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Modelling and forecasting the volatility of the exchange rate between the US dollar and the Nigeria naira note

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Abstract

This study investigates the volatility of the US dollar-Nigeria naira exchange rate using the Generalised Autoregressive Conditional Heteroskedasticity (GARCH) model for in-depth research and forecasting. The secondary data for this study was obtained from the Central Bank of Nigeria and covers the years January 2000 to December 2023. According to descriptive statistics, there are significant fluctuations in the mean exchange rate (231.0) and standard deviation (157.4). The fluctuation of the naira is evident from the least recorded exchange rate of 113.70 and the highest rate of 1233.00. The exchange rate series is shown to be non-stationary at both the level and the first difference, necessitating the application of the GARCH model to account for autoregressive conditional heteroskedasticity (ARCH) effects. The series was shown to be non-stationary using the Augmented Dickey-Fuller test, with a p-value greater than 0.05. The model selection technique indicates that, when compared to the other models tested, the GARCH model has the lowest Akaike Information Criterion (AIC) value of 9.815702, making it the most competitive model for this dataset. The findings show that external economic variables such as interest rates and inflation have a significant influence on exchange rate volatility, with implications for risk management and economic policy development. This study concludes with suggestions for adopting measures to stabilise the naira, regularly monitoring key economic indicators, and employing GARCH models to forecast exchange rates. This research extends our understanding of exchange rate dynamics and provides critical information to investors and policymakers navigating the complexities of Nigerian currency volatility.

Keywords: Exchange rate, volatility, Nigerian, ARCH, GARCH

1. Introduction

In financial economics, volatility is a significant and inherent metric that measures the levels of variation in trading prices over a given time. It is significant in determining the level of risk and controlling the investment procedures and economic policies. Simply put, investors and especially financial analysts tend to view volatility as a risk and uncertainty indicator in the financial markets and as such, high volatility is usually seen as having a tendency to lead to losses (Zivot, 2017) ^[53]. This is more so in the foreign exchange markets because the changing exchange rates are a threat to export and import business, foreign investment and the overall economic equilibrium. Exchange rates are the price of one currency in relation to other currency and it is one of the most crucial variables that define the condition of economy in a country. In the case of Nigeria, it is relevant to consider the exchange rate between the Nigerian naira and the US dollar because Nigeria heavily relies on oil exports, and the dollar is the most powerful reserve currency in the world. As an illustration, fluctuations in the naira value may present mind-boggling inflation risks, balance of trade, and foreign direct investment in Nigeria (Adewuyi, 2019; Alam & Rahman, 2012, Zubair *et al.*, 2022) ^[1, 6, 56]. Historically, the Nigerian economy has had numerous oil shocks and most of them were linked to currency issues. The naira has historically appreciated and depreciated over time due to one major factor or the other that can be categorized into external forces such as oil prices, domestic factors and politics among others. As an example, Naira floated easily in 1970s during oil boom, unfortunately, it was accompanied by unbalanced economy and inconsistent policy measures that resulted in long period of depreciation of Naira (Ewa & Asher, 2011) ^[27]. This volatility has not only reduced the purchasing power of the consumers but has also destabilized business entities that engage in imports and exports whereby it is difficult to

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predict future costs and revenues using the current rates. Since the exchange rates are not predictable due to the different factors that affect the value of a currency, then it is only wise to have good models to analyze the changes in the exchange rates. The Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model offers a more complicated process of volatility modeling because it takes into consideration the time dependent volatility and the volatility clustering that is typical of financial time series data (Engle, 2001) ^[24]. As has been mentioned earlier, this model is particularly appropriate in modeling exchange rates since it can explain the conditional heteroskedasticity that is usually observed in currency markets. Moreover, one needs to understand the factors that lead to variability in the exchange rate in order to respond to the interests of the policymakers and investors better.

Using the GARCH model, this paper aims to provide a comprehensive analysis of the USD/NGN exchange rate volatility between 2000 and 2023 so that users can make the right investment decisions and policymakers can make effective policy decisions based on the findings of this study. Lastly, the findings of this study will contribute to the literature knowledge on the exchange rate fluctuations.

2. Literature Review

2.1 Conceptual Review

Exchange rate is a term that is used to describe the value of the currency of one country in relation to the currency of another (Bagh *et al.*, 2017) ^[10]. The volatility of the exchange rate, which is generally understood as the risk of unexpected changes in the value of currencies, has a significant impact on the economic policy of a country (Meniago and Eita, 2017) ^[36]. Moreover, the currency rate may appreciate or depreciate. The exchange rate increases when a unit of foreign currency is exchanged with less domestic currency and decreases when a unit of foreign currency is exchanged with more domestic currency. Nevertheless, the exchange rate can be determined in two ways:

- The nominal exchange rate
- The actual exchange rate

The nominal exchange rate is the amount of local currency needed to obtain a unit of foreign currency. That is, the nominal exchange rate is the domestic price expressed in foreign currency. The actual exchange rate is the price of imported products in relation to domestic goods. In other words, it is the price-adjusted exchange rate.

2.1.1 Volatility in Foreign Exchange Rate, Export Performance, and Economic Growth

The positive or negative fluctuations are not good to the manufacturers of export products as they increase risk and uncertainty in international transactions. The International Monetary Fund (IMF) found out that these fluctuations involve undesirable macroeconomic events like inflations, despite the positive impact of exchange rate fluctuations on export trade in European Union countries as reported (Onyekachi, 2012) ^[43]. discussed the effects of these changes on foreign direct investment and observed that low exchange rates are beneficial to importation of production, machinery and production export during high foreign currency rates. Moreover, Froot and Stein (1991) ^[28] found significant evidence of a weak host country increase inward model since depreciation (a fall in currency rate) renders a host country less expensive. Dike (2018) ^[23] argues that exchange rate depreciation in host countries is likely to increase foreign

direct investment inflows, and a high real exchange rate increases the incentives of foreign businesses to produce at home and export rather than invest in a host country and export. The exchange rate leads to contraction of the exporting manufacturing sector. To sustain export performance, the real exchange rate of a country currency must be depreciated and this can be achieved by injecting money into the economy, and it is possible to maintain export performance by depreciating the real exchange rate of a country currency, which has an allocative effect in the economy (Lama and Medina, 2010) ^[35]. Adubi and Okunmadewa (1999) ^[2] argue that Nigeria as a developing nation is supposed to gain through the rise in export conversion prices due to devaluation of the currency. Obadan (2006) ^[40] discovered that exchange rate influences the export rate of a country with its associated risk impacting positively on the exporters when devalued and poor outcome of the flotation. The structural adjustment program was adopted in 1986 and the main aim of the program was to reform the production base of the economy with a positive bias towards agricultural export production. This shift enabled the gradual depreciation of the Nigerian naira, and a rise in local prices of agricultural exports that stimulated local production. Srou (2006) ^[51] cites diversification of export base of a country as one of the reasons given by the developing nations to change foreign exchange rates and regimes, which in turn increases local production, employment, income and economic growth, concluding that foreign exchange rate is a determinant of export trade and economic growth in Nigeria. Dubi and Okunmadewa (1999) ^[2].

2.1.2 Exchange Rate Policy in Nigeria

The major issues in the debate on exchange rates and their management in Nigeria are increased volatility, the continued overvaluation of the real exchange rate despite the constant nominal depreciation, and the continued quest to find a market-determined rate mechanism in a situation where the government is the major provider of foreign exchange. One of the main objectives of the Nigerian monetary policy is to stabilize the exchange rate. Over the years, the focus of exchange rate policy has largely been on maintaining nominal exchange rate stability. The nominal exchange rate is an indicator of economic welfare of the common people and a declining rate indicates economic problems. The other characteristic feature of the exchange system is the huge premium, which indicates the extent of market distortions. This is due to the fixed regime until the mid-1980s, the managed float of the SAP era, the re-fixing of the official rate under the Abacha government (1994-1998) and the resulting huge discrepancy between the official and parallel (free) market rates. With the high demand of foreign currency to import goods and other uses, and the fact that forex at the official rate was well regulated with strict paperwork requirements, the parallel market expanded (Ajao, 2015) ^[4].

Another characteristic of the regime is volatility in the real exchange rate (RER). The standard deviation of the increase in real exchange rate was 4% in 1961-1970. The standard deviation in the period 1991-2000, which was a period of rising liberalization, was 35 percent, and Nigeria had one of the most erratic RER regimes in the developing world. The RER was more stable during the fixed-nominal-exchange-rate system (1961-1985), but with the introduction of large oil profits and fiscal irresponsibility, rising domestic price inflation, and unsuccessful efforts to stabilize the nominal exchange rate, extensive volatility began. The uncertainty

(measured by volatility) is a major cause of concern as it deters investment by the private sector. A major issue that policymakers face is how to prevent RER overvaluation and exchange rate premia in a market-determined nominal exchange rate regime, especially when the government is the main provider of foreign money. The Central Bank has been conducting numerous experiments to establish the official nominal rate which is a controlled float. In 1999-2001, CBN returned to the pre-reform system of selling foreign exchange at a predetermined rate in the interbank foreign exchange market (IFEM) and the interbank market was split into the IFEM and the open inter-bank market where banks traded amongst themselves at freely negotiated exchange rates (the NIFEX). The Bureau de Change and the parallel market of foreign exchange are open markets, where no documentation is necessary in the foreign exchange transactions.

In the year 2000, the exchange rate depreciated in all markets. The Naira lost an average of 6.5 percent on the Interbank Foreign Exchange Market (IFEM) to N101.65 against one US dollar. This depreciation was mainly blamed on a sharp rise in foreign exchange demand due to the rise in government spending. The overall demand of foreign exchange at the IFEM in that year was 6.9 billion dollars as compared to 4.9 billion dollars in 1999. At the same time, the parallel market depreciated by 30 percent between December 1999 and May 2001, which led to a 20 percent widening of the differential between the parallel market and the IFEM rate. In April 2001, a foreign exchange crisis broke out due to excess liquidity occasioned by fiscal expansion. The Central Bank of Nigeria (CBN) made a slight change in the IFEM rate, which worsened the crisis unintentionally. To address the situation, the government sold off a lot of foreign currency, which drained foreign reserves. This and other stricter monetary policies saw the parallel market exchange rate increase to an average of N133 in the remaining part of 2001 with a 21 percent gap between the official and parallel market rates. In 2002, the Central Bank reintroduced the Dutch Auction System (DAS) which had been tried during the Structural Adjustment Program (SAP) in the mid-1980s but had failed. With the current civilian government having removed the fixed (nominal) exchange rate that existed during the Abacha regime, the premium between the parallel and official rates has reduced to 9.83 percent as compared to 28.98 percent. The DAS brought the premium down to approximately 7.8 percent, which is still quite high compared to the rates in most other developing countries, which are below 2 percent. Recent trends in the exchange rate policy in Nigeria have witnessed an increase in the average rate of the Naira against the US dollar to N128 to a dollar at the Dutch Auction System (DAS) in 2006. Between 2006 and December 2008, the exchange rates were relatively stable with mild appreciation being experienced throughout 2007 and most of 2008. This stability and appreciation was achieved due to large inflows of foreign exchange and a conscious policy not to allow massive appreciation leading to large reserves. Remarkably, the convergence of rates in different segments of the foreign exchange market was witnessed for the first time. It is anticipated that the exchange rate regime will continue to serve as a significant shock absorber to the economy, maintaining internal and external balance (Soludo, 2009) ^[50].

2.1.3 Factors Explaining Exchange Rate Volatility

The Naira has since the mid-1980s been depreciating in value, with its value against the US dollar dropping to N424 in 2022, despite the attempts by the rule of law to maintain a stable

exchange rate. It is therefore important to first learn the triggers that affect the exchange rate. Although there is no consensus on the exact causes of exchange rate volatility, this literature points out a number of factors that cause this phenomenon. Most of these factors tend to be country specific. The most frequently cited determinants are trade openness, capital flows, economic growth rate, level of financial development, external reserves, external indebtedness, and the current exchange rate regime. The way and the degree to which each of these factors affects the exchange rate movements is dependent on the economic conditions that prevail in each country (Stancik, 2006) ^[52]. The literature is generally in agreement that the exchange rates of countries in transition, especially emerging market economies are more prone to the effects of these factors (Stancik, 2006; Al Samara, 2009) ^[52, 5]. This section will explore a literature review of the determinants of exchange rate volatility, and how these different factors lead to the volatility that is observed.

- **Capital Flows**

International capital flow includes long-term and short-term capital. The long-term capital such as foreign direct investment is usually viewed as sustainable whereas the short-term capital, which is mainly foreign portfolio investment (FPI), is viewed as temporary (Rashid and Hussain, 2013) ^[44]. The inflow of capital usually causes the appreciation of the domestic currency (Corden, 1994; Oaikhenan and Aigheyisi, 2015) ^[21, 39], whereas the outflow of capital causes depreciation of the currency. As a result, inflow and outflow of capital into and out of an economy causes the exchange rate of the local currency against the currencies of the trading partners to fluctuate. The magnitude of these exchange rate fluctuations, which are caused by capital flows, depends on the nature of the capital and the financial market depth. In a situation where short-term (temporary) capital is predominant, which is known to be volatile in nature, the chances of exchange rate volatility are more than in a situation where long-term (sustainable) capital is dominant (Jean-Louis, 2009) ^[32]. Kapur (2007) ^[34] blames the high volatility of the exchange rate on what he refers to as destabilizing capital flows. Sudden slowdown in the rate of inflow of private capital into the emerging market economies and a gradual transition of large current account deficits to small deficits or small surpluses can also lead to volatility in the real exchange rate (Calderon and Kubota, 2012) ^[14]. It is important to note that the effect of capital flows on exchange rate volatility varies among countries with well-developed financial markets and those with poorly developed financial markets. It is a well-known fact that the international capital flows are likely to cause greater volatility in the exchange rates of the developing or transitional economies than in the industrialized economies. This has been attributed to the fact that the financial markets in most developing/emerging market economies are underdeveloped (Chit and Judge, 2008; Saborowski, 2009) ^[18, 45].

- **Trade Openness**

The volatility in the exchange rates can also be explained by the level of trade openness. According to Calderon and Kubota (2012) ^[14], the effect of trade openness on exchange rate volatility depends on the extent to which an economy is integrated into the global market. Basically, the more open an economy is, the less volatile

the exchange rate of its currency is, as pointed out by Stancik (2006) ^[52]. Nevertheless, the success of trade openness in reducing exchange rate volatility depends on the extent of flexibility in the adjustment of aggregate prices as suggested by Cavallo (2008) ^[16]. These structural linkages between the flexibility of aggregate prices and exchange rate volatility increase the volatility in less open economies. The issue is further compounded by the fact that policy actions to stabilize the exchange rate can unintentionally create more volatility in inflation, output and interest rates. Within the framework of a small open economy, as explained by Alfaro (2009) ^[7] and the sticky price model by Gali and Monacelli (2005) ^[29], a trade-off is required in the process of attaining stability in the exchange rate and maintaining stable inflation and output gap. This kind of economy can be compared to a balloon: when one tries to minimize volatility in one area, it is transferred to another area as Cho and West (2003) ^[19] argue.

• **Fiscal Deficit**

Apriorist reasoning assumes that there is a positive correlation between exchange rate volatility and fiscal deficit. In more basic terms, it is believed that large fiscal deficits cause large exchange rate fluctuations (Avila, 2011) ^[9]. This view is consistent with the available empirical evidence, which indicates that higher inflation and fiscal deficit are associated with greater volatility in nominal effective exchange rates (Canales-Kriljenko and Habermeier, 2009, Zubair *et al.*, 2022) ^[15, 56]. The argument is that when government deficits as a percentage of GDP increase, not only do interest rates and exchange rate volatility increase, but also other important macroeconomic indicators move in the wrong direction (Ussher, 1998) ^[53]. In the case of the Nigerian economy, Iyoha and Oriakhi (2002) ^[31] found that the nominal shocks that caused the fluctuations in the naira-dollar exchange rates in the 1978 - 1985 period were due to fiscal deficits. Also, Ogunleye (2008) ^[41] blamed the high volatility in the real exchange rate to high spending due to the oil windfall in the same time.

• **Economic Growth**

Numerous theoretical and empirical studies have been conducted on the effects of exchange rate volatility on different aspects of the economy such as investment, productivity, trade, capital flows, and economic growth (Schnabel, 2007; Aliyu, 2009; Aghion *et al.*, 2009; Boar, 2010; Shehu and Youtang, 2012) ^[47, 8, 3, 12, 49]. It is however well known that there is a reciprocal causal relationship between economic growth and exchange rate volatility which means that economic growth can also be a cause of exchange rate volatility. The exchange rates of highly developed economies and the emerging markets and developing countries have been compared and it has been observed that the currencies of the industrialized countries are more stable (Calderon and Kubota, 2012) ^[14]. This difference is explained by the well-developed and stable financial systems, the free access to international capital markets, the highly liquid currencies, the independence of the central bank, and the highly open economies of the industrialized countries. These characteristics are said to promote faster economic growth and protection of these economies against external shocks, including fluctuations in major macroeconomic variables, like the exchange rate. This implies that there is an inverse relationship between

economic growth and volatility of the real exchange rate, which means that high and possibly increasing economic growth rate is likely to reduce volatility in the exchange rate (Bastourre and Carrera, 2007) ^[11]. Nonetheless, economic growth has been associated with increased productivity, which is a cause and effect of economic growth, and has been associated with less volatility in exchange rates (Sanusi, 2004) ^[46].

2.1 Theoretical Review

Although there are many exchange rate theories, we will only examine two common ones in this paper: the Quantity Theory of Money, and the Purchasing Power Parity Theory.

2.2.1 The Quantity Theory of Money (QTM)

According to the monetarist school of thought, the Quantity Theory of Money (QTM) is a simple model of calculating the long-run equilibrium exchange rate. The proponents of the QTM state that a change in the amount of money only affects the price level and does not affect the real sector of the economy (Nyoni, 2018) ^[38]. In international economics, or the international version of the QTM, a rise in money supply is reflected by a corresponding increase in the exchange rate. Oleka *et al* (2014) ^[42] point out that the exchange rate may be viewed as determined by the demand of money, which is positively affected by the rate of real economic growth and negatively affected by the inflation rate. It is therefore a fact that the development of the real economy is a major factor in determining the currency position of a country. According to Mustapha *et al.*, (2021) ^[37], one of the limitations of the international QTM is that it fails to explain the changes in the real exchange rate, but only the nominal exchange rate.

2.2.2 Purchasing Power Parity Theory (PPPT)

This economic theory, which is used to determine the relative value of currencies and determine the adjustments that need to be made in the exchange rate between two countries, seeks to harmonize the exchange rate with the purchasing power of the currency (CBN, 2011). It is also referred to as the Purchasing Power Parity Theory (PPPT) and it recognizes the importance of the level and trends of inflation in determining the exchange rates of currencies in both emerging and developed economies (Scott, 2008) ^[48]. PPPT assumes that the value of a currency will fall when there is high inflation in the country or when there is fear of an increase in the level of inflation. This loss is explained by the erosive effect of inflation on the purchasing power, which results in the reduced demand of that specific currency. Interestingly, as Oleka *et al.*, (2014) ^[42] point out, a currency can sometimes appreciate in the face of inflationary pressures because of the anticipation that the central bank will act by increasing short-term interest rates to reverse the inflationary trend. Obadan (2006) ^[40] points out that PPPT holds that the equilibrium exchange rate between two interchangeable paper currencies is the equality of their purchasing power, which is basically their relative prices.

3. Material and Method

3.1 Data

The data used was monthly data on the Nigerian Naira exchange rate against that of the US dollar provided by the Central Bank of Nigeria at www.cenbank.org. The data employed is between January 2000 and December 2022. The naira to the US dollar exchange rate series has 276 observations. Gujarati (2004) ^[30] and Christoffersen *et al.*, (2014) ^[20] state that such unstable series cannot be used to make any additional statistical inferences due to their

implications. This non-stationarity requires the transformation of the series.

3.2 Stationarity Test

Many standard statistical methods require time series data to be stationary. A time series is said to be stationary when all the statistical characteristics of the time series like mean, variance, autocorrelation etc. remain constant over time. Unless stationarity is confirmed and corrected, non-stationary data may yield spurious regressions and invalid inferences when used directly in estimation. There are a number of unit root tests that are applied to test stationarity of a time series. The most common tests are the Dickey-Fuller test and augmented Dickey-Fuller (ADF) test. Both tests test the null hypothesis that there is a unit root (the series is non-stationary) against the alternative that there is no unit root (the series is stationary). In this research, the ADF test will be employed.

Augmented Dickey-Fuller Test

The ADF adds lags of the dependent variable to the DF regression to account for serial correlation:

$$\Delta y_t = \alpha + \beta_t + \gamma y_{t-1} + \sum_{i=1}^p \delta_i \Delta y_{t-i} + \epsilon_t \quad (1)$$

Where:

Δy_t is the first difference of the series.

α is a constant (drift term).

β_t is a deterministic trend.

γ is the coefficient of interest, testing for a unit root.

δ_i are the coefficients for the lagged differences.

ϵ_t is the error term.

p is the number of lagged difference terms.

Hypotheses

Null Hypothesis (H_0): The series has a unit root (non-stationary).

Alternative Hypothesis (H_1): The series is stationary.

3.3 GARCH Model in Time Series Analysis

Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model is a model that is applied to predict and forecast the volatility of time series data, especially in finance. It explains the phenomena of volatility clustering where large movements in the value of assets are followed by even larger movements.

3.4 The GARCH(p,q) model

GARCH(p,q) model may be simply considered as an extension of GARCH (1,1) model with p and q being the number of lagged conditional variance and squared residual included respectively. The GARCH(1, 1) model has two primary equations, one of the return series and the other one is the conditional variance. The simple GARCH(1, 1) model, proposed by (Bollerslev, 1986), is given by:

Mean Equation

$$r_t = \mu + \epsilon_t \quad (2)$$

Where:

r_t = Return at time t

μ = Mean return

ϵ_t = Residual (error term) at time t .

Variance Equation

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^q \alpha_i \epsilon_{t-i}^2 + \sum_{j=1}^p \beta_j \sigma_{t-j}^2 \quad (3)$$

Where:

σ_t^2 = conditional variance at time t .

α_0 = constant term.

α_i = coefficients for the lagged squared residuals (the ARCH terms).

β_j = coefficients for the lagged conditional variances (the GARCH terms).

ϵ_{t-i}^2 = squared residuals from past periods.

σ_{t-j}^2 = lagged conditional variances.

3.4.1 The Threshold GARCH (TGARCH) Model

The asymmetric or the Threshold GARCH (TGARCH) model is a model that is based on the GARCH model but has an extra element of the asymmetric effects. Nevertheless, it clarifies that negative and positive shocks of the same magnitude influence volatility differently: this is a common occurrence in financial markets (Zakoian, 1994). Mathematically, the TGARCH(1,1) model is given by:

$$\sigma_t^2 = \omega + \alpha \epsilon_{t-1}^2 + \gamma I(\epsilon_{t-1} < 0) \epsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \quad (4)$$

Where,

σ_t^2 is the conditional variance.

ω is a constant term.

α measures the effect of the squared past error term on current volatility (response to magnitude of shocks).

γ measures the asymmetry effect (response to the sign of shocks).

β captures the effect of past volatility on current volatility.

$I(\epsilon_{t-1} < 0)$ is an indicator function that takes the value 1 if $\epsilon_{t-1} < 0$ (negative shock) and 0 otherwise.

3.4.2 The EGARCH Model

The Exponential GARCH (EGARCH) model, developed by Nelson (1991), is a popular model for capturing volatility dynamics in financial time series. Unlike traditional GARCH models, EGARCH can model asymmetries and does not require non-negativity constraints on its parameters. The EGARCH(1,1) model is given by:

$$\log(\sigma_t^2) = \omega + \beta \log(\sigma_{t-1}^2) + \alpha \left(\frac{\epsilon_{t-1}}{\sigma_{t-1}} \right) + \gamma \left(\left| \frac{\epsilon_{t-1}}{\sigma_{t-1}} \right| - \sqrt{\frac{2}{\pi}} \right) \quad (5)$$

Where:

σ_t^2 is the conditional variance at time t .

ω, β, α , and γ are parameters to be estimated.

$\frac{\epsilon_{t-1}}{\sigma_{t-1}}$ is the standardized residual from the previous period.

γ captures the asymmetry or leverage effect.

3.4.3 The PGARCH Model

The Power GARCH (PGARCH) model is an extension of the GARCH model that introduces flexibility in modeling the volatility of time series data by allowing for power transformations of the conditional standard deviation.

$$\sigma_t^\delta = \alpha_0 + \sum_{i=1}^q \alpha_i |e_{t-i}|^\delta + \sum_{j=1}^p \beta_j \sigma_{t-j}^\delta \quad (6)$$

Where:

σ_t is the conditional standard deviation at time t .

δ is the power parameter, which is estimated from the data.

α_0, α_i , and β_j are parameters to be estimated.

e_{t-i} are the residuals from previous periods.

3.5 Model Selection Criteria

In statistical modeling, using the right model is essential to getting precise and broadly applicable findings. The Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC) are two often used criteria for model selection. In order to avoid overfitting and choose which model is best for the data, both criteria seek to strike a balance between model fit and complexity.

3.5.1 The Akaike Information Criterion (AIC)

Based on information theory, the Akaike Information Criterion is used to assess models by taking into account their complexity and probability. The AIC formula is:

$$AIC = 2k - 2\ln(L) \quad (7)$$

where k is the number of parameters in the statistical model, and L is the maximized value of the likelihood function for the estimated model. While lower values of AIC suggest a better model, higher values imply a bad model. Models with more parameters to control over fitting are penalized by the AIC, which favors less complicated but equally well-fitting models.

3.5.2 The Bayesian Information Criterion (BIC)

The Bayesian Information criteria, often referred to as the Schwarz Information Criterion (SIC), is a model selection criteria that makes use of Bayesian concepts. BIC is computed as follows:

$$BIC = \ln(n)k - 2\ln(L) \quad (8)$$

Where n represents the number of observations in the dataset, k is the number of parameters in the model, and L indicates the maximum value of the likelihood function for the model. In the same way as AIC, a lower BIC value is desirable. However, compared to the AIC, the BIC penalty for the number of parameters increases proportionately with the log of the sample size, making it even less tolerant of over fitting.

3.6 Diagnostic tests

The assumption of constant error variance is essential for reliable statistical inference in econometrics and finance econometrics. Heteroskedasticity is the outcome when this assumption is broken, meaning that the variability of mistakes varies over time or between observations. For successful model estimate, heteroskedasticity must be addressed and diagnosed. The ARCH (Autoregressive Conditional Heteroskedasticity) LM (Lagrange Multiplier) test is a useful technique for identifying heteroskedasticity in time series data.

3.6.1 Heteroskedasticity

When the variance of the residuals or errors in a regression model is not constant, heteroskedasticity occurs. This variation may vary over time or consistently with the explanatory variable's level. Because such a condition departs from the premise of homoskedasticity (constant variance of

errors), it may compromise the validity of statistical estimates and tests.

3.6.2 Autoregressive Conditional Heteroscedastic-Lagrange Multiplier (ARCH -LM) Test

Engle (1982) developed the ARCH LM test, which is intended to detect autoregressive conditional heteroskedasticity in time series data. Volatility clustering, in which intervals of high volatility are followed by additional high volatility and vice versa, is a characteristic of this kind of heteroskedasticity. The model can be specified as;

$$\hat{\varepsilon}_t^2 = \alpha_0 + \alpha_1 \hat{\varepsilon}_{t-1}^2 + \alpha_2 \hat{\varepsilon}_{t-2}^2 + \dots + \alpha_p \hat{\varepsilon}_{t-p}^2 + \vartheta_t \quad (9)$$

where $\hat{\varepsilon}_t^2$ are the squared residuals from the original model, and $\alpha_1, \alpha_2, \dots, \alpha_p$ are the coefficients to be estimated.

3.7 Measures of Forecast Accuracy

Evaluating the precision of predictions derived from time series models is a crucial yet challenging undertaking. This section provides two forecasting performance measures to address this crucial modeling stage. These are the Mean Absolute Error (MAE) and the Root Mean Square Error (RMSE).

3.7.1 Root Mean Square Error

The estimate of forecasting error deviation is known as the root mean square error, or RMSE. The square root of the squared difference between the observed and predicted values is obtained and used to calculate it. An improved model estimate is indicated by a smaller RMSE.

$$RMSE = \sqrt{\frac{1}{n} \sum_{t=1}^n (\sigma_t^2 - \hat{\sigma}_t^2)^2} \quad (10)$$

3.7.2 Mean Absolute Error

The MAE is regularly used for examining the suitability of time series model. The MAE is express by:

$$MAPE = \frac{1}{n} \sum_{t=1}^n \left| \frac{\sigma_t^2 - \hat{\sigma}_t^2}{\sigma_t^2} \right| \quad (11)$$

4. Results

4.1 Descriptive Statistics

Table 4.1 presents the descriptive statistics of the exchange rate. The data comprises of 264 observations. The mean (average) naira to US dollar is 231.0351 which represents the central tendency of the data. The Minimum value of the variable is 113.7000 while the maximum values are 1233.000. The standard deviation of the variables is 157.3866, which measures the amount of variation or dispersion in the data. The skewness value is 2.914398 which indicates the asymmetry of the distribution of the variables suggesting that the distribution is skewed to the right, meaning that the tail on the right side of the distribution is longer or stretched compared to the left side. The kurtosis value is 15.28093 which indicates the shape of the distribution of the of the variables.

Table 4.1: Descriptive Statistics of Nigeria exchange rate

Variable	N	Min	Max	Mean	Variance	Std. Dev.	Skewne	Kurtosi
US Dollars	264	113.7	1233.0	231.035	24770.5418	157.386	2.9143	15.2809

4.2 Time plot

Figure 4.1 depicts the graph of exchange rate from naira to us dollar the data. The above graph reveals that there is steady

increase in the us dollar over the years. The us dollar data started increasing and moving upwardly from 2002 to its highest peak in 2023.

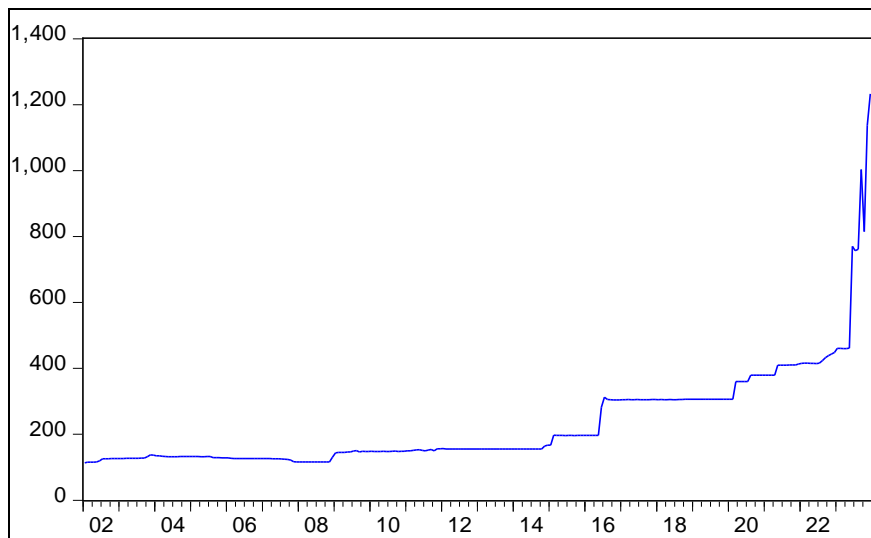


Fig 4.1: Graph of Exchange Rate from Naira to US dollar.

Figure 4.2 shows that the Us Dollar Data is now stationary after the second differencing (i.e., the mean, variance and covariance of the series are constant over time).

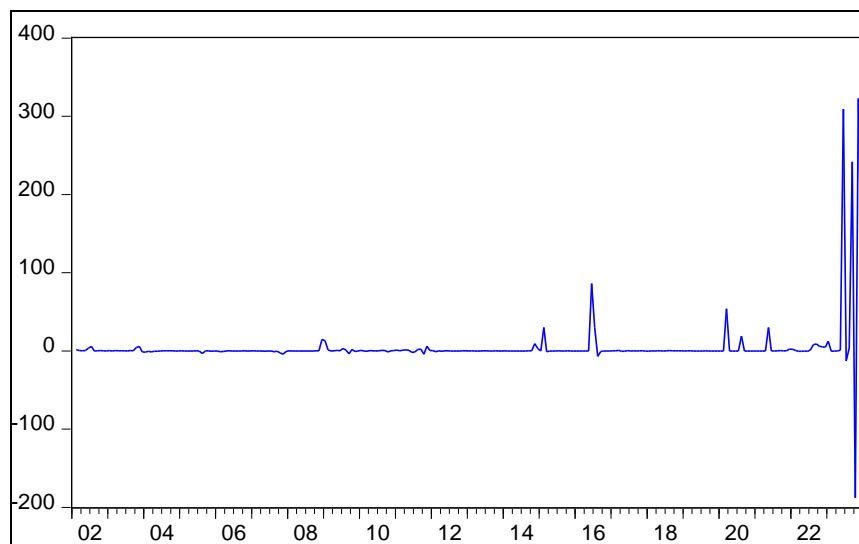


Fig 4.2: Graph of Second Differenced Us Dollar Data

4.3 Augmented Dickey Fuller (ADF) Test for Stationarity

In figure 4.1 looking at the seasonal and increment pattern of the data, it is seen that the US dollar data is not stationary. Hence, the US dollar data is differenced. Also, Augmented

Dickey fuller test further confirmed that the data is stationary at level but were stationary at 5% and 10% level of significance after the second difference.

Table 4.2: Augmented Dickey Fuller (ADF) Test

ADF Test	t-statistics			CV 5%			CV 10%		
	I(0)	I(1)	I(2)	I(0)	I(1)	I(2)	I(0)	I(1)	I(2)
US dollar	2.58	-0.27	-18.85	-2.873	-2.87	-2.87	-2.57	-2.57	-2.57
				FTR	FTR	Reject	FTR	FTR	Reject

CV = Critical Value at 3 lags; FTR - Fail to reject = unit root; Reject = no unit root.

4.4 Autoregressive Conditional Heteroscedasticity (ARCH) Model

H_0 = There is no ARCH effect in the US Dollar data

H_1 = There is ARCH effect in the US Dollar data

Decision Rule: Reject the null hypothesis if the p-value < α -value; otherwise, do not reject H_0 .

Conclusion: Since p-value < 0.0001 therefore null hypothesis is rejected. Therefore, conclude that there is ARCH effect in the US Dollar data at 5% level of significance.

Table 4.3: Autoregressive Conditional Heteroscedasticity (ARCH) Model for US Dollar data

ARCH = C(2) + C(3)*RESID(-1)^2				
Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	148.0409	0.147397	1004.369	0.0000
Variance Equation				
C	0.144797	0.087418	1.656374	0.0976
RESID(-1)^2	1.185979	0.184552	6.426263	0.0000
R-squared	0.279131	Mean dependent var		231.0351
Adjusted R-squared	0.269131	S.D. dependent var		157.3866
S.E. of regression	178.0022	Akaike info criterion		9.800997
Sum squared resid	8333095.	Schwarz criterion		9.841633
Log likelihood	-1290.732	Hannan-Quinn criter.		9.817325
Durbin-Watson stat	2.038053			

Dependent Variable: US_DOLLAR

4.5 Generalised Autoregressive Conditional Heteroscedasticity (GARCH) Model Efficiency

Table 4.4 shows model efficient statistics and goodness of fit criteria of GARCH models fitted for the US Dollar data. The best model for modelling the US Dollar data in the GARCH model would be the one with the highest value of log likelihood, lowest value of Akaike Information Criterion (AIC) and Schwarz Criterion (SC). Based on the given goodness of fit statistics, it appears that GARCH model has

the minimum value of AIC (9.815702) and minimum SC (9.869883).

Therefore, GARCH model is the best model for modelling among the four-model chosen for modelling the US Dollar exchange rate to naira. It has the highest Log-likelihood value of -1291.673, which indicates that the model offers a better fit to the US Dollar exchange rate to naira than any other model. Therefore, GARCH model is the best model for modelling and forecasting out of the four-model chosen for modelling the US Dollar exchange rate to naira

Table 4.4: Model Efficient Statistics and Goodness of Fit Criteria of GARCH models Fitted for US Dollar data.

Model	GARCH	T-GARCH	E- GARCH	P- GARCH
R-squared	0.796715	0.297722	0.2164060	0.2163060
Adjusted R-squared	0.776715	0.297722	0.2151045	0.2151058
Log likelihood	-1291.673	-1324.974	-1772.543	-1772.581
Durbin-Watson stat	2.034849	1.897508	1.915388	2.015388
AIC	9.815702	10.07556	13.45866	13.51313
Schwarz criterion	9.869883	10.14328	13.51284	13.48072

Note: AIC is the Akaike info criterion and SC is the Schwarz criterion

Table 4.5 displays the General Autoregressive Conditional Heteroscedasticity (GARCH) model for US dollar data. The equation of the model is $GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*GARCH(-1)$. The p-value is less than 0.05 level of significance which shows that the

GARCH(1,1) parameters are highly significant. The sum of the coefficient of the GARCH model parameters (0.778071, 0.220204) is very close to 1 which reveals that the shocks to the conditional variance is relatively persistent over time.

Table 4.5: General Autoregressive Conditional Heteroscedasticity (GARCH) Model for US Dollar data

GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*GARCH(-1)				
Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	132.0926	0.523230	252.4562	0.0001
Variance Equation				
C	0.430085	0.495474	0.868028	0.3854
RESID(-1)^2	0.778071	0.386767	2.011729	0.0442
GARCH(-1)	0.220204	0.023911	9.209341	0.0000
R-squared	0.796715	Mean dependent var		231.0351
Adjusted R-squared	0.776715	S.D. dependent var		157.3866
S.E. of regression	186.0037	Akaike info criterion		9.815702
Sum squared resid	9099111.	Schwarz criterion		9.869883
Log likelihood	-1291.673	Hannan-Quinn criter.		9.837474
Durbin-Watson stat	2.034849			

The graphical representation depicted in Figure 4.3 provides the forecast plot of the US Dollar data in exchange rate with GARCH model.

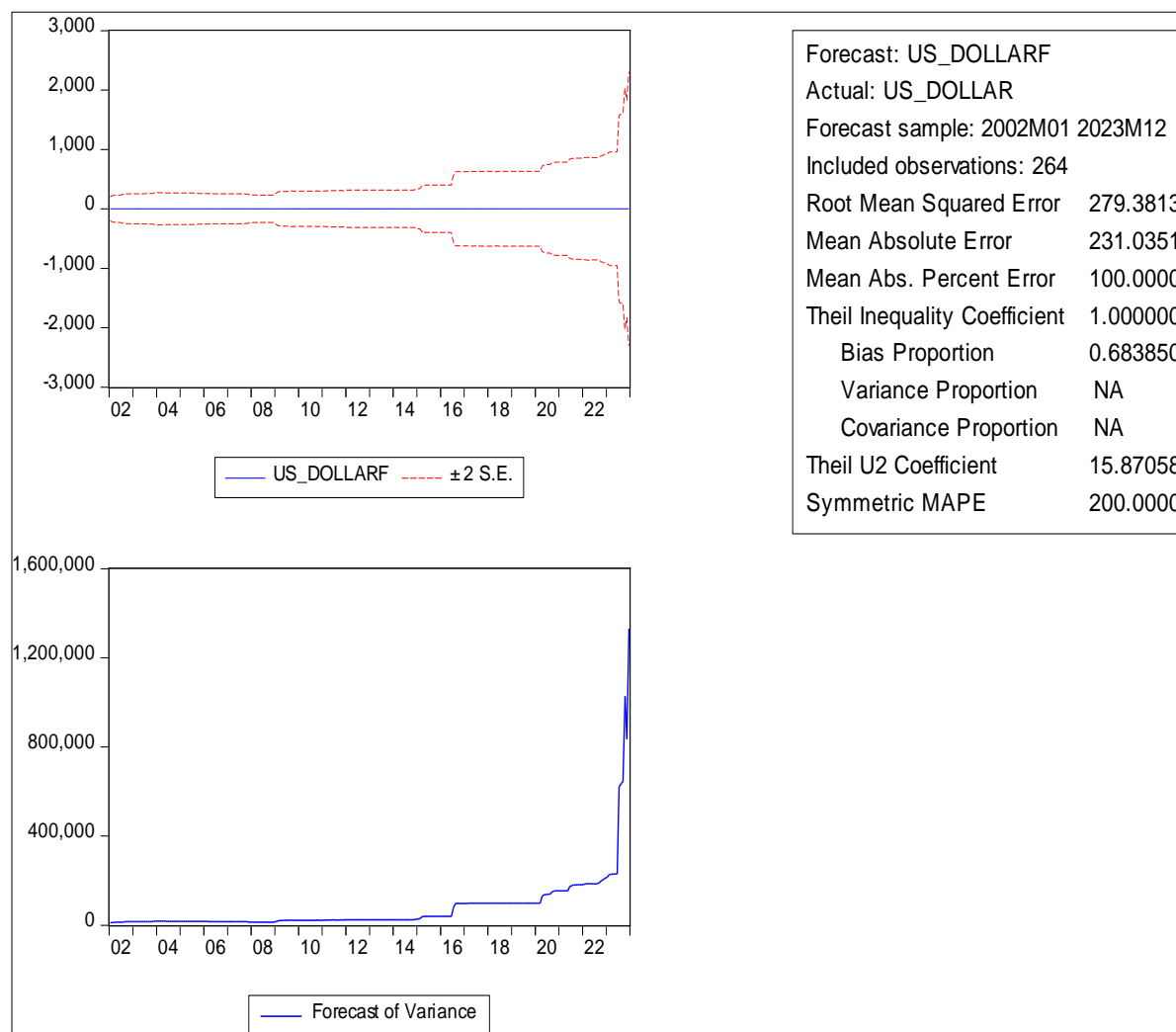


Fig 4.3: Forecast Plot of US Dollar data with GARCH

4.6 Discussion of Findings

This study was designed to apply the Autoregressive Conditional Heteroscedasticity (ARCH) and Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model to the Exchange rate of Naira to US Dollar. The data used for this study is secondary data sourced from Central Bank of Nigeria Annual, websites from January 2002 to December 2023. The exchange rate data was analysed using the Autoregressive Conditional Heteroscedasticity (ARCH) and Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model to know the best model to fit the exchange rate data. The data were not stationary at level; therefore, the data were differenced twice $I(2)$. After the second differencing, the data is stationary at 5% and 10% level of significance. The model efficient statistics and goodness of fit criteria of the distributions fitted to the US dollar data revealed that Gamma distribution is the best distribution for modelling the US Dollar data with minimum value of log-likelihood, AIC and BIC among the three-distribution chosen for modelling the Exchange rate data. The result based on Akaike Information Criterion is in line with the study of Eric-Jan & Simon (2004) and the Schwarz Criterion is in line with Degiannakis & Xekalaki, 2007 study. On the basis of the goodness of fit statistics selected in this study, it appears that GARCH model has the minimum value of AIC (9.815702) and minimum SC (9.869883) which is in line with the study of Miazhynskaia & Dorffner, 2006 and Nagaraj et (2020). Therefore, GARCH model is the best

model for modelling among the four-model chosen for modelling the US Dollar exchange rate to naira.

5. Summary, Conclusion, and Recommendations

5.1 Summary

This research aims to improve the accuracy and reliability of volatility forecasts for the US dollar and the Nigerian naira, resulting in more informed decision-making for businesses, investors, and policymakers operating in the volatile world of international finance. Secondary data from archived of Central Bank of Nigeria Annual was used in this study. The descriptive analysis revealed that mean of US Dollar is 231.0, 316 with standard deviation of 157.4. The Minimum value of the variable is 113.7000 while the maximum values are 1233.000. The Minimum value of the variable is 113.7000 while the maximum values are 1233.000. The series was not stationary at level and first difference. ARCH effect test indicates presence of ARCH in the series. In addition, model efficiency for the best generalized autoregressive conditional heteroskedasticity (GARCH) model family depicts GARCH is the model with the lowest Akaike information Criterion (AIC).

5.2 Conclusion

This study has carefully examined volatility model of US dollar to naira exchange rate from the perspective of GARCH family of modelling. Based on the results of the analyses GARCH model is the most competitive model among the family of GARCH models considered in this study.

5.3 Recommendations

Based on the results and findings of this study, the following recommendations were made; GARCH should be the appropriate statistical model used in forecasting US dollar to Naira exchange rate, also the empirical probability distribution underlying exchange rate should be determined and use as innovation distribution in modelling volatility of US dollar to Naira exchange rate.

5.4 Suggestions for Further Studies

Given the results and limitations of this study, various routes for further research are proposed. To begin, researchers may want to look at other volatility forecasting models like Exponential Generalised Autoregressive Conditional Heteroskedasticity (EGARCH) and Threshold GARCH (TGARCH). A comparison of these models to the GARCH model used in this work might provide insight into their relative ability at capturing exchange rate volatility.

Furthermore, increasing the dataset to cover a larger variety of currencies or include additional economic variables may improve the robustness of the results. future studies should look at how macroeconomic factors like inflation, interest rates, and geopolitical events affect exchange rate volatility. This would result in a better understanding of the fundamental causes of currency movements. This method would provide a more thorough understanding of the elements driving exchange rate evolution in various circumstances.

Finally, qualitative research that includes expert viewpoints from financial analysts, economists, and policymakers may improve the study by offering contextual insights into the complexity of exchange rate behaviour.

5.5 Contribution to Knowledge

This study adds to the current body of knowledge in various ways. First, it improves knowledge of exchange rate fluctuation between the US dollar and the Nigerian naira by providing empirical facts that policymakers and investors can use. Second, by using the GARCH model, the research proves the efficacy of sophisticated statistical approaches in predicting volatility, which may then be extended to other financial markets. Finally, the results emphasise the significance of precise volatility forecasting in decision-making processes, showing the value of statistical research in real financial applications. This study serves as the basis for future research in financial statistics and exchange rate analysis.

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