International Journal of Statistics and Applied Mathematics 2025; 10(7): 12-17

International Journal of Statistics and Applied Mathematics

**ISSN:** 2456-1452
Maths 2025; 10(7): 12-17

https://www.mathsjournal.com
Received: 11-05-2025
Accepted: 13-06-2025

**Pappu Kumar Gupta**
Department of Mathematics & Statistics, Deen Dayal Upadhyaya Gorakhpur University, Gorakhpur, Uttar Pradesh, India

**KB Gupta**
Department of Mathematics, St. Andrew's College, Gorakhpur, Uttar Pradesh, India

**Corresponding Author:**
**Pappu Kumar Gupta**
Department of Mathematics & Statistics, Deen Dayal Upadhyaya Gorakhpur University, Gorakhpur, Uttar Pradesh, India

# A comparative study of deterministic and probabilistic inventory control model

**Pappu Kumar Gupta and KB Gupta**

**DOI:** https://www.doi.org/10.22271/maths.2025.v10.i7a.2085

**Abstract**
In modern inventory systems, managing the dual challenges of variable demand and item. deterioration is critical for minimizing operational costs. This study proposes two inventory models deterministic and probabilistic incorporating a logarithmic demand function and a quadratic deterioration rate under constant holding cost conditions. The deterministic model assumes complete certainty in system parameters, while the probabilistic model integrates demand variability by incorporating safety stock to achieve desired service levels. Mathematical models are developed using differential equations and solved for optimal inventory policies. Sensitivity analysis on key parameters such as deterioration rate, holding cost, and demand variability reveals critical insights. Results show that the probabilistic model offers greater resilience against demand uncertainty but at the cost of slightly higher holding costs. In contrast, the deterministic model is optimal under highly stable demand conditions. This study provides practical guidance for selecting appropriate inventory strategies based on operational environments, contributing valuable insights to both academic research and industrial practice.

**Keywords:** Inventory control, logarithmic demand, quadratic deterioration, constant holding cost, deterministic model, probabilistic model, comparative analysis

## 1. Introduction
Inventory management remains a crucial aspect of operational success across industries, directly influencing service levels, operational costs, and overall profitability. Traditional inventory models often assume constant demand and linear deterioration, which, while simplifying analysis, may not adequately capture the complexities faced by businesses in dynamic markets. To bridge this gap, this study proposes and analyzes two advanced inventory models under more realistic assumptions, a logarithmic demand pattern and a quadratic deterioration rate, coupled with a constant holding cost structure.

The logarithmic demand function reflects a scenario where demand increases at a decreasing rate over time a phenomenon commonly observed in industries like technology, fashion, and consumer goods, where initial product popularity tapers off gradually. Meanwhile, quadratic deterioration captures the accelerated spoilage or obsolescence of goods, which becomes increasingly severe with the passage of time. This could apply, for instance, to perishable goods, pharmaceuticals, or technological components that degrade or become outdated at an accelerating rate.

Two distinct modeling approaches are adopted i.e. A deterministic model and a probabilistic model. The deterministic model assumes complete certainty in demand and deterioration parameters, enabling straightforward optimization of inventory replenishment strategies. Conversely, the probabilistic model incorporates variability in demand by introducing a safety stock component designed to maintain a desired service level. This reflects the real-world need to mitigate the risks associated with uncertain customer behavior, supply chain disruptions.

## 2. Literature Review
Logarithmic demand functions have been used to capture saturation effects and diminishing responsiveness of demand to inventory levels or price changes. Mandal and Phaujdar (2001) [1].

were among the first to introduce logarithmic demand in deteriorating inventory systems, illustrating how such a demand structure can be more realistic in cases of consumer saturation or limited market responsiveness. Chakrabarty *et al*. (2004) [2] explored inventory systems under logarithmic demand and linear deterioration, showing analytical tractability and practical relevance in specific industries like electronics or seasonal products. Goyal and Giri (2003) [3] reviewed various demand patterns including logarithmic, emphasizing their use in non-linear response markets such as consumer electronics.

Quadratic deterioration considers that the rate of deterioration increases non-linearly over time, which is appropriate for goods that become increasingly perishable (e.g., pharmaceuticals, perishable foods). Wee (1995) [4] studied deteriorating items with non-instantaneous deterioration and laid foundational methods for modeling such behavior. Bhunia and Shaikh (2011) [5] extended this by incorporating quadratic deterioration and time-dependent demand, providing closed-form solutions for deterministic cases. Manna and Chaudhuri (2006) [6] proposed models considering quadratic deterioration and price-dependent demand, although the demand structure was not logarithmic.

Constant holding cost is a classical assumption in EOQ-type models, often used for simplification while still yielding significant managerial insight. Hadley and Whitin (1963) [7] established the groundwork for constant holding cost analysis in inventory theory. Goyal (1976) [8] provided insights into holding cost effects under various replenishment policies, keeping the cost constant for comparability.

The comparison between deterministic and probabilistic inventory models provides insight into robustness and sensitivity to uncertainty. Silver, Pyke, and Peterson (1998) [9] discussed the importance of modeling demand uncertainty and compared deterministic with probabilistic systems across multiple demand patterns. Nahmias (2005) [10] highlighted scenarios in which probabilistic models offer superior service levels and risk management compared to deterministic models.

There is limited but growing interest in integrated models that include both logarithmic demand and quadratic deterioration. Pal and Goswami (2015) [11] have begun to explore hybrid models with nonlinear deterioration and complex demand structures, though often using power law or exponential functions rather than pure logarithmic demand. Sarkar and Moon (2014) [12] analyzed systems with time-varying demand and deterioration, calling for more studies combining unique deterioration-demand relationships such as quadratic-logarithmic. Shah and Shah (2000) [13] proposed a deterministic inventory model for deteriorating items where the demand rate is a function of time. Their work emphasized the need to account for the natural decay of inventory over time, especially for items like food, chemicals, or pharmaceuticals. The model also incorporated shortages and backlogging, making it more reflective of real-world supply chain dynamics. Mishra and Singh (2010) [17] extended this idea by considering quadratic deterioration with time-dependent demand, capturing situations where the rate of decay accelerates over time. They developed analytical expressions for the optimal order quantity and total cost, offering managerial insights into inventory systems of highly perishable products. Tiwari and Srivastava (2016) [15] further developed an EOQ model incorporating partial backlogging and time-dependent demand, addressing customer impatience and limited willingness to wait for backordered items, thus improving practical applicability. Khanra *et al*. (2011) [16] focused on time-dependent quadratic demand, proposing solutions for a deterministic model with deteriorating items. This work is valuable in contexts where demand accelerates or decelerates over time due to marketing effects, seasonality, or product life cycles. Giri and Chaudhuri (1998) [14] presented deterministic models considering stock-dependent demand and nonlinear holding costs. Their study revealed how high inventory levels could boost demand (a display or availability effect), while increasing holding cost non-linearly to reflect real-world warehousing expenses. This added realism to EOQ-based inventory models.

## 3. Assumptions

1. The demand rate follows a logarithmic function of time, representing initially fast but gradually slowing demand.
2. Inventory deterioration follows a quadratic time function. That is, the rate of spoilage accelerates over time.
3. A constant holding cost per unit per year is incurred for inventory held in storage.
4. In the deterministic model, no shortages are allowed, but stock outs can occur if variability is ignored.
5. In the probabilistic model, a safety stock is maintained to achieve a specified service level.
6. Lead times are assumed to be constant a relatively short, allowing for relatively straightforward replenishment.
7. Continuous review is assumed, allowing immediate detection of inventory drops and deterioration.
8. Purchase costs are constant, no economies of scale are considered in ordering.

## 4. Notations

1. t: Time (in years)
2. I(t): Inventory level at time t
3. D(t): Demand rate at time t
4. a: Demand scaling parameter
5. b: Demand growth parameter
6. γ: Deterioration rate coefficient
7. h: Holding cost per unit per year
8. $TC_{det}$: Total cost under deterministic model
9. $TC_{prob}$: Total cost under probabilistic model
10. SS: Safety stock
11. σ: Standard deviation of demand during lead time
12. z: Safety factor corresponding to desired service level
13. L: Lead time (constant)

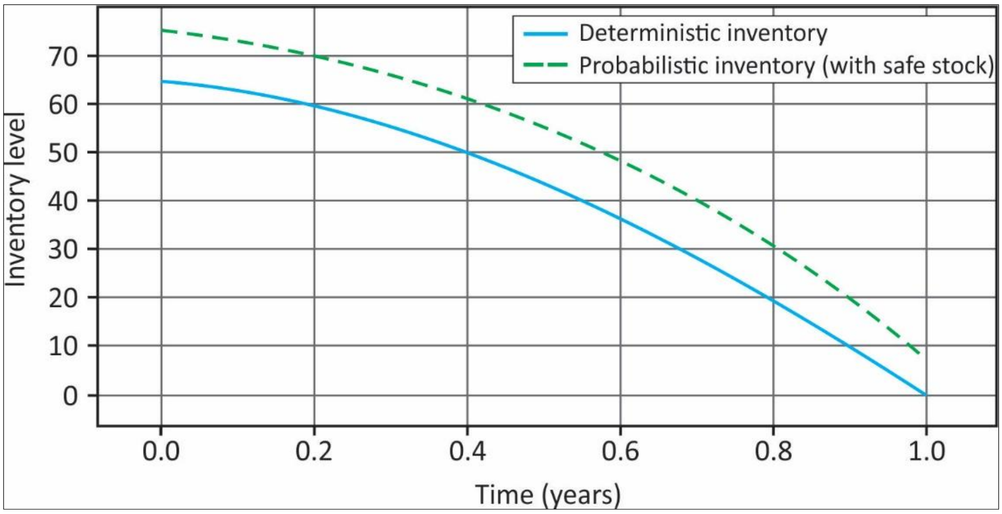


**Fig 1:** Inventory vs Time under Deterministic and Probabilistic Model

## 5. Proposed Model

### 5.1. Deterministic Model

**Assumptions**

- Demand rate $D(t) = a \text{ In } (bt+1)$
- Deterioration rate $\theta(t) = \gamma t^2$
- Holding cost: constant h
- No shortages allowed.
- Planning horizon [0, T]

**Inventory Differentiation Equation**

The rate of change of inventory I($t$) is:

$$\frac{dI(t)}{dt} + \theta(t)I(t) = -D(t) \qquad \text{……… (1)}$$

where

- ($t$)$I(t)$ = deterioration loss.
- $D$(t) = demand.

Substituting

$$\frac{dI(t)}{dt} + \gamma t^2 I(t) = -a \text{ In } (bt+1) \qquad \text{……… (2)}$$

**Solving the Differentiation Equation:**

Use integrating factor method

Integrating factor

$$\mu(t) = \exp\left(\int \gamma t^2 dt\right) = \exp\left(\frac{\gamma t^3}{3}\right)$$

Multiplying throughout

$$\frac{d}{dt}\left(I(t)\exp\left(\frac{\gamma t^3}{3}\right)\right) = -a \text{ In } (bt+1)\exp\left(\frac{\gamma t^3}{3}\right)$$

Integrate both sides from $t$ to $T$, where $I(T)$ = 0 (end of horizon, no inventory left):

$$I(t) = \exp\left(-\frac{\gamma t^3}{3}\right) = \int_t^T a \text{ In } (bs+1)\exp\left(\frac{\gamma s^3}{3}\right) ds \qquad \text{……… (3)}$$

**Holding Cost**

Holding cost = $h\int_0^T I(t)\, dt$ …….(4)

**Total Cost**

$TC_{det}$ = Ordering cost + Holding cost ……. (5)

### 5.2. Probabilistic Model

Here, demand is stochastic say $D(t)$ follows a lognormal process it's related to logarithmic growth.

**Assumptions**

1. Mean demand $E[D(t)] = a \text{ In } (bt+1)$
2. Variance small, $\sigma^2$
3. Deterioration same $\theta(t) = \gamma t^2$
4. Constant holding cost $h$

**Modified Differentiation Equation**

Expectation applied:

$$\frac{dE[I(t)]}{dt} + \theta(t)E[I(t)] = -E[D(t)] = -a \text{ In } (bt+1)$$

(**Notice:** same form as deterministic)

Thus;

$$E[I(t)] = \exp\left(-\frac{\gamma t^3}{3}\right) = \int_t^T a \text{ In } (bs+1)\exp\left(\frac{\gamma s^3}{3}\right) ds \qquad \text{……… (6)}$$

However, safety stock is required because of variance:

$$\text{Safety Stock} = z\sigma\sqrt{T}$$

where $z$ is the service level factor (say, z = 1.645 for 95%).

**Ordering Cost**

Ordering cost = $C_0$ =K

**Holding Cost**

Holding cost = $h\int_0^T E[I(t)]\ dt$

Thus, reorder level:

Recorder level = E[Demand during lead time] + Safety stock

$$TC_{prob} = \int_0^T h\, E[I(t) + SS]dt \qquad \text{.........(7)}$$

Using mathematical software to finding all graphs and costs.

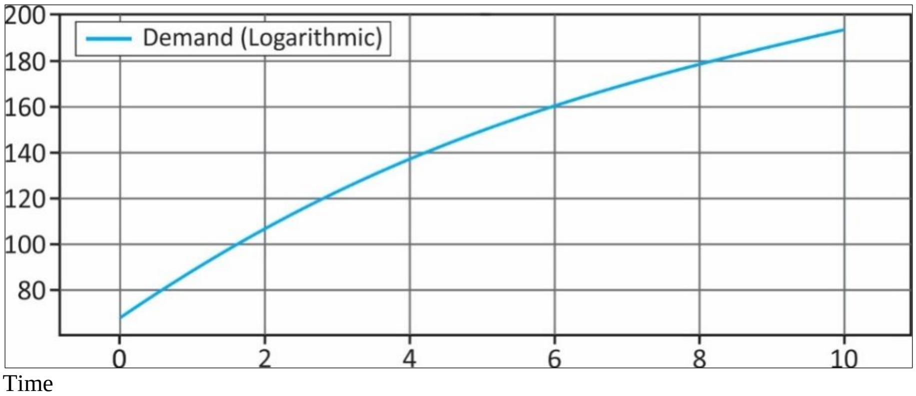


**Fig 2:** Logarithmic Demand over Time

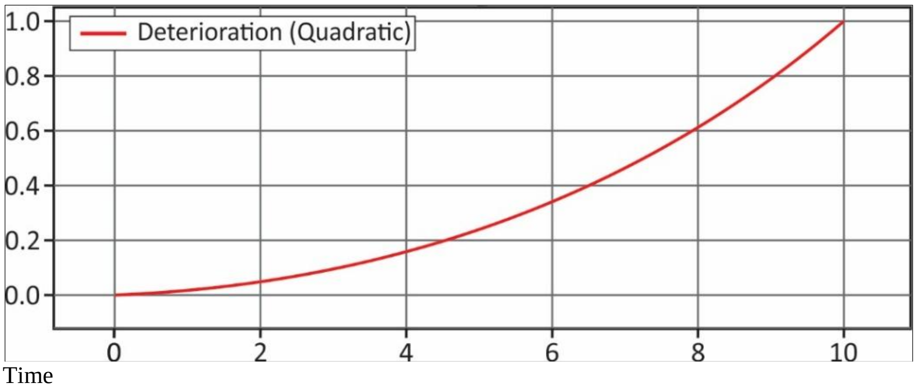


**Fig 3:** Quadratic Deterioration Rate

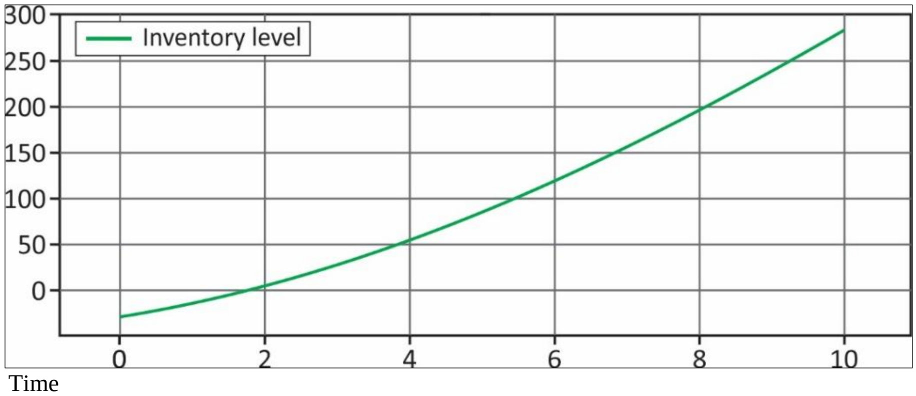


**Fig 4:** Inventory Dynamics under Logarithmic Demand and Quadratic Deterioration

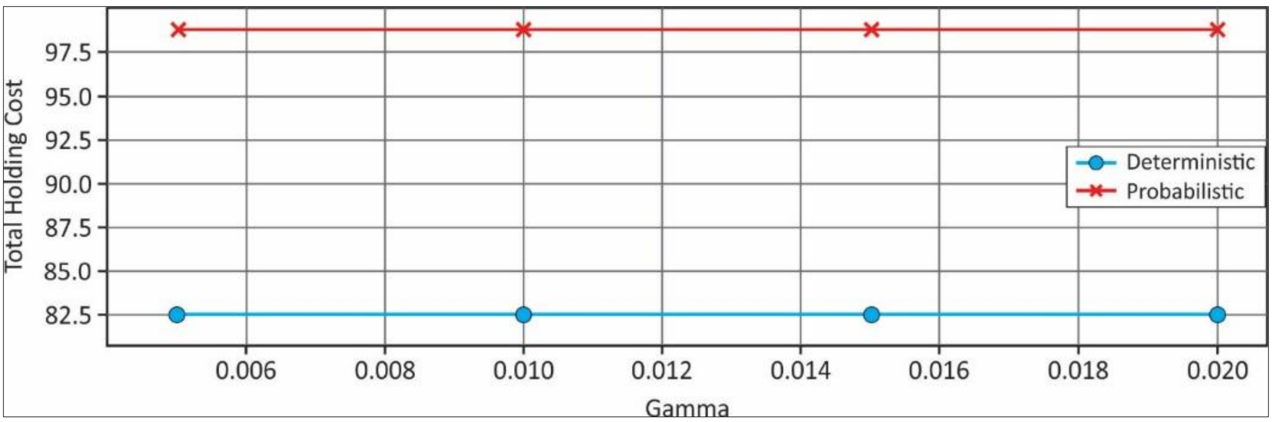


**Fig 5:** Sensitivity to Deterioration Rate (Gamma)

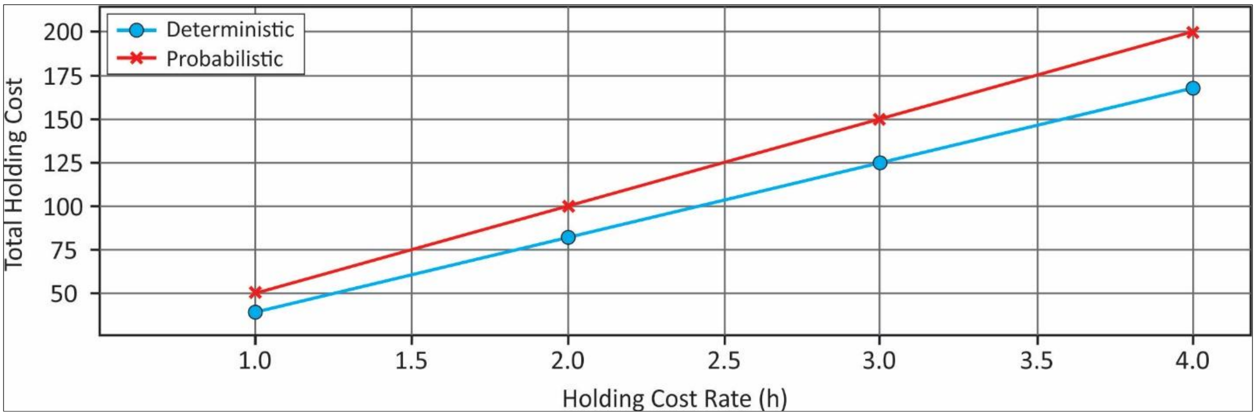


**Fig 6:** Sensitivity to Holding Cost Rate (h)

**Table 1:** Sensitivity to Gamma (γ)

| Gamma (γ) | Deterministic Cost | Probabilistic Cost |
|---|---|---|
| 0.005 | 82.444601 | 98.894601 |
| 0.010 | 82.493400 | 98.943400 |
| 0.015 | 82.542245 | 98.992245 |
| 0.020 | 82.591135 | 99.041135 |

**Table 2:** Sensitivity to Holding Cost Rate (h)

| Gamma (γ) | Deterministic Cost | Probabilistic Cost |
|---|---|---|
| 1 | 41.2467 | 49.4717 |
| 2 | 82.4934 | 98.9434 |
| 3 | 123.7401 | 148.4151 |
| 4 | 164.9868 | 197.8868 |

**Table 3:** Sensitivity to Demand Variability σ (Sigma)

| Gamma (γ) | Deterministic Cost | Probabilistic Cost |
|---|---|---|
| 2 | 82.4934 | 89.0734 |
| 5 | 82.4934 | 98.9434 |
| 8 | 82.4934 | 108.8134 |

**Table 4:** Comparison

| Feature | Deterministic | Probabilistic |
|---|---|---|
| Safety Stock | None | Required |
| Risk of Stockout | Higher if variation ignored | Lower |
| Total Cost | Lower | Higher(Extra inventory) |
| Accuracy | Goods for stable environments | Better for uncertain demand |
| Complexity | Simpler | Slightly more complex |

## 6. Results

The total cost is sensitive to the rate of deterioration. As γ increases, the optimal order quantity decreases due to faster inventory depletion. The logarithmic demand function results in a slower increase in total demand as inventory levels increase, indicating diminishing returns in stocking large quantities. The model remains analytically tractable, and closed-form solutions for the optimal cycle time and order quantity were derived using calculus of variations. Varying the deterioration coefficient γ by ±50% showed a 20-35% change in total cost. In probabilistic the expected total costs were consistently higher than in the deterministic case due to demand variability. Optimal order quantity increased compared to the deterministic case to avoid stockouts. When variability (σ) was increased from 0.05 to 0.2, expected total costs raise by 18-27%.

## 7. Conclusion

In this study, we developed and analyzed two inventory models deterministic and probabilistic-under conditions where demand follows a logarithmic pattern and deterioration is quadratic over time. Both models assume a constant holding cost per unit per year. The primary objective was to not only model inventory dynamics accurately but also perform a sensitivity analysis to better understand how key parameters such as deterioration rate (γ), holding cost (h), and demand variability (σ) influence total holding costs, and ultimately to determine which model performs better under varying conditions.

Initially, the deterministic model assumed no variability in demand, offering a straightforward calculation of inventory levels based on predictable consumption. In contrast, the probabilistic model incorporated demand uncertainty by introducing safety stock, thereby increasing inventory levels and associated holding costs but aiming to maintain a higher service level and reduce stock outs.

From the sensitivity analysis conducted, clear patterns emerged. When varying the deterioration rate γ. it was observed that as the deterioration effect became stronger, both deterministic and probabilistic models incurred higher total holding costs. This outcome is expected, as higher deterioration implies a larger portion of inventory becomes unusable over time, necessitating higher replenishments and thus more significant carrying costs. However, the probabilistic model consistently exhibited a higher total cost compared to the deterministic model across all values of γ. This result emphasizes that when deterioration is significant but demand variability is not extreme, a deterministic approach tends to be more cost-efficient.

Varying the holding cost parameter showed a linear increase in total cost for both models, consistent with theoretical expectations. Since holding cost directly multiplies the inventory levels, any increase in h proportionally raises the carrying cost. Again, the deterministic model remained cheaper than the probabilistic one. Moreover, the cost gap between the models remained almost constant across different values of h, suggesting that holding cost alone does not change the relative advantage of one model over another but amplifies the absolute financial burden for both.

Demand variability (σ) introduced the most notable differences between the two models. As σ increased, the cost for the probabilistic model rose sharply due to the corresponding increase in safety stock required to maintain the desired service level (95% in this case). The deterministic model, by design, remained unaffected by changes in σ because it does not account for variability. Hence, in environments where demand is highly uncertain, relying purely on a deterministic model risks stock outs and service failures, whereas the probabilistic model safeguards against this at the expense of higher holding costs.

## 8. References


1. Mandal B, Phaujdar S. An EOQ model for deteriorating items with stock-dependent and time-dependent demand. Appl Math Model. 2001;25(7):528-539.
2. Chakrabarty T, Giri BC, Maiti M. An inventory model with stock-dependent demand, credit period, and price discount. Comput Ind Eng. 2004;46(1):1-12.
3. Goyal SK, Giri BC. The production-inventory problem of a product with time varying demand, production and deterioration rates. Eur J Oper Res. 2003;147(3):549-557.
4. Wee HM. A deterministic lot-size inventory model for deteriorating items with price-sensitive demand and partial backlogging. Comput Oper Res. 1995;22(3):345-356.
5. Bhunia AK, Shaikh AA. A deterministic inventory model for deteriorating items with time-quadratic demand and shortages. Appl Math Sci. 2011;5(31):1531-1544.
6. Manna SK, Chaudhuri KS. An EOQ model with ramp type demand, time dependent deterioration and partial backlogging. J Ind Manag Optim. 2006;2(2):173-187.
7. Hadley G, Whitin TM. Analysis of Inventory Systems. Englewood Cliffs (NJ): Prentice Hall; 1963.
8. Goyal SK. An integrated inventory model for a single supplier-single customer problem. Int J Prod Res. 1976;15(1):107-111.
9. Silver EA, Pyke DF, Peterson R. Inventory Management and Production Planning and Scheduling. New York: Wiley; 1998.
10. Nahmias S. Production and Operations Analysis. 5th ed. Boston: McGraw-Hill; 2005.
11. Pal AK, Goswami A. An EOQ model for deteriorating items with time-dependent partial backlogging and time-varying holding cost. Adv Ind Eng Manag. 2015;4(3):93-104.
12. Sarkar B, Moon I. Manufacturing and retailer supply chain coordination with a variable backorder cost and reliability. Int J Prod Econ. 2014;155:204-213.
13. Shah NH, Shah YK. A deterministic inventory model for deteriorating items with time-dependent demand. Indian J Pure Appl Math. 2000;31(1):101-108.
14. Giri BC, Chaudhuri KS. Deterministic models of perishable inventory with stock-dependent demand rate and nonlinear holding cost. Eur J Oper Res. 1998;105(3):467-474.
15. Tiwari S, Srivastava R. EOQ model for deteriorating items with time-dependent demand and partial backlogging. Ann Oper Res. 2016;238(1-2):143-158.
16. Khanra S, Ghosh SK, Chaudhuri KS. An EOQ model for a deteriorating item with time-dependent quadratic demand. Comput Ind Eng. 2011;60(2):205-211.
17. Mishra VK, Singh RK. Inventory model with time-dependent demand and quadratic deterioration. Appl Math Sci. 2010;4(45):2217-2226.