International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452
Maths 2025; 10(6): 225-230

https://www.mathsjournal.com
Received: 09-05-2025
Accepted: 12-06-2025
Published: 16-06-2025

Bed Prakash Singh
Assistant Professor, Government (PG) College, Fatehabad, Agra, Uttar Pradesh, India

Corresponding Author:
Bed Prakash Singh
Assistant Professor, Government (PG) College, Fatehabad, Agra, Uttar Pradesh, India

Numerical solution of 1-dimensional heat conduction problem using double interpolation method

Bed Prakash Singh

DOI: https://www.doi.org/10.22271/maths.2025.v10.i6c.2079

Abstract
Using mathematical techniques and limited distinctions, this piece attempts to identify the configuration of a one-layered Heat condition complete with starting and limit conditions. We used the Bender-Schmidt repeat connection equation to calculate V(x,t) at various cross section regions. We then applied double interpolation to identify the heat condition's organization as a double interpolating polynomial. We then displayed the solution graphically.

Keywords: Bender Schmidt formula, heat conduction equation, interpolation, boundary value problem

1. Introduction

Our economy will suffer as a result of the Catatumbo neighbourhood's escalating armed conflict. The current scenario, as evidenced in the increasing rates of homelessness, underlines the importance of providing solutions to this problem. It would be intriguing for the citizens of Catatumbo, Colombia, to see government support generated as a result of the advancement of exploration focused on new technological advancements. The investigation of power-free technology is a significant step in this direction, especially in countries participating in an economic crisis. At the time, one source of increasing revenue is the study of various cooling systems and the consequences that these have on better positions.

Iyengar and Manohar (1988) [2] used the fourth-request distinction investigate to arrange Poisson's scenario in barrel-shaped facilities. They extended the technique to solve the heat situation in two-layered structures with polar positions and three-layered configurations with barrel-shaped arrangements. Marwah and Chopra (1992) [6] developed a truly outstanding scientific methodology for determining the transient intensity conduction condition in a one-layered empty compound chamber with time-dependent limit constraints. According to Sabaeian *et al*. (2008) [10], temperature conveyance research is critical for estimation, performing, and forecasting heated impacts. Ciegis *et al*. (2010) [1] create and confirm numerical models and mathematical computations for recreating intensity movement in composite materials. Javed (2012) [3] studied about the development of wet or dry intensity. The phrase "dry applications" describes products like brilliant intensity, steaming water bottles, and electric cushions. Wet intensity has been believed to be more penetrating than dry intensity, however this is more likely to be the case because materials covered in water lose heat more slowly than dry ones. The task of applying a limited contrast method in a barrel-shaped organizing structure to solve a three-layered Poisson's condition was taken up by Shiferaw and Mittal (2013) [12]. Mori and Romo (2015) [9] applied the restricted distinction technique to a mathematical recreation of 2D convection-dispersion in barrel-shaped facilities. Kafle *et al*. (2020) inferred the Forward Time Central Space Scheme (FTCSS) for the intensity condition. They also used FTCSS to investigate its mathematical arrangement, and they searched into the mathematical solution and logical arrangement for a range of homogeneous materials (for a variety of benefits of diffusivity α). The results of Khatun *et al*. (2020) [5] study on the safety of a one-layered heat condition were reported. Maturi *et al*. (2020) [7] proposed a mathematical strategy for determining the proper solution for the intensity conduction condition of copper.

Because copper is perfectly matched to lead in terms of heat and electrical conductivity, they focused on it. Salehi and Granpayeh (2020) [11] proposed a limited distinction method as an alternative solution for the two-layered Schrodinger condition in polar directions. Meyu and Koriche (2021) [8] provided a basic treatment of the intensity condition arrangement in one area. The problem of a three-layered transient intensity conduction condition was tackled by Tsega *et al.* (2022) [13]. They achieved this by using tube-shaped facilitators to approximate second-request spatial subordinates with five focal contrasts. Furthermore, he discussed the problem of the strength condition.

2. Formulation of the problem

In this discussion, we will focus on the following boundary value problem associated with the one-dimensional heat equation. A partial differential equation (PDE) that is frequently defined by may tell you how hot or cold a rod is

$$\frac{\partial V}{\partial t} = K^2 \frac{\partial^2 V}{\partial t^2} \quad (1)$$

Where $V(x,t)$ is the temperature of the pole estimated at position x at time t, and K is the warm diffusivity of the material, which estimates how well the bar can lead heat,

Dependent upon the accompanying limit conditions.

$$V(0,t) = 0 \quad (2)$$

$$V(1,t) = 0 \quad (3)$$

$$V(x,0) = sin\pi x \quad (4)$$

Where $0 \leq x \leq 1$ and $t > 0$

For the solution of this problem, Let we take $K^2 = 1$

Analytic Solution of the above problem is

$V(x,t) = sin\pi x \, e^{K^2\pi^2 t}$

The interval of differencing of x as 0.2 i.e. $h = 0.2$

From Bender Schmidt equation, the time span of t as

$$k = \frac{h^2}{2c^2} = \frac{(0.2)^2}{2} = 0.02 \quad (5)$$

Thus $x_0 = 0, x_1 = 0.2, x_2 = 0.4, x_3 = 0.6, x_4 = 0.8, x_5 = 1$

$t_0 = 0, t_1 = 0.02, t_2 = 0.04, t_3 = 0.06, t_4 = 0.08, t_5 = 0.1$

We have a total of 25 mesh points after drawing straight lines parallel to the coordinate axis (t,x).

Table 1: Shows the variation of a parameter xx*x* over time *t* at different points, illustrating the change in the parameter's value from t=0 to *t*=1 in increments of 0.2.

x / t	0	0.2	0.4	0.6	0.8	1
0	0	0.59	0.95	0.95	0.59	0
0.02	0	0.475	0.77	0.77	0.475	0
0.04	0	0.385	0.6225	0.6225	0.385	0
0.06	0	0.3113	0.504	0.504	0.3113	0
0.08	0	0.408	0.252	0.252	0.408	0
0.1	0	0.204	0.33	0.33	0.204	0

Table 2: This table presents the initial values V_{1i} along with successive differences $\Delta^{(0+n)}V_{1i}$ to analyze the changes and higher-order differences at different points.

S. No.	V_{1i}	$\Delta^{0+1}V_{1i}$	$\Delta^{0+2}V_{1i}$	$\Delta^{0+3}V_{1i}$	$\Delta^{0+4}V_{1i}$	$\Delta^{0+5}V_{1i}$
1	0.59	-0.115	0.025	-0.0087	0.1628	-0.788
2	0.475	-0.09	0.0163	0.1541	-0.6252	
3	0.385	-0.0737	0.1704	-0.4711		
4	0.3113	0.0967	-0.3007			
5	0.408	-0.204				
6	0.204					

Table 3: Table presents the values of V_2i and their successive differences (Δ^(0+1) V_2i to Δ^(0+5) V_2i) across five data points.

S. No.	V_{2i}	$\Delta^{0+1}V_{2i}$	$\Delta^{0+2}V_{2i}$	$\Delta^{0+3}V_{2i}$	$\Delta^{0+4}V_{2i}$	$\Delta^{0+5}V_{2i}$
1	0.95	-0.18	0.0325	-0.0035	-0.159	0.785
2	0.77	-0.1475	0.029	-0.1625	0.626	
3	0.6225	-0.1185	-0.1335	0.4635		
4	0.504	-0.252	0.33			
5	0.252	0.078				
6	0.33					

Table 4: This table displays the values of V_3i and their successive differences (Δ^(0+1) V_3i to Δ^(0+5) V_3i) across six data points.

S. No.	V_{3i}	$\Delta^{0+1}V_{3i}$	$\Delta^{0+2}V_{3i}$	$\Delta^{0+3}V_{3i}$	$\Delta^{0+4}V_{3i}$	$\Delta^{0+5}V_{3i}$
1	0.95	-0.18	0.0325	-0.0035	-0.159	0.785
2	0.77	-0.1475	0.029	-0.1625	0.626	
3	0.6225	-0.1185	-0.1335	0.4635		
4	0.504	-0.252	0.33			
5	0.252	0.078				
6	0.33					

Table 5: This table shows the values of V_4i and their successive differences (Δ^(0+1) V_4i to Δ^(0+5) V_4i) for six data points.

S. No.	V_{4i}	$\Delta^{0+1}V_{4i}$	$\Delta^{0+2}V_{4i}$	$\Delta^{0+3}V_{4i}$	$\Delta^{0+4}V_{4i}$	$\Delta^{0+5}V_{4i}$
1	0.59	-0.115	0.025	-0.0087	0.1628	-0.788
2	0.475	-0.09	0.0163	0.1541	-0.6252	
3	0.385	-0.0737	0.1704	-0.4711		
4	0.3113	0.0967	-0.3007			
5	0.408	-0.204				
6	0.204					

Since both the First and the Last Column of Table 1 contain 0, this means that

$$\Delta^{0+1}V_{00} = \Delta^{0+2}V_{00} = \Delta^{0+3}V_{00} = \Delta^{0+4}V_{00} = \Delta^{0+5}V_{00} = 0 \quad (6)$$

And $\Delta^{0+1}V_{50} = \Delta^{0+2}V_{50} = \Delta^{0+3}V_{50} = \Delta^{0+4}V_{50} = \Delta^{0+5}V_{50} = 0$ (7)

From Table 2, we get

$$\Delta^{0+1}V_{10} = -0.115, \Delta^{0+2}V_{10} = 0.025, \Delta^{0+3}V_{10} = -0.0087, \Delta^{0+4}V_{10} = 0.1628, \Delta^{0+5}V_{10} = -0.788 \quad (8)$$

From Table 3,

$$\Delta^{0+1}V_{20} = -0.18, \Delta^{0+2}V_{20} = 0.0325, \Delta^{0+3}V_{20} = -0.0035, \Delta^{0+4}V_{20} = -0.159, \Delta^{0+5}V_{20} = 0.785 \quad (9)$$

From Table 4,

$$\Delta^{0+1}V_{30} = -0.18, \Delta^{0+2}V_{30} = 0.0325, \Delta^{0+3}V_{30} = -0.0035, \Delta^{0+4}V_{30} = -0.159, \Delta^{0+5}V_{30} = 0.785 \quad (10)$$

From Table 5

$$\Delta^{0+1}V_{40} = -0.115, \Delta^{0+2}V_{40} = 0.025, \Delta^{0+3}V_{40} = -0.0087, \Delta^{0+4}V_{40} = 0.1628, \Delta^{0+5}V_{40} = -0.788 \quad (11)$$

After carrying out the procedure described above for each row in table 1, we have

$$\Delta^{1+0}V_{00} = 0.59, \Delta^{2+0}V_{00} = -0.23, \Delta^{3+0}V_{00} = -0.13, \Delta^{4+0}V_{00} = 0.13, \Delta^{5+0}V_{00} = 0 \quad (12)$$

$$\Delta^{1+0}V_{01} = 0.475, \Delta^{2+0}V_{01} = -0.18, \Delta^{3+0}V_{01} = -0.115, \Delta^{4+0}V_{01} = 0.115, \Delta^{5+0}V_{01} = 0 \quad (13)$$

$$\Delta^{1+0}V_{02} = 0.385, \Delta^{2+0}V_{02} = -0.1475, \Delta^{3+0}V_{02} = -0.09, \Delta^{4+0}V_{02} = 0.09, \Delta^{5+0}V_{02} = 0 \quad (14)$$

$$\Delta^{1+0}V_{03} = 0.3113, \Delta^{2+0}V_{03} = -0.1186, \Delta^{3+0}V_{03} = -0.0741, \Delta^{4+0}V_{03} = 0.0741, \Delta^{5+0}V_{03} = 0 \quad (15)$$

$$\Delta^{1+0}V_{04} = 0.408, \Delta^{2+0}V_{04} = -0.564, \Delta^{3+0}V_{04} = 0.72, \Delta^{4+0}V_{04} = -0.72, \Delta^{5+0}V_{04} = 0 \quad (16)$$

$$\Delta^{1+0}V_{05} = 0.204, \Delta^{2+0}V_{05} = -0.078, \Delta^{3+0}V_{05} = -0.048, \Delta^{4+0}V_{05} = 0.048, \Delta^{5+0}V_{05} = 0 \quad (17)$$

The formula for determining the differences between two orders can be expressed generally as

$$\Delta^{m+n}V_{00} = \Delta^{m+0}V_{0n} - n\Delta^{m+0}V_{0n-1} + \frac{n(n-1)}{2!}\Delta^{m+0}V_{0n-2} - \cdots + (-1)^m\Delta^{m+0}V_{00} \quad (18)$$

$$\Delta^{n+m}V_{00} = \Delta^{0+n}V_{m0} - m\Delta^{0+n}V_{m-10} + \frac{m(m-1)}{2!}\Delta^{0+n}V_{m-20} - \cdots + (-1)^m\Delta^{0+n}V_{00} \quad (19)$$

$$\Delta^{1+1}V_{00} = \Delta^{1+0}V_{01} - \Delta^{1+0}V_{00} = 0.475 - 0.59 = -0.1150 \quad (20)$$

$$\Delta^{1+2}V_{00} = \Delta^{1+0}V_{02} - 2\Delta^{1+0}V_{01} + \Delta^{1+0}V_{00} = 0.385 - 2\times 0.475 + 0.59 = 0.0250 \quad (21)$$

$$\Delta^{2+1}V_{00} = \Delta^{2+0}V_{01} - \Delta^{2+0}V_{00} = -0.18 - (-0.23) = -0.1150 \quad (22)$$

$$\Delta^{3+1}V_{00} = \Delta^{3+0}V_{01} - \Delta^{3+0}V_{00} = -0.115 - (-0.13) = 0.0150 \quad (23)$$

$$\Delta^{1+3}V_{00} = \Delta^{1+0}V_{03} - 3\Delta^{1+0}V_{02} + 3\Delta^{1+0}V_{01} - \Delta^{1+0}V_{00} = 0.3113 - 3\times 0.385 + 3\times 0.475 - 0.59 = -0.0087 \quad (24)$$

$$\Delta^{2+2}V_{00} = \Delta^{2+0}V_{02} - 2\Delta^{2+0}V_{01} + \Delta^{2+0}V_{00} = -0.1475 - 2\times(-0.18) + (-0.23) = -0.0175$$

$$\Delta^{1+4}V_{00} = \Delta^{1+0}V_{04} - 4\Delta^{1+0}V_{03} + 6\Delta^{1+0}V_{02} - 4\Delta^{1+0}V_{01} + \Delta^{1+0}V_{00} \quad (25)$$

$$= 0.408 - 4\times 0.3113 + 6\times 0.385 - 4\times 0.475 + 0.59 = 0.1628$$

$$\Delta^{4+1}V_{00} = \Delta^{4+0}V_{01} - \Delta^{4+0}V_{00} = 0.115 - 0.13 = -0.0150 \quad (26)$$

$$\Delta^{3+2}V_{00} = \Delta^{3+0}V_{02} - 2\Delta^{3+0}V_{01} + \Delta^{3+0}V_{00} = -0.09 - 2\times(-0.115) + (-0.13) = 0.0100 \quad (27)$$

$$\Delta^{2+3}V_{00} = \Delta^{2+0}V_{03} - 3\Delta^{2+0}V_{02} + 3\Delta^{2+0}V_{01} - \Delta^{2+0}V_{00}$$
$$= -0.1186 - 3\times(-0.1475) + 3\times(-0.18) - (-0.23) = 0.0139 \quad (28)$$

Interpolating polynomials in two variables up to the difference of the fifth degree requires the following formula:

$$V(x,t) =$$

$$V_{00} + \left[\frac{(x-x_0)}{h}\Delta^{1+0}V_{00} + \frac{(t-t_0)}{k}\Delta^{0+1}V_{00}\right]$$

$$+\frac{1}{2!}\left[\frac{(x-x_0)(x-x_1)}{h^2}\Delta^{2+0}V_{00} + \frac{2(x-x_0)(t-t_0)}{hk}V_{00} + \frac{(t-t_0)(t-t_1)}{k^2}\Delta^{0+2}V_{00}\right]$$

$$+\frac{1}{3!}\left[\frac{(x-x_0)(x-x_1)(x-x_2)}{h^3}\Delta^{3+0}V_{00} + \frac{3(x-x_0)(x-x_1)(t-t_0)}{h^2k}\Delta^{2+1}V_{00} + \frac{3(x-x_0)(t-t_0)(t-t_1)}{hk^2}\Delta^{1+2}V_{00} + \frac{(t-t_0)(t-t_1)(t-t_2)}{k^3}\Delta^{0+3}V_{00}\right]$$

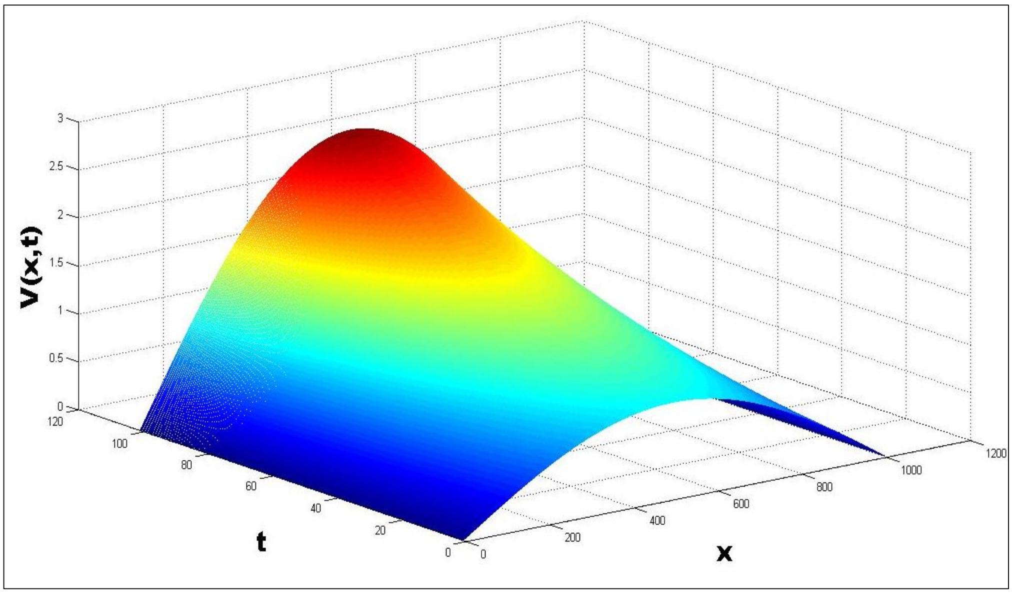
$$+\frac{1}{4!}\left[\begin{array}{c}\frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)}{h^4}\Delta^{4+0}V_{00} + \frac{4(x-x_0)(x-x_1)(x-x_2)(t-t_0)}{h^3k}\Delta^{3+1}V_{00} + \frac{6(x-x_0)(x-x_1)(t-t_0)(t-t_1)}{h^2k^2}\Delta^{2+2}V_{00} + \\ \frac{4(x-x_0)(t-t_0)(t-t_1)(t-t_2)}{hk^3}\Delta^{1+3}V_{00} + \frac{(t-t_0)(t-t_1)(t-t_2)(t-t_3)}{k^4}\Delta^{0+4}V_{00}\end{array}\right]$$

$$+\frac{1}{5!}\left[\frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{h^5}\Delta^{5+0}V_{00} + \frac{5(x-x_0)(x-x_1)(x-x_2)(x-x_3)(t-t_0)}{h^4k}\Delta^{4+1}V_{00} + \frac{10(x-x_0)(x-x_1)((x-x_2)(t-t_0)(t-t_1)}{h^3k^2}\Delta^{3+2}V_{00} + \right.$$
$$\left.\frac{10(x-x_0)(x-x_1)(t-t_0)(t-t_1)(t-t_2)}{h^2k^3}\Delta^{2+3}V_{00} + \frac{5(x-x_0)(t-t_0)(t-t_1)(t-t_2)(t-t_3)}{hk^4}\Delta^{1+4}V_{00} + \frac{(t-t_0)(t-t_1)(t-t_2)(t-t_3)(t-t_4)}{k^5}\Delta^{0+5}V_{00}\right] \quad (29)$$

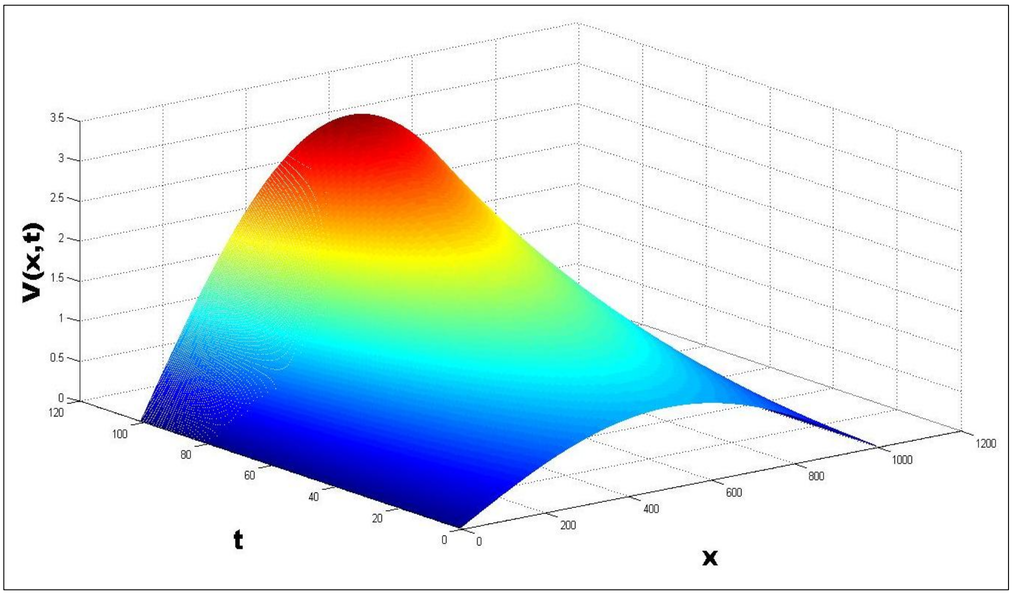
After changing the values of the various operators in equation (29) and simplifying the equation, we get the following:

$$V(x,t) = 2.95x - 2.875x(x-t-0.22) - x[2.7083(x-0.2)(x-0.4) + 71.875t(x-0.2) - 156.25t(t-0.02)] +$$
$$[3.3854x(x-0.2)(x-0.4)(x-0.6) + 15.6250xt(x-0.2)(x-0.4) - 273.4375x(x-0.2)(t-0)(t-0.02) -$$
$$906.2500xt(t-0.02)(t-0.04)] - [19.5312xt(x-0.2)(x-0.4)(x-0.6) - 260.4167xt(x-0.2)(x-0.4)(t-0.02) -$$
$$3619.8xt(x-0.2)(t-0.02)(t-0.04) - 211980x(t-0.02)(t-0.04)(t-0.06)] \quad (30)$$

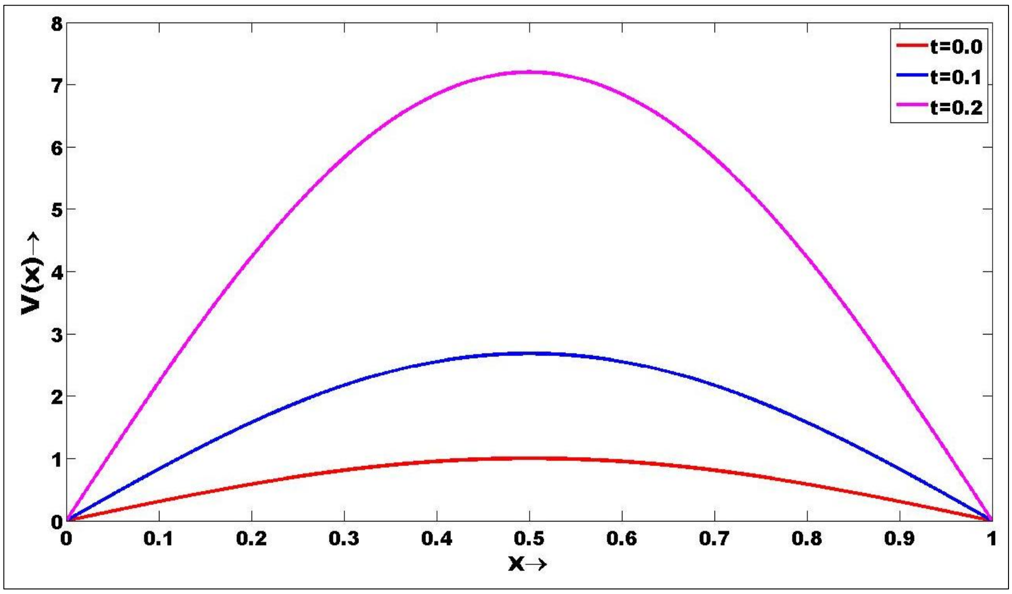
3. Graphical Solution



Graph 1: The space-time graph of numerical solutions of heat conduction equation by double interpolation method for $m = 25$



Graph 2: The space-time graph of analytic solution of heat conduction equation



Graph 3: Numerical solution of heat equation in different values of t

4. Closing Comments

This study introduces a mathematical strategy known as the double interpolation method methodology. This method is used to estimate mathematical configurations of key one-layered heat conditions. Currently, just a few lattices in the proposed technique are focused on guaranteeing the required precision. Because the limit circumstances are considered in a way that is natural, this strategy is quite beneficial for tackling limit value problems. The proposed method is also quite simple to implement, and the mathematical results show that it is extremely effective for the mathematical organization of the stated problem. Furthermore, the concept can be used to various situations involving fractional differentials.

5. References

1. Ciegis R, Jankeviciute G, Suboc O. Numerical simulation of the heat conduction in composite materials. Math Modelling Anal. 2010;15:9-22.
2. Iyengar SR, Manohar R. High order difference methods for heat equation in polar cylindrical coordinates. J Comput Phys. 1988;77(2):425-438.
3. Javed S. Thermal Modelling and Evaluation of Borehole Heat Transfer [Dissertation]. Sweden: Chalmers University of Technology, 2012.
4. Kafle J, Bagale LP, DJKC. Numerical solution of parabolic partial differential equation by using finite difference method. J Nepal Phys Soc. 2020;6(2):67-65.
5. Khatun RM, Ali SMD. Stability analysis of finite difference schemes for heat equation with various thermal conductivity. Int J Sci Eng Res. 2020;11:1275-1282.
6. Marwah J, Chopra MG. Transient heat transfer in a slab with heat generation. Defence Sci J. 1992;32:143-149.
7. Maturi D, Alsulami N, Alaidarous E. finite difference approximation for solving transient heat conduction equation of copper. Adv Pure Math. 2020;10:350-358.
8. Meyu GK, Koriche KA. Analytical solution vs. numerical solution of heat equation flow through rod of length 8 units in one dimension. Int J Appl Math Theor Phys. 2021;7(2):53-61.
9. Mori CNT, Romao EC. Numerical simulation by finite difference method of 2D convection-diffusion in cylindrical coordinates. Appl Math Sci. 2015;9(123):6157-6165.
10. Sabaeian M, Nadgaran H, Mousave L. Analytical solution of the heat equation in a longitudinally pumped cubic solid-state laser. Optica Soc Am. 2008;47(13):2317-2325.
11. Salehi M, Granpayeh N. Numerical solution of the Schrödinger equation in polar coordinates using the finite difference time-domain method. J Comput Electron. 2020;19(1):91-102.
12. Shiferaw A, Mittal R C. Fast finite difference solutions of the three dimensional Poisson's equation in cylindrical coordinates. Am J Comput Math. 2013;3(4):356-361.
13. Tsega EG. Numerical solution of three-dimensional transient heat conduction equation in cylindrical coordinates. J Appl Math. 2022;2022:1993151:1-8.