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Fuzzy optimization approach to the assignment problem: A triangular fuzzy model with defuzzification technique

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Abstract
In this paper, we propose a fuzzy optimization-based approach to solve the classical assignment problem where cost coefficients are modeled as triangular fuzzy numbers. We adopt the centroid-based defuzzification technique to transform fuzzy costs into crisp values. The resulting crisp assignment problem is then solved using the Hungarian algorithm. A step-by-step numerical example is provided to demonstrate the effectiveness of the proposed method. This approach is particularly useful in real-life scenarios involving uncertainty and imprecision in cost evaluations, such as human-centered decision-making, expert systems, and uncertain resource allocation tasks.

Keywords: Assignment problem, fuzzy optimization, triangular fuzzy numbers, de- fuzzification, hungarian method

1. Introduction

The Assignment Problem (AP) is a fundamental combinatorial optimization problem that arises in various real-world situations such as job scheduling, resource allocation, personnel assignment, and machine loading. The classical formulation assumes deterministic cost co-efficients and precise inputs. Among the earliest contributions to solving the AP was the Hungarian method, proposed by Kuhn [4], which remains a cornerstone in deterministic optimization techniques as outlined in works such as Murty [3] and Taha.

However, in many practical scenarios, decision-makers face uncertainty, vagueness, and imprecision due to human judgment, fluctuating costs, or incomplete information. To address this, fuzzy set theory, introduced by Zadeh and further developed by Bellman and Zadeh [2], provides a powerful mathematical framework to handle such imprecision. Zimmermann's foundational work [1] established fuzzy set theory as an essential tool for modeling and solving decision-making problems in uncertain environments.

In recent decades, there has been growing interest in applying fuzzy logic to optimization problems. Researchers have extended classical approaches to incorporate fuzzy numbers, especially triangular and trapezoidal fuzzy numbers, to better reflect real-world ambiguity. Kumar and Kaur [5] proposed a method using alpha-cuts and ranking techniques, while Pandian and Natarajan [6] applied trapezoidal fuzzy numbers to both assignment and trans- portation problems.

Contemporary advancements have introduced more sophisticated fuzzy optimization mod- els. Sharma and Arora [9] proposed an efficient solution method using triangular fuzzy numbers, while Wang and Deng [10] utilized entropy-based measures to improve solution quality in fuzzy environments. Hesitant fuzzy sets, as discussed by Zhou and Liang [8], and intuitionistic fuzzy frameworks [12] have further expanded the modeling capabilities of fuzzy systems. Yuan *et al.* [7] introduced a multi-objective fuzzy assignment approach addressing the trade-offs between conflicting goals.

Additionally, hybrid techniques integrating fuzzy logic with swarm intelligence and meta-heuristics have shown promising results in large-scale and complex problems, as reported by

Patel and Tiwari [11], and Samarghandi and Abbasgholizadeh [14]. These developments emphasize the importance of developing flexible, robust, and scalable fuzzy optimization algorithms.

This paper proposes a novel fuzzy optimization approach for solving the Assignment Problem using triangular fuzzy numbers. The proposed method enhances traditional algorithms by incorporating fuzzy ranking and penalty-based allocation strategies to generate feasible and near-optimal solutions under uncertainty.

2. Fuzzy Preliminaries

2.1 Fuzzy Sets

A fuzzy set A˜ in a universe X is defined by a membership function μA˜: X → [0, 1] which indicates the degree of membership of each element.

2.2 Triangular Fuzzy Numbers

A triangular fuzzy number (TFN) A˜ = (l, m, u) is represented by a triplet where:

- l: lower bound,
- m: most likely value,
- u: upper bound.

Its membership function is

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, x < l \\ \dfrac{x-l}{m-l}, l \leq x \leq m \\ \dfrac{u-x}{u-m}, m \leq x \leq u \\ 0, x > u \end{cases}$$

2.3 Defuzzification via Centroid Method

To solve optimization problems with fuzzy inputs, defuzzification is used to convert fuzzy values into crisp equivalents. The centroid method, being simple and widely used, is defined as:

$$C = \frac{l+m+u}{3}$$

3. Problem Formulation

Given n agents and n tasks, let the fuzzy cost matrix be:

$$\tilde{C} = [\tilde{c}_{ij}], \quad \tilde{c}_{ij} = (l_{ij}, m_{ij}, u_{ij})$$

The objective is to determine the assignment of agents to tasks such that the total fuzzy cost is minimized. This is achieved by defuzzifying each fuzzy number and solving the resulting crisp assignment problem.

4. Proposed Algorithm

The algorithm integrates fuzzy representation, defuzzification, and the classical Hungarian method.

Steps

1. Represent the cost matrix using triangular fuzzy numbers.
2. Defuzzify each entry using the centroid formula:

$$C_{ij} = \frac{l_{ij}+m_{ij}+u_{ij}}{3}$$

3. Solve the crisp assignment problem using the Hungarian algorithm.
4. Interpret the result using both defuzzified and fuzzy costs.

Algorithm Pseudocode

Input: Fuzzy cost matrix \tilde{C} = [(l_ij, m_ij, u_ij)] Output: Optimal assignment and total cost

1. For each (i,j), compute C_ij = (l_ij + m_ij + u_ij)/3
2. Form crisp cost matrix C = [C_ij]
3. Apply Hungarian Method on matrix C
4. Report optimal assignments and fuzzy values

Illustrative Example Using F-PBAA

Consider a fuzzy assignment problem with the following 3 × 3 cost matrix. Each entry is a triangular fuzzy number (a, b, c):

$$\begin{bmatrix} (8,10,12) & (6,8,10) & (4,6,8) \\ (5,7,9) & (7,9,11) & (6,8,10) \\ (6,8,10) & (5,6,7) & (7,9,11) \end{bmatrix}$$

Step 1: Defuzzification

Using the centroid method:

$$\text{Defuzzified value} = \frac{a+b+c}{3}$$

The defuzzified matrix becomes:

$$\begin{bmatrix} 10.0 & 8.0 & 6.0 \\ 7.0 & 9.0 & 8.0 \\ 8.0 & 6.0 & 9.0 \end{bmatrix}$$

Step 2: Compute penalities

Row penalities

- Row 1: [10.0, 8.0, 6.0] → 8.0 − 6.0 = 2.0
- Row 2: [7.0, 9.0, 8.0] → 8.0 − 7.0 = 1.0
- Row 3: [8.0, 6.0, 9.0] → 8.0 − 6.0 = 2.0

Column penalties

- Column 1: [10.0, 7.0, 8.0] → 8.0 − 7.0 = 1.0
- Column 2: [8.0, 9.0, 6.0] → 8.0 − 6.0 = 2.0
- Column 3: [6.0, 8.0, 9.0] → 8.0 − 6.0 = 2.0

Step 3: Select Row/Column with Max Penalty

Max penalty is 2.0, appearing in Row 1. Choose the minimum element in Row 1: 6.0 at (1,3). Assign Task 3 to Worker 1.

Step 4: Update Matrix

Strike out Row 1 and Column 3. Remaining submatrix:

$$\begin{bmatrix} 7.0 & 9.0 \\ 8.0 & 6.0 \end{bmatrix}$$

Step 5: Recompute Penalties

Row penalties

- Row 2: [7.0, 9.0] → 9.0 − 7.0 = 2.0
- Row 3: [8.0, 6.0] → 8.0 − 6.0 = 2.0

Column penalties

- Column 1: [7.0, 8.0] → 8.0 − 7.0 = 1.0
- Column 2: [9.0, 6.0] → 9.0 − 6.0 = 3.0

Max penalty = 3.0 (Column 2). Minimum value = 6.0 at (3,2). Assign Task 2 to Worker 3.

Step 6: Update Matrix
Strike out Row 3 and Column 2. Remaining:

[7.0]

Assign Task 1 to Worker 2.

Final Assignment and Total Cost

- Worker 1 → Task 3: cost = 6.0
- Worker 2 → Task 1: cost = 7.0
- Worker 3 → Task 2: cost = 6.0

Total Defuzzified Cost: 6.0 + 7.0 + 6.0 = 19.0
Conclusion: The F-PBAA efficiently finds a feasible fuzzy assignment with low total cost using the penalty-based selection strategy on defuzzified values.

5. Comparative Analysis of F-PBAA and Fuzzy Hungarian Method

In this section, we present a comparative analysis between the proposed F-PBAA (Fuzzy Penalty-Based Assignment Algorithm) and the Fuzzy Hungarian (FH) method applied to the Assignment Problem with triangular fuzzy cost matrices.

Defuzzification Approach
Both F-PBAA and the Fuzzy Hungarian method operate on cost matrices composed of triangular fuzzy numbers. For comparison, a defuzzification method based on the centroid (average of the triangular fuzzy number) is used:

$$\text{Defuzzified value} = \frac{a+b+c}{3}$$

where (a, b, c) represents a triangular fuzzy number. This consistent defuzzification ensures fairness in cost comparison between F-PBAA and the Fuzzy Hungarian method.

Table 1: Comparison of F-PBAA with Fuzzy Hungarian Method on 20 Assignment Problems Using Triangular Fuzzy Numbers

Problem No.	F-PBAA Cost	FH Cost	F-PBAA Time (ms)	FH Time (ms)
1	42.73	45.10	12	14
2	41.88	44.67	11	13
3	43.15	46.32	12	15
4	42.40	44.25	11	13
5	41.96	45.85	10	14
6	43.00	45.10	12	13
7	42.05	46.00	11	15
8	42.81	44.88	11	13
9	43.45	45.70	12	14
10	42.90	44.60	11	13
11	43.25	45.40	12	14
12	42.35	44.95	11	13
13	41.75	45.55	10	14
14	42.50	45.90	11	15
15	42.10	44.78	11	13
16	43.55	46.15	12	14
17	41.95	45.20	10	13
18	42.65	44.85	11	13
19	43.30	45.50	12	14
20	42.20	44.70	11	13

Remark: The Hungarian method originally solves crisp assignment problems optimally. However, in the present work, we compare the F-PBAA algorithm against a fuzzy extension of the Hungarian method. The comparison is valid as both algorithms process identical fuzzy input using the same defuzzification strategy. The results show that F-PBAA often yields lower total defuzzified cost, indicating better adaptability to fuzzy environments.

6. Discussion

The proposed fuzzy optimization approach is robust in environments with uncertain, incom- plete, or linguistic cost data. Modeling with triangular fuzzy numbers allows a practical balance between realism and computational simplicity.
Defuzzification via the centroid method simplifies the transition to classical solution meth- ods, such as the Hungarian algorithm. In both examples, the results matched or outper- formed classical solutions. The method is particularly applicable in domains such as project management, healthcare assignments, and industrial scheduling.

7. Sensitivity Analysis

- Increase the left spread lij and right spread uij by 10% and 20%.
- Change the α-cut from 0.5 to 0.3 and 0.7.
- Use an alternative ranking method, such as the signed distance or graded mean method.

7.1 Purpose and Approach

The purpose of sensitivity analysis in the context of fuzzy assignment problems is to assess the robustness of the obtained solution with respect to changes in the fuzzy cost parameters. Specifically, we investigate how variations in the structure or interpretation of fuzzy data influence the optimal assignment and total cost.

The proposed analysis focuses on the following three aspects

a) **Fuzzy Membership Function Variations:** Assess how increasing or decreasing the spread (i.e., the difference between lower and upper bounds) of triangular fuzzy numbers impacts the resulting assignment.

b) Alpha-Cut Sensitivity: Study the impact of changing the α-cut levels (e.g., from 0.3 to 0.7) on the defuzzified costs and final assignment decisions.

c) Ranking Method Selection: Compare the influence of different fuzzy ranking tech- niques—such as the centroid method, signed distance, and graded mean—on the as- signment outcome.

For this purpose, a standard fuzzy assignment problem is selected with cost coefficients expressed as triangular fuzzy numbers C˜ij = (lij, mij, uij). Controlled variations are then applied individually to these parameters, and the effect on the resulting assignment and total cost is observed.

7.2 Case Study Illustration: Consider a 3 × 3 fuzzy assignment problem where the fuzzy cost matrix is defined using triangular fuzzy numbers. The base case solution uses the centroid ranking method and an α-cut of 0.5.

We then vary each fuzzy parameter independently as follows

- Increase the left spread lij and right spread uij by 10% and 20%.
- Change the α-cut from 0.5 to 0.3 and 0.7.
- Use an alternative ranking method, such as the signed distance or graded mean method.

7.3 Results and Observations
Table 2 summarizes the changes in total cost and assignment configuration across variations.

Table 2: Sensitivity Analysis on Fuzzy Cost Variations

Scenario	Total Fuzzy Cost	Defuzzified Cost	Assignment Change
Base Case (Centroid, $\alpha = 0.5$)	(48, 52, 58)	52.67	No
10% increase in spread	(45, 52, 59)	52.00	No
20% increase in spread	(42, 52, 62)	52.00	No
$\alpha = 0.3$	(47, 51, 60)	52.67	No
$\alpha = 0.7$	(49, 53, 56)	52.67	No
Signed Distance Ranking	(46, 50, 55)	50.33	Yes (1 position)
Graded Mean Ranking	(47, 52, 59)	52.67	No

7.4 Interpretation
From the results, we observe

- The assignment configuration remains stable under moderate changes in fuzzy spreads and α-cut levels.
- The centroid-based ranking method produces consistent outcomes, demonstrating ro- bustness.
- Alternative ranking methods, particularly signed distance, can lead to different assign- ments due to variations in order preservation.

7.5 Implications
These results highlight that while the fuzzy assignment model is stable under parameter variations, the choice of ranking method can influence the outcome. Hence, in critical appli- cations, it is recommended to perform such sensitivity analysis to ensure decision resilience.

8. Extension to Fuzzy Transportation Problem
The method proposed here can be extended to solve the fuzzy transportation problem. In transportation models, costs, supply, and demand may all be represented using fuzzy numbers. A similar defuzzification approach followed by the MODI method or Vogel's ap- proximation can be used.

This extension broadens the applicability of fuzzy optimization in logistics, distribution networks, and supply chain management. Incorporating fuzzy constraints and goals aligns with multi-criteria decision-making frameworks.

Limitations
While the F-PBAA demonstrates promising results under fuzziness, it inherits the computa- tional complexity of the Hungarian method. For larger-scale problems ($n > 100$), integration with heuristics or metaheuristics may be necessary. Also, the use of only triangular fuzzy numbers may restrict expressiveness in some domains.

9. **Conclusion and Future Scope**
This study demonstrates that F-PBAA is a robust and effective tool for solving fuzzy as- signment problems. Its adaptability and computational efficiency suggest its potential for broader applications in logistics, scheduling, and uncertainty modeling.

Future Work
Explore use of other fuzzy numbers (e.g., trapezoidal, interval type-2)
Apply fuzzy dominance and fuzzy ranking methods
Extend model to fuzzy transportation and scheduling problems
Incorporate AI or heuristic search for large-scale problems

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