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Anju Dhall
Department of Maths, GGDSD College, Palwal, MDU, Rohtak, Haryana, India

Seema Bansal
Department of Maths, Vaish College, Bhiwani, CBLU Bhiwani, Haryana, India

Corresponding Author:
Anju Dhall
Department of Maths, GGDSD College, Palwal, MDU, Rohtak, Haryana, India

Stochastic modeling of PLC system with hardware redundancy and subject to PM with pre-specific time

Anju Dhall and Seema Bansal

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Abstract
This paper examines the reliability of a Programmable Logic Controller (PLC) system with hardware redundancy by performing preventive maintenance on the standby unit after it has operated for a specific period. There are possibilities of two scenarios: either the active unit undergoes a software upgrade with a certain probability, or it suffers a hardware failure independently with another probability after its preventive maintenance (PM). If any issue arises, a serviceman is immediately available to handle both software and hardware repairs as well as maintenance. After repairs, the system is assumed to return to full working condition. Hardware and software failures are modeled using exponential distributions, while other events follow arbitrary distributions. The analysis uses the Regenerative Point Graphical Technique (RPGT) combined with a semi-Markov process to efficiently evaluate reliability metrics and system behavior. Results include tabulated expressions for key parameters such as Mean Time to System Failure (MTSF), system availability & profit.

Keywords: Preventive maintenance (PM), Redundancy, Software(s/w), Hardware (h/w), Programmable Logic Controller (PLC)

1. Introduction

Programmable Logic Controllers (PLCs) are modern automation devices that offer significant advantages over traditional control systems. These include compact design, energy efficiency, and high reliability. As a result, PLCs are widely used (particularly in China) in industries such as steel manufacturing, chemical processing, transportation, quality inspection, and process monitoring. Designed for durability, PLCs can operate in harsh industrial environments involving extreme temperatures, heavy vibrations, high humidity, and electrical interference. They are commonly used to control and monitor a large number of sensors and actuators. In addition to serving as specialized digital computers, PLCs are also applied across various other control system domains and industries.

Despite robust design, no equipment is entirely failure-proof. Operational failures can lead to significant costs in time, money, and even safety risks. Therefore, maintenance especially preventive maintenance plays a critical role in ensuring long-term equipment performance. Regular preventive maintenance helps reduce the likelihood of failures and mitigate their impact. Over the past few decades, extensive research has been conducted on stochastic modeling of PLC systems, given their realistic representation of real-world conditions and practical relevance. Systems incorporating standby units have proven effective in delivering more reliable and long lasting performance. Notable researchers like Yu Jiang [5] and Ling Qi [7] have studied the reliability of PLC systems with redundancy. Recently, Malik [9] has establish a model with different probability of repair and replacement in spite of Preventive maintenance may be taken as caution.

However, many of these studies rely on abstract system models, which may not meet the precision demands required for mission critical PLC applications. Moreover, accurate reliability modeling must account for the interaction between software and hardware components in a probabilistic framework.

This study focuses on analyzing the reliability of a PLC system that includes component-level redundancy and scheduled preventive maintenance. The system experiences independent failures in both hardware and software. Preventive maintenance for hardware is performed after a predefined operational time, referred to as the maximum repair time ‘t’. A serviceman is available to immediately conduct inspections, maintenance, and repairs. After maintenance or repair, the system is considered as good as new. Hardware and software failure times follow an exponential distribution, while maintenance, inspection, and repair times follow general (arbitrary) distributions with different probability density functions. All random variables are assumed to be statistically independent, and switching devices are considered perfect.

Using a semi-Markov process and the Regenerative Point Graphical Technique (RPGT), the study derives steady-state expressions for key reliability metrics, including transition probabilities, mean sojourn times, Mean Time to System Failure (MTSF), system availability, server busy periods (due to maintenance/inspection and repair), expected number of serviceman visits, and a profit function. Numerical analyses are also provided, based on specific parameter values and associated costs, to assess the behavior of crucial performance indicators.

2. Notations

PLC/HCs	The unit is in operative/ hardware cold standby mode
ϵ/ϵ'	Constant failure rate of hardware and software.
p/q	The probability that system has hardware /software failure.
$\phi(t)/\Phi(t)$	Pdf/cdf of hardware repair time.
$\omega(t)/\varpi(t)$	Pdf/cdf of software up gradation time.
$\psi(t)/\Psi(t)$	Pdf/cdf of maintenance time for hardware component.
$\aleph_0$	The rate for which hardware component undergoes for preventive maintenance
$\dot{q_{ij}}(t)/\dot{Q_{ij}}(t)$	Pdf/cdf of direct transition time from a regenerative state regenerative state (0,t].
$\dot{q_{ij.k}}(t)/\dot{Q_{ij.k}}(t)$	pdf/cdf of first passage time for a regenerative state S_i to regenerative state S_j or to failed state S_j visiting state S_k once in (0,t]
$M_i(t)$	Probability that the system is up initially in state $S_i \in$ E is up at the time “t” without visiting to any other regenerative state.
δ_i	The mean sojourn time spent in state $S_i \in$ E before transition to any other state
δ_i'	The total unconditional time spent in state before transition to any Other regenerative state given that the system entered regenerative state i at time t=0
${f^z}_i$	Fuzziness measure of the i-state
n_i	Expected time spend while doing a job,given that the system entered regenerative state i at time t=0
m_{ij}	Contribution to mean sojourn time in state S_i when system transits directly to state S_j(S_i,$S_j \in$ E) $\delta_i = \sum m_{ij}$ so that $m_{ij} = \int t dQ_{ij}(t) = -q_{ij}^{*\prime}(0)$
$'$	Derivative of function
Pdf/cdf	probability density function/cumulative density function
$HUPm/HUPM$	Hardware component of PLC is under/continuously preventive maintenance
$HFUr/HFUR$	Hardware component of PLC is under/continuously repair
$SFWug/SFWUG$	Software component of PLC is waiting under/continuously for upgradation
$HFWr/HFWR$	Hardware component of PLC is waiting under/continuously repair from previous stage.

The possible transition states of the system models are shown in Figure 1: Stage 0: (PLC, Hcs); Stage I: (PLC, HUPm); Stage II: (PLC, HFUr); Stage III: (SFUg, Hcs); Stage IV: (HFWr, HWPM); State V: (SFWUg, HUPM); Stage VI: (HFUR, HFWr); Stage VII: (HFUR, HFWUg); Stage VIII: (HFUR, HWPm); State IX: (HUPm, HWPM)

State Transition Diagram

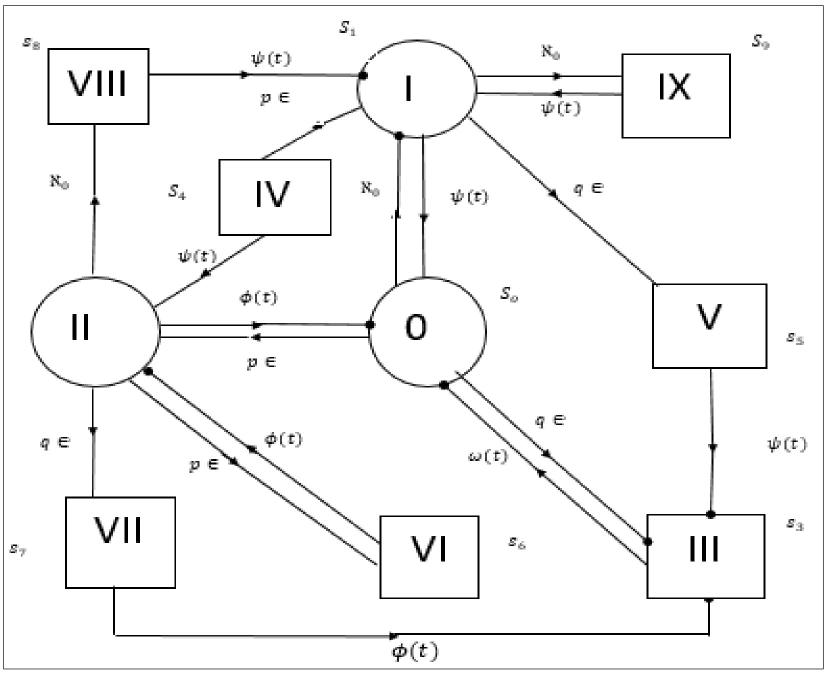

Fig 1: State transition diagram of the PLC system

3. Transition Probabilities

Simple probabilistic considerations yield the following expressions for the non-zero elements

$$\dot{p_{ij}} = Q_{ij}(\infty) = \int_0^\infty \dot{q}_{ij}(t)dt \text{ as}$$

$\dot{q_{ij}}(t)$	$\dot{P_{ij}} = \dot{q_{ij}^*}(0)\ \phi(t) = \alpha e^{-\alpha t}, \omega(t) = \beta e^{-\beta t} \psi(t) = \gamma e^{-\gamma t}$
$\dot{q_{01}} = \aleph_0 e^{-(p\in+q\in'+\aleph_0)}$ $\dot{q_{02}} = p \in e^{-(p\in+q\in'+\aleph_0)}$ $\dot{q_{03}} = q \in' e^{-(p\epsilon+q\in'+\aleph_0)}$	$\dot{p_{01}} = \frac{\aleph_0}{p \in +q \in' + \aleph_0}$ $\dot{p_{02}} = \frac{p \in}{p \in +q \in' + \aleph_0}$ $\dot{p_{03}} = \frac{q \in'}{p \in +q \in' + \aleph_0}$
$\dot{q_{10}} = e^{-(p\in+q\in'+\aleph_0)}\psi(t)$ $\dot{q_{14}} = p\epsilon e^{-(p\in+q\epsilon'+\aleph_0)}\overline{\Psi(t)}$ $\dot{q_{15}} = q\epsilon' e^{-(p\epsilon+q\epsilon'+\aleph_0)}\overline{\Psi(t)}$ $\dot{q_{19}} = \aleph_0 e^{-(p\epsilon+q\epsilon'+\aleph_0)}\overline{\Psi(t)}$	$\dot{p_{10}} = \psi^*(p \in +q \in' + \aleph_0)$ $\dot{p_{14}} = \frac{p\epsilon}{p\epsilon + q\epsilon' + \aleph_0}[1 - \psi^*(p\epsilon + q\epsilon' + \aleph_0)]$ $\dot{p_{15}} = \frac{q\epsilon'}{p\epsilon + q\epsilon' + \aleph_0}[1 - \psi^*(p\epsilon + q\epsilon' + \aleph_0)]$ $\dot{p_{19}} = \frac{\aleph_0}{p\epsilon + q\epsilon' + \aleph_0}[1 - \psi^*(p\epsilon + q\epsilon' + \aleph_0)]$
$\dot{q_{20}} = e^{-(p\epsilon+q\epsilon'+\aleph_0)}\phi(t)$ $\dot{q_{26}} = p\epsilon e^{-(p\epsilon+q\epsilon'+\aleph_0)}\overline{\Phi(t)}$ $\dot{q_{27}} = q\epsilon' e^{-(p\epsilon+q\epsilon'+\aleph_0)}\overline{\Phi(t)}$ $\dot{q_{28}} = \aleph_0 e^{-(p\epsilon+q\epsilon'+\aleph_0)}\overline{\Phi(t)}$	$\dot{p_{20}} = \phi^*(\dot{a}\epsilon + \dot{b}\epsilon' + \aleph_0)$ $\dot{p_{26}} = \frac{p\epsilon}{p\epsilon + q\epsilon' + \aleph_0}[1 - \phi^*(p\epsilon + q\epsilon' + \aleph_0)]$ $\dot{p_{27}} = \frac{p\epsilon'}{p\epsilon + q\epsilon' + \aleph_0}[1 - \phi^*(p\epsilon + q\epsilon' + \aleph_0)]$ $\dot{p_{28}} = \frac{\aleph_0}{p\epsilon + q\epsilon' + \aleph_0}[1 - \phi^*(p\epsilon + q\epsilon' + \aleph_0)]$
$\dot{q_{30}} = \omega(t)$	$\dot{p_{30}} = \omega^*(0)$
$\dot{q_{42}} = \dot{q_{53}} = \dot{q_{91}} = \psi(t)$	$\dot{p_{42}} = \dot{p_{53}} = \dot{p_{91}} = \psi^*(0)$
$\dot{q_{62}} = \dot{q_{73}} = \dot{q_{81}} = \phi(t)$	$\dot{p_{62}} = \dot{p_{73}} = \dot{p_{81}} = \phi^*(0)$

It is clear that Summations of all the terms of $\dot{p_{ij}}$ in each box is equal to 1.

4. Mean Sojourn Times

The mean sojourn times (μ_i) in the state S_i are

$$\delta_0 = \int_0^\infty P(T > t)dt = \sum_{\substack{i=0 \\ j=1,2,3}} m_{ij} = \frac{1}{p \in +q \in' + \aleph_0}$$

$$\delta_1 = \sum_{\substack{i=1 \\ j=0,4,5,9}} m_{ij} = \frac{1}{p \in +q \in' + \aleph_0 + \gamma},$$

$$\delta_2 = \sum_{\substack{i=2 \\ j=0,6,7,8}} m_{ij} = \frac{1}{p \in +q \in' + \aleph_0 + \alpha}$$

$$\delta_3 = \sum_{\substack{i=3 \\ j=0}} m_{ij}$$

$$\delta_1' = \sum_{\substack{i=1 \\ j.k=0,1.9,2.4,3.5}} m_{ij.k} = \frac{p \in +q \in' + \aleph_0 + \gamma^2}{\gamma^2(p \in +q \in' + \aleph_0 + \gamma)}$$

$$\delta_2' = \sum_{\substack{i=2 \\ j.k=0,1.8.,2.6,3.7}} m_{ij.k} = \frac{p \in +q \in' + \aleph_0 + \alpha^2}{\alpha^2(p \in +q \in' + \aleph_0 + \alpha)}$$

5. MTSF (Mean Time to System Failure)

The regenerative un-failed states to which the system can transit before entering any failed state are i=0,1,2 ;(k_1,k_2=Nil i=0) the mean time to system failure (MTSF) is given by

$$MTSF = \frac{(\dot{0}-\dot{0})\delta_0+(\dot{0}-\dot{1})\delta_1+(\dot{0}-\dot{2})\delta_2}{1-(\dot{0},\dot{1},\dot{0})-(\dot{0},\dot{2},\dot{0})} = \frac{\delta_0+p\dot{_{01}}\delta_1+p\dot{_{02}}\delta_2}{1-(\dot{0},\dot{1},\dot{0})-(\dot{0},\dot{2},\dot{0})}$$

$$MTSF = \frac{Num.}{Den.}; Num. = \frac{1}{p \in +q \in' + \aleph_0} + \frac{1}{\beta} + \frac{p \in +q \in'}{(p \in +q \in' + \aleph_0 + \gamma)(p \in +q \in' + \aleph_0)}$$

$$\text{Den.} = 1 - \frac{\aleph_0 \gamma}{(p\in+q\in'+\aleph_0+\alpha)(p\in+q\in'+\aleph_0)} - \frac{\alpha p \in}{(p\in+q\in'+\aleph_0+\alpha)(p\in+q\in'+\aleph_0)} n$$

$$c_{y1} = (\dot{1},\dot{9},\dot{1}); c_{y2} = (\dot{1},\dot{4},\dot{2},\dot{8},\dot{1}); c_{y3} = (\dot{2},\dot{6},\dot{2}); c_{y4} = (\dot{2},\dot{8},\dot{1},\dot{4},\dot{2})$$

$$\mathrm{k}_1 = (\dot{0},\dot{0}); k_2 = (\dot{0},\dot{1}); k_3 = (\dot{0},\dot{2},\dot{8},\dot{1}); k_4 = (\dot{0},\dot{2}); k_5 = (\dot{0},\dot{1},\dot{4},\dot{2}); k_6 = (\dot{0},\dot{3})$$

$$\mathrm{k}_7 = (\dot{0},\dot{1},\dot{5},\dot{3}); \mathrm{k}_8 = (\dot{0},\dot{1},\dot{4},\dot{2},\dot{7},\dot{3}); \mathrm{k}_9 = (\dot{0},\dot{2},\dot{7},\dot{3}); \mathrm{k}_{10} = (\dot{0},\dot{2},\dot{8},\dot{1},\dot{5},\dot{3})$$

Table 1: MTSF V/s. hardware failure Rate (ϵ)

ϵ ↓	ℵ0=0.001,α=2,ϒ=0.03,p=0.6,q=0.4,,β=5,ϵ'=0.001	α=3	ℵ0=0.002	ϒ=0.4	β=7	p=0.4,q=0.6	ϵ'=0.003
0.01	2255.182106	2290.713	2104.956	2283.635	2255.1821	1596.946	1950.42
0.02	1930.939247	2030.106	1753.28	1968.418	1930.9392	1505.042	1591.204
0.03	1597.137197	1750.708	1431.632	1633.288	1597.1372	1393.05	1288.401
0.04	1302.794076	1487.754	1164.144	1333.27	1302.7941	1271.817	1046.724
0.05	1061.886699	1257.502	951.1549	1086.044	1061.8867	1150.02	857.9089
0.06	871.0781006	1063.613	784.2679	889.744	871.0781	1033.584	710.9846
0.07	721.5505824	903.4885	653.731	735.8749	721.55058	925.9378	596.1086
0.08	604.2625	772.2651	551.0557	615.2849	604.2625	828.6177	505.4775
0.09	511.6359266	664.8128	469.5589	520.1812	511.63593	741.9065	433.1986
0.1	437.7854897	576.5441	404.1873	444.4747	437.78549	665.3449	374.9015

6. Availability (Steady state)

The regenerative state at which system is available are i=0,1,2 and j=0,1,2,3.

$$A_0 = N \div D$$

$$\mathrm{N} = \mathrm{k}_1 \mathrm{f}^z{}_0 \delta_0 + \left[\frac{k_2}{1 - c_{y1} - \frac{c_{y2}}{1-c_{y3}}} + \frac{k_3}{\left\{1 - c_{y1} - \frac{c_{y2}}{1-c_{y3}}\right\}\{1 - c_{y3}\}}\right] \mathrm{f}^z{}_1 \delta_1 + \left[\frac{k_4}{1 - c_{y3} - \frac{c_{y4}}{1-c_{y1}}} + \frac{k_5}{\left\{1 - c_{y1} - \frac{c_{y2}}{1-c_{y3}}\right\}\{1 - c_{y3}\}}\right] \mathrm{f}^z{}_2 \delta_2$$

$$\mathrm{N} = \delta_0\left[(1 - c_{y1})(1 - c_{y2}) - c_{y2} + k_2 \delta_1\{(1 - c_{y2})(k_3)\} + \{k_4 \delta_2\}\{(1 - c_{y1}) + k_5\}\right.$$

$$\mathrm{D} = \mathrm{k}_1 \delta_0 + \left[\frac{k_2}{1 - c_{y1} - \frac{c_{y2}}{1-c_{y3}}} + \frac{k_3}{\left\{1 - c_{y1} - \frac{c_{y2}}{1-c_{y3}}\right\}\{1 - c_{y3}\}}\right] \delta_1' + \left[\frac{k_4}{1 - c_{y3} - \frac{c_{y4}}{1-c_{y1}}} + \frac{k_5}{\left\{1 - c_{y1} - \frac{c_{y2}}{1-c_{y3}}\right\}\{1 - c_{y3}\}}\right] \delta_2'$$

$$+ \left[k_6 + \frac{k_7}{1 - c_{y1} - \frac{c_{y2}}{1-c_{y3}}} + \frac{k_8}{\left\{1 - c_{y1} - \frac{c_{y2}}{1-c_{y3}}\right\}\{1 - c_{y3}\}} + \frac{k_9}{1 - c_{y3} - \frac{c_{y4}}{1-c_{y1}}} + \frac{k_{10}}{\left\{1 - c_{y1} - \frac{c_{y2}}{1-c_{y3}}\right\}\{1 - c_{y3}\}}\right] \delta_3$$

$$\mathrm{D} = \delta_0\left[(1 - c_{y1})(1 - c_{y2}) - c_{y2} + k_2 \delta_1'\{(1 - c_{y2}) + k_3)\} + \left\{\{k_4 \delta_2'\}\{(1 - c_{y1}) + k_5\right\}\right.$$

$$+\{k_6(1 - c_{y1})(1 - c_{y2}) - c_{y2} + \{k_7(1 - c_{y2}) + k_8 + k_9(1 - c_{y1}) + k_{10}\}\delta_3'\}$$

$$N = \frac{(p \in +q \in' + \gamma)(p \in +q \in' + \aleph_0 + \gamma)(q \in' + \aleph_0 + \gamma) - p \in \aleph_0(p \in +q \in' + \aleph_0 + \gamma) + \aleph_0(q \in' + \aleph_0 + \gamma)(p \in +q \in' + \aleph_0 + \alpha) + p \in \aleph_0(p \in +q \in' + \aleph_0 + \gamma) + (p \in +q \in' + \aleph_0 + \alpha)(p \in \aleph_0 + p \in (p \in +q \in' + \gamma))}{(p \in +q \in' + \aleph_0 + \gamma)(p \in +q \in' + \aleph_0)(p \in +q \in' + \aleph_0 + \beta)(p \in +q \in' + \aleph_0 + \alpha)}$$

And D

$$(\beta(\alpha^2\gamma^2(p\in+q\in'+\gamma)(p\in+q\in'+\aleph_0+\gamma)(q\in'+\aleph_0+\gamma)-p\in\aleph_0(p\in+q\in'+\aleph_0+\gamma)+\alpha^2(p\in+q\in'+\aleph_0+\gamma)(p\in+q\in'+\aleph_0+\gamma^2)$$

$$\aleph_0(q\in'+\aleph_0+\alpha)(p\in+q\in'+\aleph_0+\alpha)+p\in\aleph_0(p\in+q\in'+\aleph_0+\gamma)+\gamma^2(p\in+q\in'+\aleph_0+\alpha^2)(p\in+q\in'+\aleph_0+\gamma)$$

$$p\in\aleph_0+p\in(p\in+q\in'+\gamma)+(\aleph_0(q\in'(p\in+q\in'+\aleph_0+\alpha)(p\in(q\in'(p\in+q\in'+\aleph_0+\gamma))+$$

$$q\in'(p\in+q\in'+\gamma)((q\in'+\aleph_0+\gamma)-p\in\aleph_0)\alpha^2\gamma^2(p\in+q\in'+\aleph_0+\gamma)$$

$$=\overline{\beta\alpha^2\gamma^2(p\in+q\in'+\aleph_0+\gamma)(p\in+q\in'+\aleph_0)(p\in+q\in'+\aleph_0+\beta)(p\in+q\in'+\aleph_0+\alpha)}$$

Table 2: Availability v/s. hardware failure Rate (ϵ)

ϵ ↓	ℵ0=0.001,α=2,ϒ=0.03,p=0.6,q=0.4,,β=5,ϵ'=0.001	α=3	ℵ0=0.002	ϒ=0.4	β=7	p=0.4,q=0.6	ϵ'=0.003
0.01	0.999656825	0.999658	0.999327764	0.99981	0.9996796	0.99968	0.99893
0.02	0.999441357	0.999451	0.998914671	0.99971	0.9994641	0.99954	0.99832
0.03	0.999226075	0.99925	0.998510166	0.9996	0.9992488	0.99939	0.99773
0.04	0.999009165	0.999053	0.998111138	0.99949	0.9990319	0.99925	0.99715
0.05	0.998790324	0.998861	0.997716902	0.99938	0.9988131	0.99911	0.99659
0.06	0.998569281	0.998672	0.997326828	0.99925	0.998592	0.99896	0.99603
0.07	0.998345784	0.998487	0.996940337	0.99912	0.9983685	0.99882	0.99548
0.08	0.998119608	0.998306	0.996556893	0.99899	0.9981423	0.99867	0.99494
0.09	0.997890545	0.998127	0.996176002	0.99884	0.9979133	0.99852	0.99441
0.1	0.997658406	0.997951	0.995797208	0.99869	0.9976811	0.99837	0.99389

7. Busy Period (due to inspection /PM/repair/replacement)
The regenerative state where the server is busy while doing maintenance/inspection/repair are i=1, 2, 3

$$B_0=N^1\div D$$

$$N^1=k_1\eta_0+\left[\frac{k_2}{1-c_{y1}-\frac{c_{y2}}{1-c_{y3}}}+\frac{k_3}{\left\{1-c_{y1}-\frac{c_{y2}}{1-c_{y3}}\right\}\{1-c_{y3}\}}\right]\eta_1+\left[\frac{k_4}{1-c_{y3}-\frac{c_{y4}}{1-c_{y1}}}+\frac{k_5}{\left\{1-c_{y1}-\frac{c_{y2}}{1-c_{y3}}\right\}\{1-c_{y3}\}}\right]\eta_2$$

$$N^1=\eta_0[(1-c_{y1})(1-c_{y2})-c_{y2}\}]+\eta_1\{k_2(1-c_{y2})+(k_3)\}+\{\{k_4\eta_2\}(1-c_{y1})+k_5\eta_2\}$$

$$N^1=\frac{\begin{array}{c}(p\in)(p\in+q\in'+\aleph_0+\gamma)+\aleph_0(p\in+q\in'+\aleph_0+\alpha)*\\ p\in\aleph_0(p\in+q\in'+\aleph_0+\alpha)q\in'(p\in+q\in'+\aleph_0+\gamma)+q\in'(p\in+q\in'+\aleph_0+\gamma)((q\in'+\aleph_0+\gamma))-p\in\aleph_0\end{array}}{\beta(p\in+q\in'+\aleph_0+\gamma)(p\in+q\in'+\aleph_0)(p\in+q\in'+\aleph_0+\alpha)}$$

D is already defined.

8. **Expected Number of Visits of the Server**
The regenerative state where the server visits (afresh) for the maintenance/inspection/repairs are i=1,2,3

$$V_0=N^2\div D$$

$$N^2=k_1+\left[\frac{k_2}{1-c_{y1}-\frac{c_{y2}}{1-c_{y3}}}+\frac{k_3}{\left\{1-c_{y1}-\frac{c_{y2}}{1-c_{y3}}\right\}\{1-c_{y3}\}}\right]+\left[\frac{k_4}{1-c_{y3}-\frac{c_{y4}}{1-c_{y1}}}+\frac{k_5}{\left\{1-c_{y1}-\frac{c_{y2}}{1-c_{y3}}\right\}\{1-c_{y3}\}}\right]$$

$$N^2=[(1-c_{y1})(1-c_{y2})-c_{y2}+k_2\{(1-c_{y2})+(k_3)\}+\{k_4\}\{(1-c_{y1})+k_5\}$$

$$N^2 = \frac{(p \in +q \in' + \aleph_0 + \alpha)(p \in +q \in' + \aleph_0 + \gamma)(p \in +\aleph_0) + \aleph_0 (q \in')(p \in +q \in' + \aleph_0 + \alpha) + (p \in +q \in' + \aleph_0 + \alpha) q \in' \left((q \in' + \aleph_0 + \gamma)\right) - p \in \aleph_0}{(p \in +q \in' + \aleph_0 + \gamma)(p \in +q \in' + \aleph_0)(p \in +q \in' + \aleph_0 + \alpha)}$$

D is already defined.

9. Profit Analysis

Profit of Plc model is obtained as:

$$P_0 = K_0 A_0 - K_1 B_0^r - K_2 V_0$$

Where,
K_0=Revenue per unit up-time of the system.
K_1=Cost per unit time for which server is busy due to maintenance/Up-gradation(s/w)/repair(h/w).
K_2= Cost per unit time visit of the serviceman.

Table 3: Profit V/s. hardware failure Rate (ϵ)

ϵ ↓	ℵ0=0.001,α=2,ϒ=0.03,p=0.6,q =0.4,,β=5,ϵ'=0.001	α=3	ℵ0=0.002	ϒ=0.4	β=7	p=0.4,q=0.6	ϵ'=0.003
0.01	23417.2455	23591.87	23382.60069	23421	23418.74	23458.55043	23351.86
0.02	23402.94637	23587.69	23375.17064	23409.62	23404.03	23428.99514	23347.64
0.03	23395.2566	23583.52	23367.28811	23404.71	23396.18	23415.42845	23338.68
0.04	23389.44289	23579.44	23359.32306	23401.55	23390.28	23406.92714	23328.25
0.05	23384.393	23575.44	23351.37456	23399.04	23385.18	23400.66395	23317.28
0.06	23379.70435	23571.53	23343.47049	23396.79	23380.46	23395.57728	23306.13
0.07	23375.19385	23567.69	23335.61643	23394.61	23375.92	23391.17825	23294.93
0.08	23370.76492	23563.94	23327.80958	23392.42	23371.47	23387.2099	23283.78
0.09	23366.36092	23560.25	23320.04369	23390.16	23367.05	23383.52388	23272.69
0.1	23361.94579	23556.63	23312.31115	23387.8	23362.62	23380.02826	23261.68

10. Conclusion

All reliability metrics of the PLC system such as Mean Time to System Failure (MTSF), availability, and profit are derived from the probability density functions $\phi(t) is\ \alpha e^{-\alpha t}(t); \omega(t)\ is\ \beta e^{-\beta t}; \psi(t)\ is\ \gamma e^{-\gamma t}$ are shown numerically. These results, presented numerically in the tables 1.1 to 1.3, show an inverse relationship with the hardware repair rate, the software up gradation rate, and the rate of preventive maintenance. Specifically, all these reliability measures decrease as the failure rates of the hardware or the software up gradation process increase, or as the frequency of hardware preventive maintenance (after a predetermined operation time ‘t’) rises. However, the system becomes more profitable and has higher availability, and shows a lesser MTSF when other parameters remain constant and the likelihood of hardware failure is replaced with that of software failure. Notably, the PLC system exhibits reduced MTSF, availability, and profit when the probability of software failure is higher. Therefore, it can be concluded that implementing preventive maintenance for hardware in cold standby within a PLC system featuring component-level redundancy may not be a beneficial strategy.

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