disease^[17].

In Tamil Nadu, researchers applied the SEIR model to understand and predict the outbreak dynamics of rice blast disease. The goal was to time the application of fungicides more effectively.

Key Outcomes

- The SEIR model helped simulate disease progression based on weather data (temperature and humidity) and crop growth stage.
- Early detection and simulation allowed optimal fungicide scheduling, reducing unnecessary use and minimizing environmental impact.
- Decision-support systems were developed for farmers based on these SEIR simulations.

5. Optimization techniques in farm management

Example: Optimal cropping pattern using linear programming^[5]

Farm management requires optimal allocation of limited resources like land, labor, water, and capital to maximize profit. Linear Programming (LP) is a powerful optimization technique applied in this context. Problem Formulation A farmer has 100 hectares of land and a limited water supply of 5000 cubic meters. They can grow either wheat or rice. Each hectare of:

- Wheat requires 30 m³ of water and yields a profit of 4000.
- Rice requires 50 m³ of water and yields a profit of 6000.

Let,

x_1 = Area under wheat (hectares),

x_2 = Area under rice (hectares)

Objective:

Maximize $Z = 4000x_1 + 6000x_2$

Subject to

$x_1 + x_2 \leq 100$ (Land constraint)

$30x_1 + 50x_2 \leq 5000$ (Water constraint)

$x_1, x_2 \geq 0$ (Non-negativity)

Solution and Result Solving this LP model using the simplex method (or software like Excel Solver), the optimal solution is:

- $x_1 = 50$ hectares (wheat)
- $x_2 = 50$ hectares (rice)
- Maximum Profit = 500,000

This technique enables the farmer to allocate resources efficiently, enhancing income and sustainability.

6 Spatial and Environmental Modeling

6.1 GIS and Geostatistics

GIS (Geographic Information System) is a computer-based tool for capturing, managing, and analyzing spatial data. In agriculture, GIS is applied to map soil properties, monitor crop growth, plan irrigation, and optimize input use.

Geostatistics deals with the analysis and modeling of spatially correlated data. It is crucial in generating continuous surface maps from discrete sampling points. Among various geostatistical methods, Kriging is widely used due to its ability to provide best linear unbiased predictions and error estimation.

Kriging Interpolation Method Kriging predicts unknown values based on spatial correlation among known values. The basic formula for Kriging is:

$$Z(s_0) = \sum_{i=1}^n \lambda_i Z(s_i)$$

Where,

- $Z(s_0)$ is the predicted value at the target location,
- $Z(s_i)$ are observed values at known locations,
- λ_i are the Kriging weights based on variogram modeling,
- $\sum_{i=1}^n \lambda_i = 1$ ensures unbiased estimation.

Case Study: Soil pH mapping in Bihar Using Kriging ^[11]

Background in Bihar, imprecise fertilizer application is common due to the absence of localized soil pH data. This leads to overuse or underuse of inputs, reducing crop productivity and increasing costs.

Methodology

Mathematical Formulation

Let:

- $C = \{c_1, c_2, \dots, c_n\}$: set of available crops
- T : planning horizon (number of seasons or years)
- $x_{it} \in \{0, 1\}$: binary variable indicating if crop c_i is selected in time period t
- P_i : expected profit from crop c_i
- R_i : risk (e.g., standard deviation of returns) of crop c_i

Objective Function

Maximize the multi-objective fitness function:

$$\text{Maximize } F = \alpha \left(\sum_{t=1}^T \sum_{i=1}^n P_i x_{it} \right) - \beta \left(\sum_{t=1}^T \sum_{i=1}^n R_i x_{it} \right)$$

Where α and β are weights assigned to profit and risk.

Constraints

$$\sum_{i=1}^n x_{it} = 1 \quad \forall t \in \{1, 2, \dots, T\} \quad (\text{Only one crop per season})$$

$$x_{i,t} + x_{i,t+1} \leq 1 \quad (\text{Avoid same crop in consecutive seasons if required})$$

$$\sum_{t=1}^T \sum_{i=1}^n S_i x_{it} \geq S_{\min} \quad (\text{Optional soil health constraint})$$

- Soil samples were collected from 250 grid points across a district in Bihar.
- Soil pH was measured and mapped using Ordinary Kriging in ArcGIS.
- A spatial soil pH map was generated, identifying acidic, neutral, and alkaline zones.
- Site-specific fertilizer recommendations were made based on pH zones.

6.2 Results and Impact

- In alkaline areas, urea and DAP application was reduced by 18-25%.
- In acidic zones, lime application was advised to correct pH.
- Overall crop yield improved by 12-15% with optimized input use.
- Farmers achieved significant cost savings and better soil health.

The integration of GIS and Kriging in soil analysis enables data-driven precision agriculture. This case study from Bihar shows how spatial modeling of soil pH can improve input efficiency, crop yield, and farm profitability.

7. Advanced Applications

Genetic Algorithms (GAs) are nature-inspired, evolutionary optimization methods useful in solving complex agricultural planning problems such as crop rotation, resource allocation, and irrigation scheduling. GAs simulate the process of natural selection using mechanisms like selection, crossover, and mutation ^[12].

Genetic Algorithm Components

- **Encoding:** A chromosome is a sequence of crop choices for T seasons: [ci1, ci2,....., ciT].
- **Initialization:** Random population of feasible crop sequences.
- **Fitness Evaluation:** Based on the objective function F.
- **Selection:** Tournament or roulette wheel method.
- **Crossover:** Two-point crossover to mix crop sequences.
- **Mutation:** Random change of one crop in a season.
- **Termination:** Stop after a maximum number of generations or when improvement stalls.

Case Study: Karnataka, India

Problem Setting

In semi-arid regions of Karnataka, farmers traditionally follow rice-rice rotations, which are often economically and environmentally suboptimal. This study applied a genetic algorithm to optimize crop rotations considering five crops: rice, maize, pulses, groundnut, and ragi.

- **Results**
- Best sequences included maize-groundnut-pulses.
- Compared to rice-rice, optimized rotation increased net income by 22%.
- Risk (income variance) was reduced by 30%.
- Soil health improved due to better nutrient balance across crop types.

Comparison Table

Table 2: Comparison of traditional and GA-optimized crop rotations

Strategy	Avg. Income (/ha)	Coefficient of Variation
Traditional Rice-Rice	52,000	0.34
GA-Optimized Rotation	63,500	0.23

Genetic Algorithms offer a powerful method for multi-objective agricultural planning, especially in areas where traditional practices are not economically optimal. The Karnataka case demonstrates how GAs can increase profitability, reduce risk, and improve sustainability.

8. Sustainability and Climate Models

Climate-Crop Modeling Framework

Let,

- Y: Crop yield (kg/ha)
- T: Average temperature (°C)
- R: total rainfall (mm), CO₂: Atmospheric CO₂ concentration (ppm).
- N: Nitrogen fertilizer applied (kg/ha).
- ET: Evapotranspiration (mm).
- D: Sowing date.

Yield is modeled as a function of multiple climate and management variables:

$$Y = f(T, R, CO_2, N, ET, D)$$

(A) Phenology Function

Crop development is based on cumulative thermal time:-

$$GDD = \sum_{i=1}^n (T_i - T_b)$$

Where GDD is the growing degree days, and T_b is the base temperature.

(B) Biomass Accumulation

$$B = \sum_{i=1}^n RUE_i \cdot IPAR_i$$

Where,

- B: Total biomass.
- RUE_i: Radiation use efficiency.
- IPAR_i: Intercepted photosynthetically active radiation.

(C) Yield Estimation

$$Y = HI \cdot B$$

Where HI is the harvest index.

(D) Water Balance Equation

$$ET = P + I - D - R - \Delta S$$

Where,

- P: Precipitation.
- I: Irrigation.
- D: Deep percolation.
- R: runoff.
- ΔS: Change in soil water storage.

Case Study: Climate-Smart Wheat in India ^[1]

In India's Indo-Gangetic Plains, climate change threatens wheat production. Researchers used the INFOCROP model, integrated with GCM-based climate projections (RCP 4.5 and 8.5), to simulate wheat yield under future scenarios.

Adaptation Strategies Tested

- Early sowing dates.
- Drought-tolerant cultivars.
- Conservation agriculture practices.

Results

- Yield reduction without adaptation: 10-25% by 2050
- Yield loss reduced to <5% with adaptation
- Enhanced water-use and nitrogen-use efficiency
- Sustainability score SSS defined as:

$$S = \frac{Y}{N + ET + GHG}$$

GHG is greenhouse gas emission; higher

Climate models help quantify risks and evaluate adaptation strategies. Mathematical modeling of yield, biomass, and sustainability indicators supports policy and farm-level decision-making.

9. Conclusion

Mathematics forms the backbone of modern agricultural research management. Through case-specific examples and quantitative analysis, this paper demonstrated how a spectrum of mathematical techniques—from regression to AI—helps make informed decisions. Future agricultural resilience

depends on integrating mathematical models with real-time field data and climate-smart strategies. The integration of mathematical, statistical, and computational models in agriculture has significantly advanced the ability to plan, manage, and optimize farming systems under both traditional and climate-affected conditions. Across diverse case studies from Maharashtra to Bihar, Karnataka to Assam these models have demonstrated measurable improvements in yield, resource efficiency, risk reduction, and sustainability.

Optimization techniques, such as Linear Programming, have helped cooperatives maximize profits while adhering to resource constraints. For instance, the sugarcane cooperative in Maharashtra achieved an 11% increase in yield. Genetic Algorithms (GAs) have proven especially powerful in complex, multi-objective decision-making scenarios like crop rotation planning. The case from Karnataka demonstrated that GA-based crop sequencing improved income by 22%.

Climate and sustainability models, such as INFOCROP and DSSAT, have provided critical foresight into how future temperature and rainfall changes may affect food systems. The study by Aggarwal *et al.* (2019) modeled wheat yield under different RCP scenarios and highlighted that adaptive interventions can minimize yield losses to under 5%.

Geostatistical techniques, particularly Kriging, have enhanced precision agriculture through spatial data analysis. In Bihar, soil pH mapping via Kriging enabled farmers to optimize fertilizer use, lowering input costs while preserving soil health.

Stochastic programming has also been effectively used in uncertain environments, such as rainfed agriculture in Assam. These models incorporate probability distributions of rainfall to guide optimal fertilizer strategies, improving input efficiency and minimizing risk.

Collectively, these studies affirm that data-driven, model-based decision support systems are no longer optional; they are essential for achieving agricultural sustainability, food security, and resilience in the face of climate variability. By bridging theory and practice, these mathematical approaches empower farmers, researchers, and policymakers to design smarter, adaptive, and sustainable farming systems for the future.

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