International Journal of Statistics and Applied Mathematics 2026; 11(1): 125-127

International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452
NAAS Rating (2025): 4.49
Maths 2026; 11(1): 125-127

www.mathsjournal.com
Received: 12-12-2025
Accepted: 09-01-2026

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A numerical method for solving second-kind linear Fredholm integral equations

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DOI: https://www.doi.org/10.22271/maths.2026.v11.i1b.2247

Abstract
This study presents a numerical technique for solving second-kind Fredholm integral equations via the collocation method. The modeled problem is transformed into an algebraic equation system and solved with standard collocation points. After determining the approach's uniqueness and convergence, numerical examples were used to assess its efficacy. The results indicate that the method outperforms others.

Keywords: Fredholm, integral equations, standard collocation points, approximate solution.

1. Introduction
Integral equations have sparked widespread interest in a variety of applications, including biological, physical, and engineering concerns. Several works have looked into numerical approaches for solving Fredholm integro-differential equations. Many engineering and mechanics issues can be solved using second-order Fredholm integral equations in two dimensions. In plasma physics calculations, for example, Fredholm integral equations are typically solved Mirzaee and Hadadiyan [1].

Many efforts have been conducted to create and analyze numerical approaches for solving Fredholm integral equations of the second kind, including: Adomian decompositions method by Khan and Bakodah [2], divided differences interpolation by Parandin and Gholamtabor [3], Bernstein method by Adhraa and Ayal [4], Collocation method by Ajileye *et al.* [5], Agbolade and Anake [6], Hybrid linear multistep method by Mehdiyera *et al.* [7], Chebyshev-Galerkin method by Issa and Saleh [8], Lagrange Interpolation by Shoukralla and Ahmed [9], Least-Squares Method by Al-Humedi and Shoushan [10], Chebyshev polynomials by Maadadi & Rahmoune [11], Optimal Auxiliary Function Method (OAFM) by Zada *et al.* [12], Modified Simpson's Rule by Djaidja and Khirani [13] and many other methods.

This paper introduces a new method for obtaining an approximate numerical solution to the second kind of linear Fredholm integral equation of the form

$$y(x) + \alpha \int_a^b G(x,t)\, y(t)dt = F(x), a \le x \le b \qquad (1)$$

Where $G(x,t)$ represents the Fredholm integral kernel, α is the supplied parameter, $F(x)$ is a known function, and $y(x)$ is the unknown function to be calculated.

2. Fundamental Terms and Definitions
In order to formulate the given problem, we provide certain definitions and basic ideas of integral in this part.

2.1 Definition 1: (Ajileye *et al.*, 2024) Let (a_m) be a sequence of real integers and $m < 0$. The power series in t with coefficients a_m is an equation.

$$u(x) = \sum_{m=0}^{N} a_m t^m = \theta(t)B \qquad (2)$$

Where $\theta(t) = [1 t t^2 \cdots t^N], B[a_0 a_1 \cdots a_N]^T$

Then $u(t,m) = t^m B, i = 0(1)N, m \in Z^+$

2.2 Definition 2: (Agbolade and Anake, 2017) This method identifies the essential collocation sites between intervals. For example, $[c, d]$ is defined as

$$x_i = c + \frac{(d-c)i}{M}, i = 0, 1, 2 \dots \dots M \quad (3)$$

3. Materials and Methods

In this section, we implement approximation approach for the numerical solution of Fredholm integral equations.

3.1 Method of Solution

Let the solution of equation (1) be approximated by

$$y(x) = \sum_{n=0}^{M} a_n x^n \quad (4)$$

Substituting equation (4) into equation (1) gives

$$\sum_{n=0}^{M} a_n x^n + \alpha \int_a^b G(x,t) \left(\sum_{n=0}^{M} a_n t^n\right) dt = F(x) \quad (5)$$

$$\sum_{n=0}^{M} a_n \left(x^n + \alpha \int_a^b G(x,t)(t^n) dt\right) = F(x) \quad (6)$$

Equation (6) can be rewrite in the form

$$\sum_{n=0}^{M} a_n \beta(x) = F(x) \quad (7)$$

Where

$$\beta(x) = x^n + \alpha \int_a^b G(x,t)(t^n) dt$$

Using the standard collocation points, collocate equation (6) at x_i.

$$x_i = a + \frac{(b-a)i}{M}, i = 0, 1, 2 \dots \dots M$$

$$\sum_{n=0}^{M} a_n \beta(x_i) = F(x_i) \quad (8)$$

Where,

$$\beta(x_i) = \begin{bmatrix} \beta_0(x_0) & \beta_1(x_0) & \beta_2(x_0) & \cdots & \beta_N(x_0) \\ \beta_0(x_1) & \beta_1(x_1) & \beta_2(x_1) & \cdots & \beta_N(x_1) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \beta_0(x_N) & \beta_1(x_N) & \beta_2(x_N) & \cdots & \beta_N(x_N) \end{bmatrix}, F(x_i) = \begin{bmatrix} F(x_0) \\ F(x_1) \\ \vdots \\ F(x_N) \end{bmatrix}$$

We solve the system of equations (8) for the unknown values and input the results into the approximate solution to get the numerical result.

4. Numerical Examples

To assess the accuracy and usefulness of the procedure, we provide numerical examples in this section. Let the approximate and exact solutions be denoted by $y_n(x)$ and $y(x)$ respectively. $Error_N = |y_n(x) - y(x)|$

Example 1: Consider the Fredholm integral equation

$$y(x) = \int_0^1 xt\, y(t) dt + e^{3x} + \frac{(2e^3+1)x}{9} \quad (9)$$

Exact solution: $y(x) = e^{3x}$

Solution 1

The approximate solution to equation (9) at $N = 5$ yields

$y_5 = 1.00000000000063 + 3.15243302340969x + 2.82511637499556x^2 + 10.9927934007719x^3 - 7.66395900305361x^4 + 9.78006358630955x^5$

Table 1: Exact, approximate and absolute error values for example 1

X	Exact	Our Method N=5	Error5	Error Parandin *et al.*=5
0.2	1.822118800	1.822300893	1.82093e-4	3.08e-3
0.4	3.320116923	3.320481107	3.64184e-4	7.5e-3
0.6	6.049647464	6.050193740	5.46276e-4	1.13e-2
0.8	11.02317638	11.02390475	7.2837e-4	1.51e-2
1.0	20.08553692	20.08644739	9.1047e-4	1.88e-2

The result obtained for Example 1, as shown in Table 1 revealed that our method performed better than the method proposed in the literature at the same value of N.

Example 2: Consider the Fredholm integral equation

$$y(x) = e^{2x+\frac{1}{3}} - \frac{1}{3}\int_0^1 e^{2x+\frac{5t}{3}} y(t) dt \quad (10)$$

Exact solution: $y(x) = e^{2x}$

Solution 2

The approximate solution to equation (10) at $N = 8$ yields

$y_8 = 1.00000000281580 + 1.99999058246613x + 2.00019645690918x^2 + 1.33168411254883x^3 + 0.674118041992188x^4 + 0.247802734375000x^5 + 0.117797851562500x^6 - 0.183105468750000e - 3x^7 + 0.177345275878906e - 1x^8$

Table 2: Exact, approximate and absolute error values for example 2

x	Exact	Our Method, N=8	Error8	Error Parandin *et al.*=10
0.2	1.221402758000	1.221402752000	3.40e-08	9.149e-08
0.4	1.491824698000	1.491824780000	8.20e-08	6.031e-06
0.6	1.822118800000	1.822118933000	1.33e-07	7.08e-05
0.8	2.225540928000	2.225541113000	1.85e-07	4.10e-04
1.0	2.718281828000	2.718282092000	2.67e-07	1.61e-03

In numerical Example 2, as shown in Table 2, the approximate solution at N=8 yields a better result compared to the result obtained in the literature at N=10.

Example 3: Consider the Fredholm integral equation

$$y(x) = 2\mathrm{Sin}\left(\frac{x}{2}\right) + \int_0^{2\pi} \mathrm{Sin}(x)\mathrm{Sin}\left(\frac{\pi}{2}\right) y(t) dt \quad (11)$$

Exact solution: $y(x) = 2\mathrm{Sin}\left(\frac{x}{2}\right)$

Solution 3

The approximate solution to equation (11) at N=5 yields

$y_5 = 1.421085472 \times 10^{14} + 0.999999801217200x + 0.390448258258402e - 5x^2 - 0.416818645899184e - 1x^3 + 0.257630599662662e - 4x^4 + 0.503619870869443e - 3x^5$

Table 3: Exact, approximate and absolute error values for example 3

x	Exact	Our Method, $_{N=5}$	Error$_5$	Error $_{\text{Parandin } et\ al.=10}$
0.2	0.1996668333	0.1996668639	3.06e-08	9.04e-06
0.4	0.3973386616	0.3973387225	6.09e-08	1.349e-05
0.6	0.5910404134	0.5910405039	9.05e-08	2.012e-05
0.8	0.7788366846	0.7788368039	1.193e-07	3.002e-05
1.0	0.9588510772	0.9588512244	1.472e-07	4.478e-05

The approximate result in Example 2, as shown in Table 3, confirmed the reliability of our method of solution by giving a better result compared to the result obtained in the literature at the value of N=10.

5. Conclusions

This paper investigated the collocation approach for numerically solving Fredholm integral equations. This method is dependable, efficient, and easy to compute. All computations in this paper were performed using Maple 18. Considering some problems demonstrates the method's reliability.

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