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Solving quadratics with compass: The geometry of Algebra

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Abstract
Algebra and geometry are often taught as distinct branches of mathematics, leading students to perceive symbolic manipulation and visual reasoning as unrelated skills. This study presents an innovative geometric approach to solving quadratic equations of the form $ax^2 + bx + c = 0$ $(a \neq 0)$ using only a compass, straightedge, and Cartesian plane construction. The background of the study is rooted in the need to enhance conceptual understanding and student engagement in secondary-level algebra by bridging abstract algebraic formulas with intuitive geometric representations.
The primary objective of the study is to demonstrate a generalized construction method that visualizes the roots of any quadratic equation through circle geometry, thereby linking the coefficients of the equation to spatial coordinates. Methodologically, the equation is first normalized into monic form, and two fixed points A at (0, 1) and B at $(-b/a, c/a)$ are plotted on the coordinate plane. A circle is then constructed with diameter AB, and the intersections of this circle with the x-axis represent the real solutions of the quadratic equation. The results show that this method consistently yields correct roots across a wide range of cases, including equations with positive, negative, fractional, repeated, and complex roots. The nature of the discriminant is visually interpreted through the circle's interaction with the x-axis, providing immediate geometric insight into the existence and type of roots. Classroom implementation further indicates improved student motivation, conceptual clarity, and confidence.
The study concludes that the compass-based geometric construction transforms quadratic equations from a procedural task into a meaningful visual exploration. By integrating algebraic reasoning with geometric intuition, this approach supports deeper mathematical understanding and promotes the appreciation of mathematics as both a logical and artistic discipline.

Keywords: Algebra-geometry integration, standard form, unit circle, center, tangent, coefficients, roots, discriminant

Introduction
We are taught from a young age that Algebra and Geometry are two different worlds. One is the world of shifting variables and rigid formulas like the Quadratic Formula; the other is a world of elegant curves, compasses, and visual symmetry. Here is a geometric method bridged this divide with a single, elegant stroke of a compass. This method is used to find the roots of a quadratic equation of the form $ax^2 + bx + c = 0$, $(a \neq 0)$ using only a circle and its intersections with the x-axis and this approach is to visualize how the coefficients of a quadratic equation relate to its solutions. "Can you find the solutions to an equation without doing any algebra?" Most will say no. Students often view algebra and geometry as two separate "folders" in their brains. This geometric method forces them to merge these folders into one. It turns the abstract coefficients (a, b, c) into physical coordinates, making the properties of the quadratic formula tangible. This is a bridge from abstract algebra to beautiful geometry. To generalize the method for any Quadratic Equation $ax^2 + bx + c = 0$, $(a \neq 0)$ we essentially normalize the equation by dividing by a (the lead coefficient). This aligns the geometry with the standard monic form

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

In this geometry, if the roots are α, and β, then the x-coordinate of our moving point B represents the sum of the roots (α+β), and the y-coordinate represents the product of the roots (αβ).

i.e., [$\alpha+\beta = \frac{-b}{a}$ and $\alpha\beta = \frac{c}{a}$]

Materials Needed

- Graph paper (large grid)
- Compass
- Scale
- Scientific calculator (to check c/a values)
- Colored pencils

Step-by-step construction guide

- **Goal:** To find the roots of $ax^2 + bx + c = 0$ using a compass and straightedge.
- **Preparation:** Draw a large Cartesian coordinate system. Ensure the scale is consistent. (Eg: 1 cm = 1 unit).
- **Plot Point A:** Always place the Point A at (0, 1). This is Our "Pivot" point.

By fixing A at (0, 1), we ensure that the y-coordinate of B maps directly to the constant term c/a without any extra scaling factors and also by keeping A at (0,1), we created a "Unit Circle" environment where we don't have to do any extra division or multiplication to find our points.

- **Plot Point B:** Calculate $\frac{-b}{a}$ and $\frac{c}{a}$ and Plot B = $\left(\frac{-b}{a}, \frac{c}{a}\right)$
- **Find the Diameter:** Draw a straight line segment connecting A and B.
- **Locate the Center (M):**
  - **Algebraic way:** Calculate Center $M = \left(\frac{-b}{2a}, \frac{a+c}{2a}\right)$
  - **Geometric way:** Use compass to draw two arcs of equal radius from A and B. The line connecting the intersection of these arcs bisects AB at the center M.
- **Draw the Circle:** Place the compass on point M, stretch the pencil to A, and draw the full circle.
- **(vii) The Intersection:** The circle will intersect the x-axis at exactly the points, x_1 and x_2 which satisfy the quadratic equation.
- **(viii) Identify Roots:** Mark the points where the circle crosses the x-axis. These x-values are our solutions.

If the circle (i) Slices through the x-axis (b^2-4ac > 0): Two distinct real roots

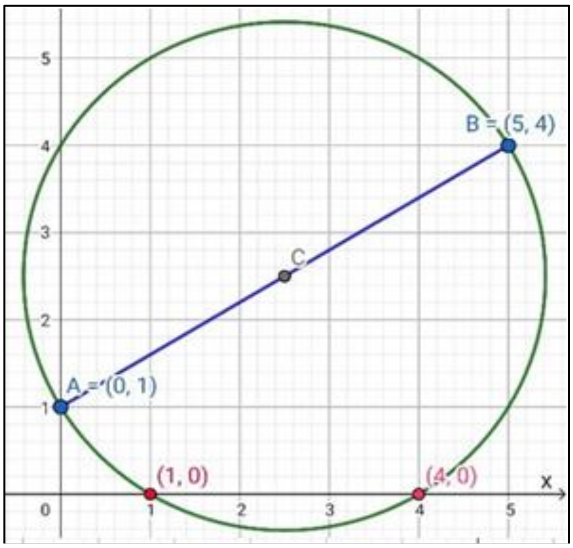
- **Kisses the x-axis (Tangent) (b^2-4ac = 0):** One repeated real root.
- **Floats above/below the axis (b^2-4ac < 0):** Two complex (imaginary) roots.
- Center is on the x-axis c/a = -1 Roots are related to the Golden Ratio or symmetry across the axis.

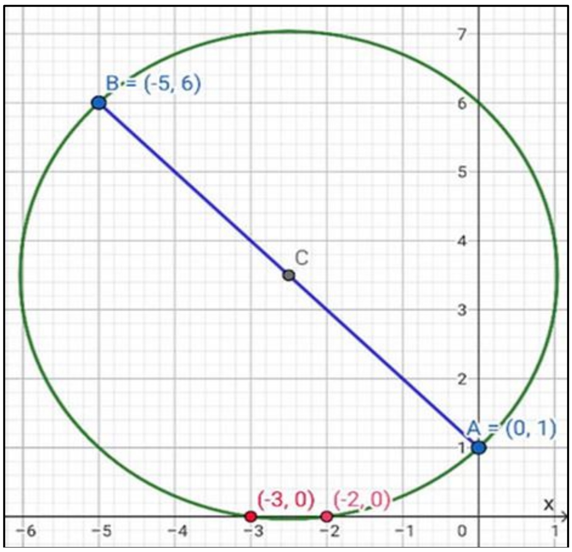
Through these four cases, the Circle demonstrates that algebra is not merely a set of rules for manipulation, but a description of geometric relationships. It serves as a bridge between the analytical and the visual, proving that every quadratic equation carries within it the blueprint of a circle.

Activity Phase	Student Action	Teacher Facilitation
Setup	Set up a grid for at least -10 to +10 on both axes.	Ensure students use a sharp pencil for accuracy.
Prediction	Calculate the Discriminant (b^2-4ac) before drawing.	Ask: "Will your circle cross, touch, or miss the axis?"
Execution	Plot A and B, find M, and swing the compass.	Check that Point A is always (0, 1), not (1,0).
Verification	Read the x-intercepts and check them in the equation.	Have students swap papers to "grade" the accuracy of the drawing.

Example 1: The Standard (Positive Integer Roots)

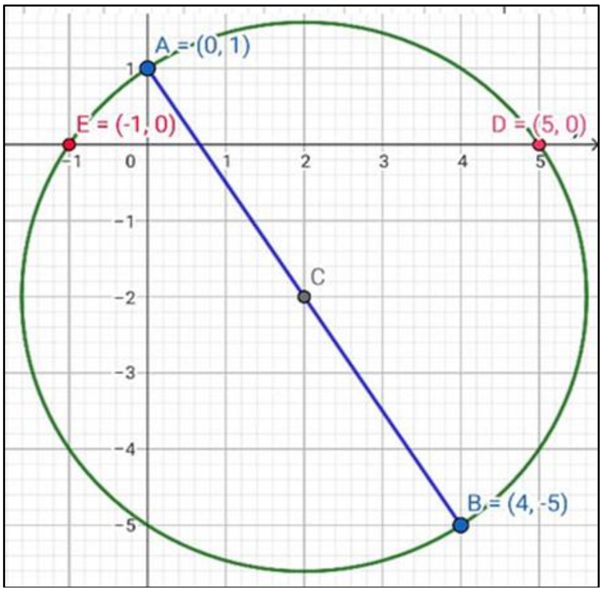
- **Equation:** x^2-5x + 4 = 0 Coefficients: a=1, b=-5, c=4 Points: A(0, 1) and B(5, 4) The Circle: Center M(2.5, 2.5).
- **The Result:** The circle will cross the x-axis at x=1 and x=4.
- **Discussion:** This is the best introductory example because the roots are easy to verify via factoring: (x-1)(x-4)=0.





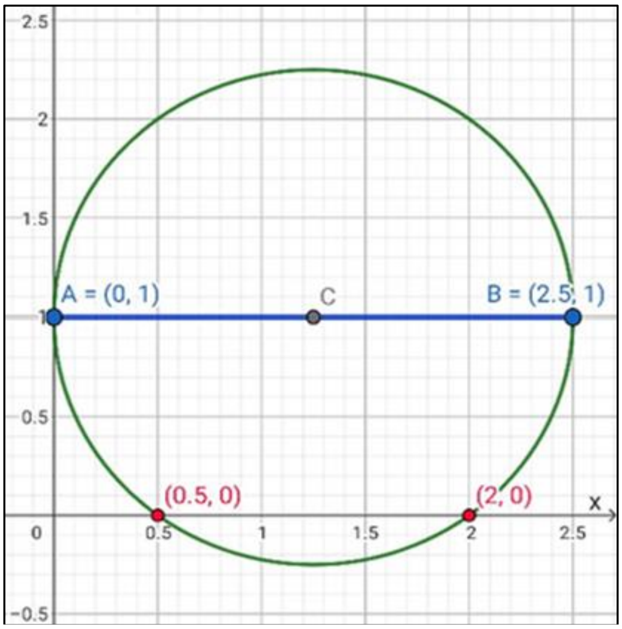
Example 2: The Negative Navigator (Negative Roots)

- **Equation:** x^2 + 5x + 6 = 0 Coefficients: a=1, b=5, c=6 Points: A(0, 1) and B(-5, 6)
- **The Circle:** The center is at (-2.5, 3.5).
- **The Result:** The circle crosses at x=-3 and x=-2.
- **Discussion:** Students often struggle with the "sign flip" in the quadratic formula. Here, they see that a positive b coefficient physically moves the circle to the left side of the y-axis.



Example 3: The Negative Shift (Opposing Roots / Sign Flip)

- **Equation:** x^2-4x-5 = 0 Coefficients: a=1, b=-4, c=-5 Points: A(0, 1) and B(4,-5)
- **The Circle:** The center is at (2,-2).
- **The Result:** The circle crosses at x=-1 and x=5.
- **Discussion:** Whenever c/a is negative, Point B drops below the x-axis, ensuring that we will always have one positive and one negative root.



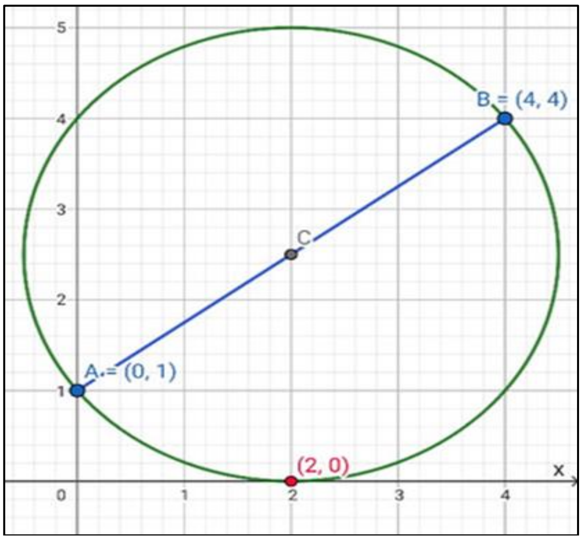
Example 4: The Fractional Force (a > 1)

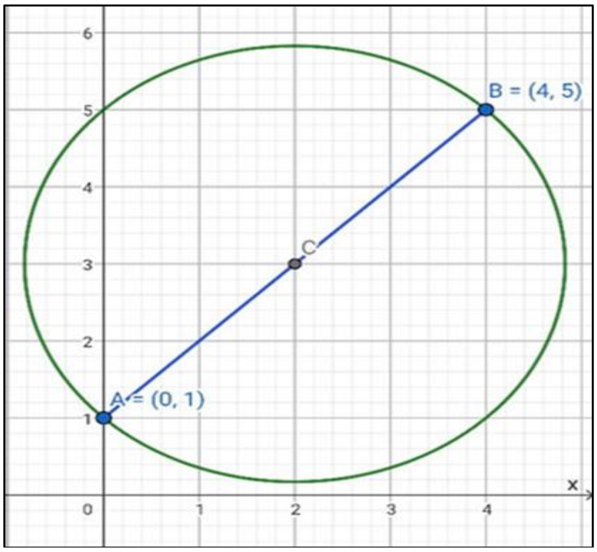
- **Equation:** $2x^2$-5x + 2 = 0 Coefficients: a=2, b=-5, c=2 **Points:** A(0, 1) and B(2.5, 1)
- **The Circle:** The center is at (1.25, 1).
- **The Result:** The circle crosses at x=0.5 and x=2.
- **Discussion:** This demonstrates that the method is universal. Even with a leading coefficient of 2, the normalized points B(-b/a, c/a) yield the correct intersections.

Example 5: The Tantalizing Tangent (Double Roots)

Equation: x^2-4x + 4 = 0 Coefficients: a=1, b=-4, c=4 Points: A(0, 1) and B(4, 4)

- **The Circle:** The center is at (2, 2.5).
- **The Result:** As the compass swings, the circle doesn't "cut" the x-axis. Instead, it drops down and barely grazes it at x = 2 before heading back up.
- **Discussion:** This is the visual proof of a Discriminant (b^2-4ac) equal to zero. There is no "inside" or "outside" the root; there is only the point of contact.





Example 6: The Floating Circle (Complex Roots)

Equation: x^2-4x + 5 = 0 Coefficients: a=1, b=-4, c=5 Points: A(0, 1) and B(4, 5)

- **The Circle:** The Center is at (2, 3).
- **The Result:** The radius of this circle is $\sqrt{(2-0)^2 + (3-1)^2}$ =
- $\sqrt{8}$ = 2.8. Since the center is at y=3 and the radius is only 2.8, the circle never reaches the x-axis.
- **Discussion:** This is a vital teaching moment. The lack of intersection visually represents the Discriminant being negative (b^2-4ac = 16-20=-4). The roots exist, but they are not "Real".

How the students enjoy this method?

In a classroom setting, the Circle usually evokes a strong positive reaction for several reasons:

- **The Magic Factor:** Students are often stunned that drawing a circle through two points (A and B) will automatically finds the answers they usually struggle to calculate.

- **Tactile Learning:** It gets students away from their screens and calculators. Using a physical compass and ruler engages fine motor skills and provides a sensory connection to the math.
- **Low Barrier to Entry:** Even a student who struggles with arithmetic can plot two points and draw a circle. It builds confidence because they can see the answer before they have to prove it with algebra.
- **Immediate Feedback:** If a student draws a circle and it hits x=2 and x=3, and their friend factors the equation to find (x-2), (x-3), the instant validation creates a "light bulb moment".

Conclusion
Here the Circle transforms the quadratic equation from a chore into a discovery. It turns the Cartesian plane into a game board where the compass is the controller of the game. If we divide the class into "Constructors" (using compasses) and "Calculators" (using the Quadratic Formula) and see who reaches the roots first. Usually, the Constructors win on simple integer problems, but the Calculators win on messy decimals, leading to a great discussion. And finally the students will enjoy the Art of Mathematics.

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